Amplifiers in systems

• Amplification
  single gain stage rarely sufficient
  add gain to avoid external noise eg to transfer signals from detector
  practical designs depend on detailed requirements
    constraints on power, space, ... cost in large systems
  e.g. ICs use limited supply voltage which may constrain dynamic range

• Noise will be an important issue in many situations
  most noise originates at input as first stage of amplifier dominates
  often refer to Preamplifier = input amplifier
  may be closest to sensor, subsequently transfer signal further away

• In principle, several possible choices
  V sensitive
  I sensitive
  Q sensitive
Voltage sensitive amplifier

• As we have seen many sensors produce current signals but some examples produce voltages - thermistor, thermocouple,...
  
  op-amp voltage amplifier ideal for these
  especially slowly varying signals - few kHz or less

• For sensors with current signals voltage amplifier usually used for secondary stages of amplification

• Signal  \[ V_{\text{out}} = \frac{Q_{\text{sig}}}{C_{\text{tot}}} \]
  
  \( C_{\text{tot}} = \) total input capacitance
  
  \( C_{\text{tot}} \) will also include contributions from wiring and amplifier
  
  \( V_{\text{out}} \) depends on \( C_{\text{tot}} \)
  
  not desirable if \( C_{\text{det}} \) is likely to vary
  
  eg with time, between similar sensors, or depending on conditions

• Noise  to be discussed more later

  contribution from amplifier, and possibly sensor

  \[ S/N = \frac{Q_{\text{sig}}}{(C_{\text{tot}} \cdot v_{\text{noise}})} \] can it be optimised?
Current sensitive amplifier

• Common configuration, eg for photodiode signals
  \[ v_{out} = -A v_{in} \]
  \[ v_{in} - v_{out} = i_{in} R_f \]
  \[ v_{out} = -\left[ A/(A+1) \right] i_{in} R_f \approx -i_{in} R_f \]

• Input impedance
  \[ v_{in} = i_{in} R_f/(A+1) \quad Z_{in} = R_f/(A+1) \]

• Effect of C & R_{in} - consider in frequency domain
  \[ v_0 = i(1/j\omega C||R_{in}) \]
  \[ = i(\omega) R/(1 + j\omega\tau) \]
  input signal convoluted with falling exponential
  increasing R_f to gain sensitivity will increase \( \tau \)
  fast pulses will follow input with some broadening

• Noise
  will later find that feedback resistor is a noise source
  contributes current fluctuations at input \( \sim 1/R_f \)
**Charge sensitive amplifier**

- Ideally, simple integrator with $C_f$
  - but need means to discharge capacitor - large $R_f$
- Assume amplifier has $Z_{in}$ very high (usual case)
  
  $$v_{out} = -Av_{in}$$
  $$v_{out} = v_{in} = \frac{i_{in}}{j\omega C_f}$$
  $$v_{out} = -\frac{[A/(A+1)]i_{in}}{j\omega C_f} \approx \frac{i_{in}}{j\omega C_f}$$
  
  $$\Rightarrow \frac{-Q}{C_f}$$

- Input impedance
  
  $$v_{in} = \frac{i_{in}}{(A+1)j\omega C_f} \quad C = (A+1)C_f \text{ at low } f$$

so amplifier looks like large capacitor to signal source

low impedance but some charge lost

$$Q_A = \frac{Q}{[1 + C_{tot}/(A+1)C_f]}$$

- e.g. $A = 10^3$, $C_f = 1\text{pF}$
  
  $$C_{tot} = 10\text{pF} \quad Q_A/Q = 0.99$$
  
  $$C_{tot} = 100\text{pF} \quad Q_A/Q = 0.90$$
Feedback resistance

- Must have means to discharge capacitor so add \( R_f \)
  
  \[
  Z_f = R_f \parallel 1/j\omega C_f
  \]

  \[
  v_{\text{out}} = -\left[A/(A+1)\right].i_{\text{in}}Z_f
  \]
  
  \[
  = i(\omega)R_f/(1 + j\omega\tau_f) \quad \tau_f = R_f C_f
  \]

  step replaced by decay with \( \sim \exp(-t/R_f C_f) \) \( \tau \) is long because \( R_f \) is large (noise)

  easiest way to limit pulse pileup - differentiate
  ie add high pass filter

- Pole-zero cancellation

  exponential decay + differentiation \( \Rightarrow \) unwanted baseline undershoot
  introduce canceling network

  \[
  v_0 = 1/(1 + j\omega\tau_f)
  \]

  \[
  v_1 = 1/(1 + j\omega\tau_f)(1 + j\omega\tau_1)
  \]

  \[
  \tau_1 = RC < \tau_f
  \]

  add resistor \( R_p \) so \( R_p C = \tau_f \)

  then

  \[
  v_1' = 1/(1 + j\omega\tau_3) \quad \text{with} \quad \tau_3 = (R \parallel R_p)C < \tau_f
  \]
Effect of finite bandwidth

- Realistic input stage of amplifier

\[ v_{out} = i_d(R_L || C_L) \quad i_d = g_m v_{in} \]

\[ A = g_m R_L \quad \text{low } f \]

\[ A = g_m / j\omega C_L \quad \text{high } f \]

(NB phase change)

\[ Z_{in} \approx 1/A \cdot j\omega C_f = C_L / g_m C_f \quad \text{resistive!} \]

Irrespective of detailed design

\[ A \approx A_0 \omega_h / j\omega \quad \omega_h = \text{gain-bandwidth product} \]

\[ = 1/A_0 \omega_h C_f \]

- Effect of \( R_{in} \)

signal current shared between \( R_{in} \) & \( C_{det} \)

\[ v_{out} = i_{in} Z_f \approx i/[j\omega C_f(1 + j\omega \tau_{rise})] \]

\[ \sim 1 - \exp(-t/\tau) \]

high frequencies in leading edge

\[ \tau = R_{in} C_{det} \]

leading edge of output pulse
Output impedance

• Usual method of varying $v_{out}$ and finding $i_{out}$ - generally messy algebra

• Current sensitive amplifier, open loop gain = $A$

\[
\begin{align*}
    v_{out} &= i_2(R_2 + R_{in}) \\
    v_{in} &= i_2R_{in} \\
    v_o &= -Av_{in} = -Ai_2R_{in} \\
    i_o &= (v_{out} - v_0)/R_o = (v_{out} - Ai_2R_{in})/R_o
\end{align*}
\]

\[
Z_{out} = v_{out}/i_{out} = R_o(R_2 + R_{in})/[R_o + R_2 + R_{in}(A+1)] \\
\approx R_o/(A+1) \\
since R_{in} \gg R_2, R_o
\]

• In general

\[
\begin{align*}
    Z_{out} &= R_o/(1+Ab) \quad \text{if voltage is sampled at output} \\
    Z_{out} &= R_o(1+Ab) \quad \text{if current is sampled at output}
\end{align*}
\]

$R_o = \text{open loop output impedance}$
Comparators

- Frequently need to compare a signal with a reference
  eg temperature control, light detection, DVM,...
  basis of analogue to digital conversion -> 1 bit

- Comparator
  high gain differential amplifier,
  difference between inputs sends output to saturation (+ or -)
  could be op-amp - without feedback - or purpose designed IC
  Sometimes ICs designed with open-collector output so add pull-up R to supply
  also available with latch (memory) function

- NB
  no negative feedback so \( v_- \neq v_+ \)
  saturation voltages may not reach supply voltages - check specs
  speed of transition

- Potential problem
  multiple transitions as signal changes near threshold
Hysteresis

- Add positive feedback (Schmitt trigger)
  - $V_{ref}$ changes as $v_{out} \rightarrow +V_S$
  - $v_{out}$ changes as $v_{out} \rightarrow +V_S$
  - $V_{ref}$ changes once transition is made
  - Preventing immediate fall
  - Positive feedback speeds transition
    - $v_{out} = A(V_{ref} - v_-)$
    - $V_{ref} > v_- \Rightarrow v_{out} = V_s$  $V_{ref} = V_{high}$
    - $V_{ref} < v_- \Rightarrow v_{out} = 0V$  $V_{ref} = V_{low}$
  - Here, signal $\Rightarrow$ logical "1": $v_{out} = 0V$

- Output depends on history
  - $V_+ = 10V$, $V_S = +5V$, $0V$
    - $R_1 = 10k\Omega$, $R_2 = 10k\Omega$, $R_3 = 100k\Omega$
  - $V_{out} = 0V$, $V_{ref} = 4.76V$
  - $V_{out} = 5V$, $V_{ref} = 5V$
  - Hysteresis $= \Delta V_{ref} = 0.24V$
Example - alarm

\[ R_3 \]
\[ R_2 \] + \[ v_{out} \]
\[ R_1 \]
\[ V_{ref} \] -
\[ R_f \]
\[ I_{\text{photo}} \]

\[ +10V \]
\[ 0V \]
Oscillators

• Basic building block of many systems
  clock or timer, signal generators, function generators, ...
  can exploit positive feedback

• Relaxation oscillator
  charge capacitor $C$ through $R$ \( \sim \exp(-t/RC) \)
  $v_-$ crosses threshold at $V_{\text{ref}}$, $V_{\text{out}} \Rightarrow \pm V_S$
  $V_{\text{ref}}$ changes sign
  etc, etc...
  square wave output: $[+V_S, -V_S]$
  Period $T = 2.2RC$

• many more types of oscillator design available
  IC classic = 555 (many versions)
  external components set period and duty cycle
Wien bridge oscillator

- Sine wave oscillators also often required
  - favourite circuit for audio test applications:
  - low harmonic distortion at $f \sim$ few kHz

Gain = real at $\omega_0 = 1/RC$
- so positive feedback
Lamp provides temperature dependent resistor
- so negative feedback controls amplitude

What values to choose for lamp resistance and $r$?

What determines amount of harmonic distortion?

$G_+ = \frac{v_{out}}{v_+} = 3$
- at $\omega \tau = 1$
Temperature controller

• A frequent requirement - similar to many other control applications
e.g. cryostat with stable temperature maintained by resistive heater, or oven, ...

• On-Off control
  \[ T < T_0 \]
  set heater to maximum power
  add hysteresis \( (T_{o1}, T_{o2}) \)
to prevent noise from switching too rapidly

  ok for central heating or domestic oven but not good for stable measurements - try to improve
**P & PD Temperature control**

• Set heater power, proportional to temperature difference (P)
  \[ W = P(T_{\text{meas}} - T_0) \]
  T still oscillates and undershoots desired value
  unstable if heat too fast

• Add control term proportional to rate of change (PD)
  \[ W = P[(T_{\text{meas}} - T_0) + Dd(T_{\text{meas}} - T_0)/dt] \]
  D too large: overshoot & ringing
  D too small: slow response
PID Temperature control

• PD can eliminate ringing & overshoot but undershoot error remains
  add integral term

• PID control

\[ W = P[(T_{\text{meas}} - T_0) + Dd(T_{\text{meas}} - T_0)/dt + I\int(T_{\text{meas}} - T_0) \, dt] \]

...good results but need to choose coefficients P, D, I empirically to ensure stability ...

...we'll later look at methods to solve such system equations using transforms...
Temperature control circuit

• Notes
• $R_1 \gg R_2$ to avoid loading
• still need heating circuit
  want $W \propto V_{\text{out}}$
• Diode ensures $W \geq 0$
  $V_{\text{diode}}$?
• Time constants to be selected
  depend on appliance chosen
  commercial devices will recommend values
• Need to consider offset currents and voltages
  null, or consider more complex circuit design
Instrumentation amplifier

• High gain, dc-coupled differential amplifier
  - single ended output
  - high input impedance
  - high CMRR
  - use to amplify small differential signals where large CM signal may be present
  - but small normal mode
    - eg strain gauge, other bridge circuits
    - "weak" voltage source

• Drawback of differential amplifier
  - relatively low input impedance
  - CMRR relies on excellent resistor matching
  - cheap op-amps may have CMRR ~80dB

\[ V_+ = V_- \approx 5V \]
\[ V_+ - V_- \approx mV \]

To measure 5mV signal with 1% error
\[ CMRR = \frac{0.05}{5000} = 100\text{dB} \]
Improved differential amplifier

• Add voltage buffers and choose precise resistors
  improves input impedance
  0.1% resistors available
  careful nulling of circuits

  still need high CMRR from output amplifier
  big demands on R precision

  often find restrictions on driving circuit
  ie source
Classic instrumentation amplifier

• Input stage differential gain

$$v_{10} - v_1 = iR_2 = v_2 - v_{20} \ (1)$$
$$iR_1 = v_1 - v_2$$
$$(v_{10} - v_{20}) - (v_1 - v_2) = 2iR_2$$
$$(v_{10} - v_{20}) = 2iR_2 + iR_1$$
$$= (v_1 - v_2)(2R_2 + R_1)/R_1$$
$$G_{\text{diff}} = 1 + 2R_2/R_1$$

• Input stage common mode gain

$$v_1 = v_{CM} + u_1$$
$$v_2 = v_{CM} + u_2$$
$$2v_{CM} = v_1 + v_2 \ \text{with signal} \ u_1 = u_2 = 0$$

From (1) $$v_{10} + v_{20} = v_1 + v_2$$
$$G_{CM} = 1$$

• Remainder is normal differential amplifier, ($G = 1$ in this case)

Instrumentation ICs available

Reduce requirements on second stage
still choose input amps for good CMRR
and null carefully
The Instrumentation Amplifier in practice

- Can add some more useful features
  - feed common mode level back as guard
  - connect to cable shield
  - reduce effects of cable capacitance, leakage currents
  - sense voltage at load
  - allows feedback to correct for losses in wiring or offset of DC conditions

![Instrumentation Amplifier Circuit Diagram]

\[ G = \left(1 + \frac{2R_2}{R_1}\right) \frac{R_4}{R_3} \]