Nuclear and Particle Physics - Lecture 24
Chain reactions

1 Introduction

We have seen that fission results in nuclei with more neutrons than are needed to lie on the beta-stability curve. These are often ejected and fission commonly results in between one and four neutrons being emitted. We have also seen that neutrons can excite nuclei and so speed up the fission rate enormously. These two facts together can allow a chain reaction to occur. This has been used to produce power from nuclear fission as a lot of energy is released. Atomic bombs and nuclear power stations work using chain reactions.

2 Chain reactions

If the neutrons produced in a nuclear fission can be used to cause further fissions, then a chain reaction occurs. If the mean time before inducing a fission for a neutron is t_f, then the number of fissions in a short time δt is $n(\delta t/t_f)$ for n neutrons present. Taking the average number of neutrons from each fission which react further as m, then this number of fissions produce $mn(\delta t/t_f)$ new neutrons. Hence, the total change to the number of neutrons is

$$\delta n = mn\frac{\delta t}{t_f} - n\frac{\delta t}{t_f} = (m-1)n\frac{\delta t}{t_f}$$

Taking δt infinitessimally small, this gives

$$\frac{dn}{dt} = (m-1)\frac{n}{t_f}$$

which gives

$$\frac{dn}{n} = (m-1)\frac{dt}{t_f}$$

so integrating

$$\ln n = (m-1)\frac{t}{t_f} + k$$

for some constant of integration k, so

$$n = e^k e^{(m-1)t/t_f} = n_0 e^{(m-1)t/t_f}$$

where n_0 is clearly the number of neutrons at time $t = 0$. Hence, the number of neutrons (and hence rate of fissions) increases or decreases exponentially, depending on the value of $m - 1$. Clearly when $m < 1$, there are not enough neutrons being produced to sustain the chain reaction indefinitely; such a situation is called subcritical. Conversely, $m > 1$ means more are produced at each step and so the reaction rate increases; this is called supercritical. The dividing line, when $m = 1$, is said to be critical.

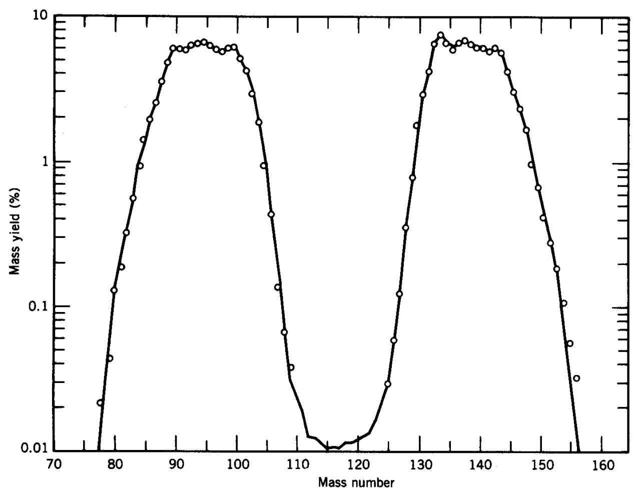
We defined m as the average number of neutrons emitted from each fission which cause further fissions. This is clearly not necessarily the number emitted from each fission. Some neutrons can react by other processes (e.g. absorption followed by gamma emission, rather than fission) or be lost from the surface of the material. One of the critical issues in sustaining a fission chain reaction is optimising the number of neutrons for the purpose required.

3 Uranium fission

The most important material for fission chain reactions is uranium. Plutonium has similar properties with regard to fission and so can also be used for a chain reaction. However, all plutonium isotopes have lifetimes of around 10^5 years as opposed to 10^9 years for uranium, so there is no natural plutonium left. To be used, it has to first be manufactured in a nuclear reaction. Hence, for practical reasons, uranium is the most commonly used material.

The main fission isotope of uranium is $^{236}_{92}$U. This is an even-even nucleus and hence benefits from the extra pairing energy associated with this. The fission barrier height for this isotope is around 6.2 MeV above the ground state. It is made by neutron absorption from $^{235}_{92}$U, which is even-odd and so is less strongly bound. This results in the binding energy difference of these two nuclei being $\Delta B_E = 6.5$ MeV, which is more than the barrier height. Hence, absorption of even a zero energy neutron by $^{235}_{92}$U is enough to give fissionable $^{236}_{92}$U.

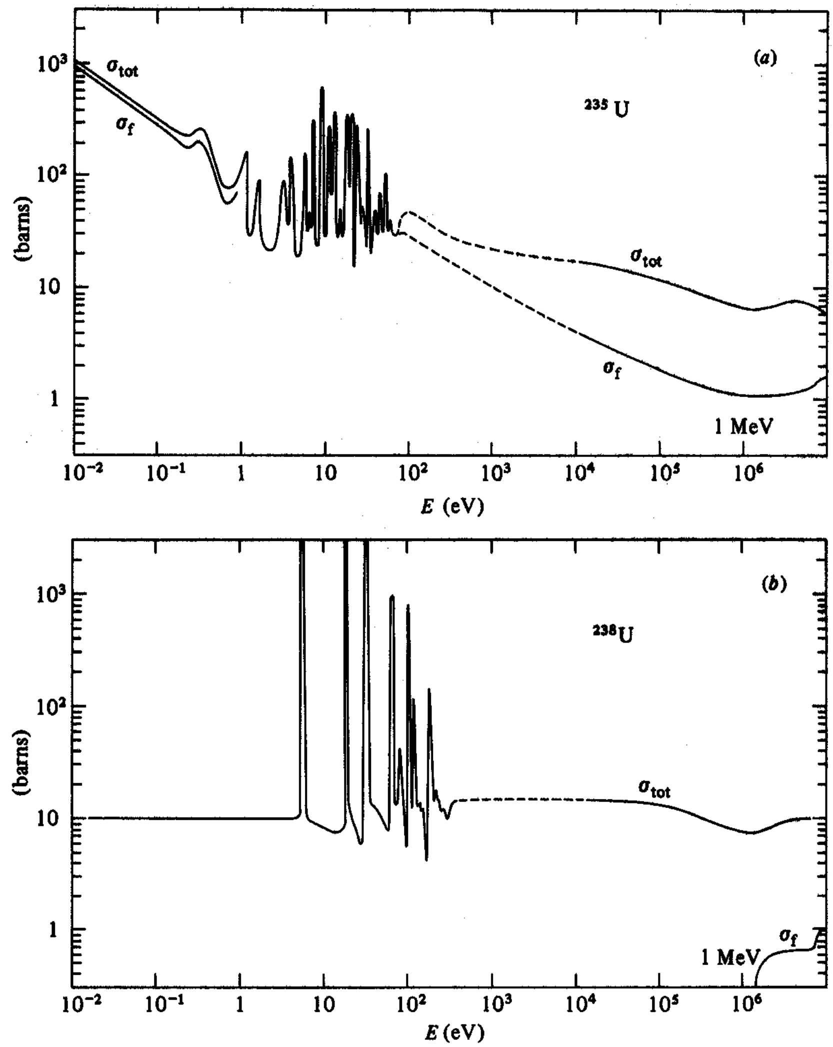
The A distribution of the two nuclei produced from this fission is shown below



Note the nuclei tend to be unequal, despite the energy release being maximum for an equal division.

Because these two nuclei are excited and neutron rich, they rapidly give off neutrons. Each $^{236}_{92}$U induced fission produces 2.5 neutrons on average and so this nucleus has the possibility of producing a supercritical chain reaction, if at least one of these neutrons on average can be made to cause a further fission. The neutrons are emitted with an average energy of around 2 MeV each. The total immediate energy release per fission is around 180 MeV although the daughter nuclei usually beta and gamma decay later, giving roughly another 20 MeV at later times.

Natural uranium is found in two isotopes, the $^{235}_{92}$U needed for the above reaction and $^{238}_{92}$U. These have naturally occuring fractions of 0.72% and 99.28%, respectively, so the required isotope is actually a very small proportion of uranium ore. Would the other, more numerous isotope be useful for fission? $^{238}_{92}$U is an even-even nucleus and so relatively strongly bound. Neutron absorption by $^{238}_{92}$U gives $^{239}_{92}$U, which is even-odd. Hence, the binding energy difference in this case is quite a bit smaller than before, i.e. by $2a_p/A^{1/2} \sim 1.5$ MeV. The actual $\Delta B_E = 4.8$ MeV, compared with a fission barrier height for $^{239}_{92}$U of 6.2 MeV, i.e. very similar to $^{236}_{92}$U, as would be expected as they have the same charge. Hence, in this case, neutrons of at least 1.4 MeV kinetic energy are needed to induce fission; lower energy ones will be absorbed and mainly produce gamma decays. The total and fission cross sections for neutrons on these two uranium isotopes are shown below.



It is seen that although the fission cross section is a reasonable proportion of the total for $^{235}_{92}$U, it is not for $^{238}_{92}$U. The latter also has several resonances with large cross sections below the fission threshold which will strongly absorb neutrons without fissioning. The main problem with getting a chain reaction from uranium is therefore that the fissionable isotope is such a small proportion of the total. Natural uranium is well below being critical, i.e. $m \ll 1$, due to the other neutron reactions in $^{238}_{92}$U.

4 The fission atomic bomb

The conceptually easiest way around this problem is to separate the uranium isotopes and use only $^{235}_{92}$U. This is the method used for atomic fission bombs, the so-called "A bomb". However, this is a very difficult thing to do in practise. By definition, the uranium isotopes have the same nuclear charge and hence extremely similar chemistry, so they cannot be separated by forming different compounds. They differ by 3 nucleons in around 240, so their masses differ only by around 1%. The main method used is a centrifuge, where a suspension of a uranium compound is spun at very high angular velocity. The centrifugal force is proportional to mass and so the heavier isotope atoms tend to migrate to the outermost parts of the container.

Assuming some approximately pure $^{235}_{92}$U can be acquired, then enough has to be assembled to go supercritical. As stated above, each fission results in around 2.5 neutrons, each of around 2 MeV. From the cross sections above, a 2 MeV neutron has an 18% chance of causing a fission, which by itself would mean $m \sim 0.45$, which is subcritical. However, the other 82% of the total cross section is mainly scattering, not absorption, where the neutron simply loses energy to the nucleus and continues. The loss of energy actually makes the neutron more likely to fission, as the fission proportion of the total cross section increases as the neutron energy drops. After several such scatters, the neutron has a high probability of causing a fission. The actual average number of scatters before fission is around six.

We can make an order of magnitude estimate of how long the neutron takes to fission and the average distance gone in this time quite simply. The mean free path is generally given by

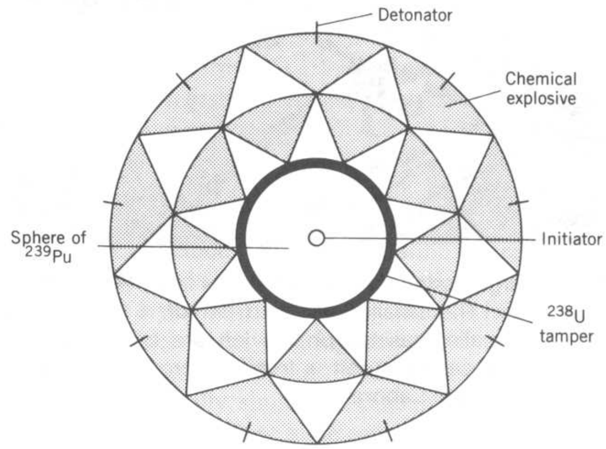
$$\langle d \rangle = \frac{1}{\rho\sigma}$$

where ρ is the number density of targets and σ is the cross section. For a neutron in $^{235}_{92}$U, it usually scatters down through energies from a few MeV to a few keV before being captured and causing fission. Take the average kinetic energy order of magnitude to be around 0.1 MeV in this range, which gives a cross section for scattering of around 10 barns. The number density of uranium is 5×10^{28} m^{-3}, so the mean free path is $\langle d \rangle \sim 2$ cm. As these neutrons are not relativistic, the kinetic energy is approximately $m_n v^2/2 = m_n c^2 \beta^2/2$ and with $m_n \sim 1000$ MeV/c^2, then $\beta \sim 0.01$. Hence, the velocity is $\sim 3 \times 10^6$ ms^{-1} and so the time between scatters is around 10^{-8} s. The average of six scatters will take of order $t_f \sim 10^{-7}$ s and with a random walk the neutron will go of order 10 cm.

Hence, the fraction of neutrons which actually cause fission is pretty high and the main factor limiting m in pure $^{235}_{92}$U is the loss of neutrons from the surface. Clearly, the bigger the volume, the smaller this effect. Hence, there is a critical sphere radius at which the loss is such as to make $m = 1$; this is clearly around the size of the distance a neutron goes and is in fact 8.7 cm. The corresponding quantity of uranium is known by (the famous phrase) the critical mass and is 52 kg. A mass larger than this will have a smaller surface loss, so that $m > 1$ and it will be supercritical. The fission rate will increase exponentially with a time constant of $t_f \sim 10^{-7}$ s and, as each fission releases around 200 MeV, there will be an enormous energy release in an extremely short time. The first atomic bombs had energy yields of around 10^{14} J; even then, only around 10% of the uranium actually fissioned before the rest was blown apart by the explosion.

Ignoring the difficulties of getting pure $^{235}_{92}$U, the main issue to overcome in an atomic bomb is how to assemble the critical mass at the required time. If not done all at once, the subcritical masses can still produce enough heat to explode or melt any mechanical structure and so prevent the completion of the critical mass. Several pieces of the uranium need to be stored separately and brought together very rapidly to get an efficient bomb. One way this is done is by using conventional explosives to implode the uranium pieces into a sphere at the centre of the bomb. The ingoing momentum is then sufficient to allow supercriticality for a long enough period before the heat generated blows the mass apart. A neutron-emitting initiator is usually placed at the centre to produce neutrons to start the chain reaction going.

The diagram below shows a similar design but for a plutonium bomb. Both the first (Trinity) atomic bomb and the one dropped later on Nagasaki were of this type.



5 Fission reactors

The more peaceful use of fission is of course in power stations. Here, the use of pure $^{235}_{92}$U is too expensive. Nuclear power stations use natural or slight enriched uranium and get round the problem of losing neutrons to the dominant $^{238}_{92}$U by using a moderator. This is a material, such as carbon or heavy water, which has a high likelihood of scattering neutrons so they can lose energy quickly. Low Z nuclei are favoured as they are light and so energy loss of the neutrons is larger. A common technique is then to have narrow uranium rods inserted into the moderator. The geometry is optimised so that the neutrons from fission, with energies around 2 MeV, have a good chance of escaping from the uranium rods into the moderator. They then scatter many times, losing energy each time and usually become thermalised, meaning they have energies corresponding to the temperature of the moderator; in practise around 1000 K, or 0.1 eV. These thermal neutrons then have some probability of scattering back into the uranium and at such low energies, the most probable reaction is $^{235}_{92}$U fission as the very high cross section at such low energies more than compensates for the small proportion of this isotope. The design of the reactor needs to balance the probabilities for losing neutrons all together, having them reenter the uranium too soon, with too high energies, and so be absorbed by the $^{238}_{92}$U resonances, having them react with the moderator, etc, and get a value of $m = 1$ so the reaction continues at a constant level. Control is provided by inserting or withdrawing control rods made of a strongly neutron-absorbing material, such as cadmium or boron, which can soak up a lot of neutrons if needed and keep $m = 1$. Practical reactor design is extremely complex and there are many different ways in which a reactor can be built, not all safely, as Chernobl showed.

One final comment is that the neutron-rich daughter nuclei tend to be highly radioactive and this material, the radioactive waste, needs to be stored until safe. As some of the materials produced have lifetimes of millions of years, this is a very difficult issue and there is as yet no agreement on a long-term solution.