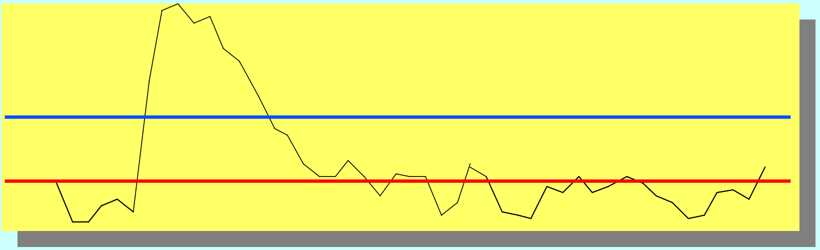
Noise

- What is NOISE?   *A definition:*

  Any unwanted signal obscuring signal to be observed

  two main origins



- EXTRINSIC NOISE   examples...

  pickup from external sources   unwanted feedback

  RF interference from system or elsewhere, power supply fluctuations

  ground currents

  small voltage differences => currents   can couple into system

  may be hard to distinguish from genuine signals   *but* AVOIDABLE

  Assembly & connections, especially to ground, are important

- INTRINSIC NOISE

  Fundamental property of detector or amplifying electronics

  Can't be eliminated but can be MINIMISED

Origins of noise in amplifying systems

•1. Thermal noise

Quantum-statistical phenomenon

Charge carriers in constant thermal motion

macroscopic fluctuations in electrical state of system

•2. Shot noise

Random fluctuations in DC current flow

originates in quantisation of charge
non-continuous current

•3. 1/f noise

Characteristic of many physical systems

least well understood noise source

commonly associated with interface states in MOS electronics

Thermal noise (i)

- Einstein (1906) , Johnson, Nyquist (1928)

  e.g. resistor: *~10^{23} possible states*

  macroscopic statistical average over micro-states

- Experimental observation

  Mean voltage $\langle v \rangle = 0$

  Variance $\langle v^2 \rangle = 4kT.R.\Delta f$ Δf = observing bandwidth

  $\sigma(v) = \sqrt{\langle v^2 \rangle} = 1.3\ 10^{-10}\ (R.\Delta f)^{1/2}$ volts at 300K

  e.g. $R = 1M\Omega$ $\Delta f = 1Hz$ $\sigma(v) = 0.13\mu V$

  gaussian distribution of fluctuations in v

  Noise power = $4kT.\Delta f$

  independent of R & q independent of f - WHITE

- Quantum effects

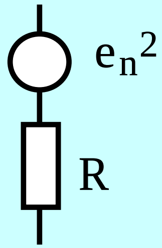
  Normal mode energies: $kT \approx hf / (e^{hf/kT} - 1)$ $kT \gg hf$

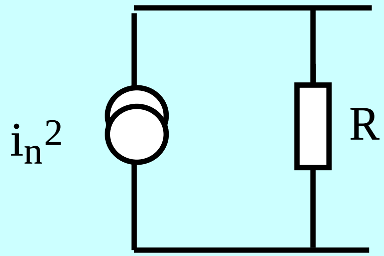
  at 300°K $kT = 0.026$ eV $hf = kT$ at $f = 6.10^{12}$ Hz

Thermal noise (ii)

- Circuit representations

  Noise generator + noiseless resistance R





- Spectral densities

  mean square noise voltage or current per unit frequency interval

  $w_V(f) = 4kTR$ (voltage) $w_I(f) = 4kT/R$ (current)

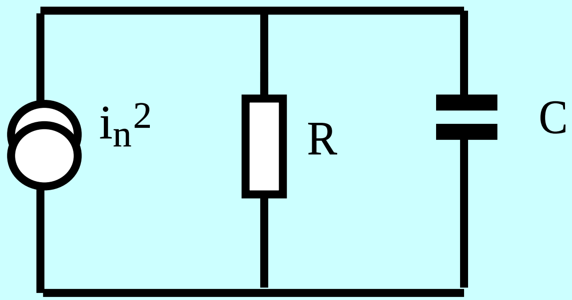
- Why not infinite fluctuation in infinite bandwidth?

  A-1: QM formula -> 0 at high f

  A-2: real components have

  capacitive behaviour (high f)

  or inductive (low f).

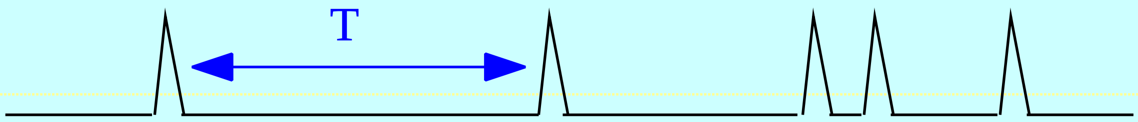


with R and C in parallel $\langle v^2 \rangle = kT/C$

Shot noise

- Poisson fluctuations of charge carrier number

  eg arrival of charges at electrode in system - induce charges on electrode



quantised in amplitude and time

- Examples

  electrons/holes crossing potential barrier in diode or transistor

  electron flow in vacuum tube

$< i_n^2 > = 2qI.\Delta f$ WHITE (NB notation e = q)

I = DC current

gaussian distribution of fluctuations in i

1/f noise

• White noise sources frequently dominate in many real systems

however frequency dependent noise is also common

• 1/f noise is a generic term for a wide range of phenomena, possibly not always related

Power spectral density

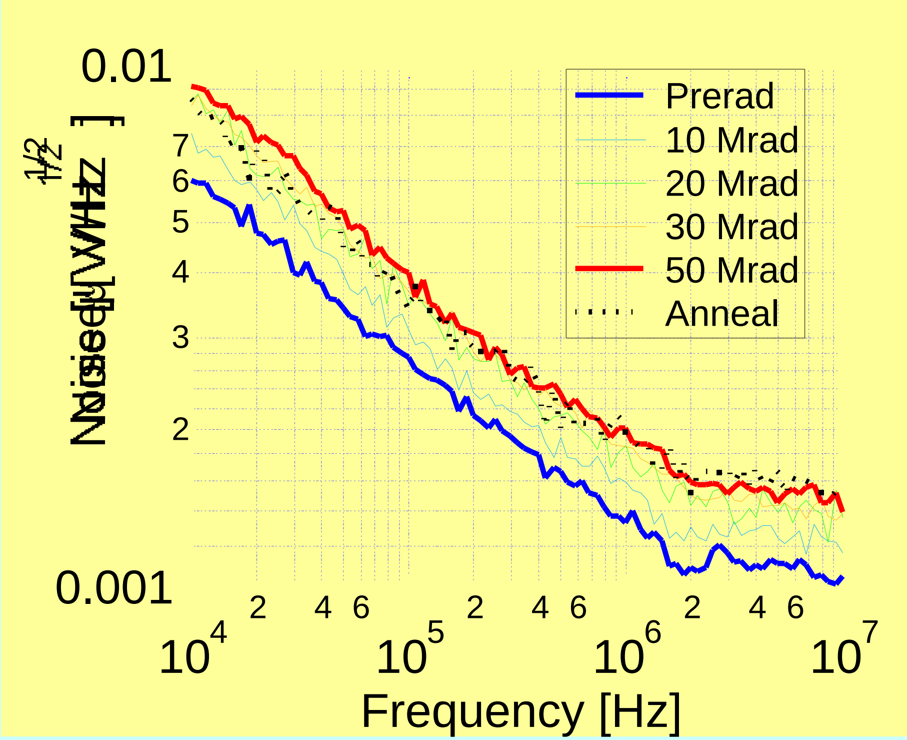
$w(f) = A_f / f^n$ n ~ 0.8-1.5 typical

• Most important for MOS FET devices,

often dominates

but can also arise in other circumstances

e.g. dielectrics,...



pMOS transistor noise spectrum

An explanation for 1/f noise

- Silicon MOS transistors are very sensitive to oxide interface

  typically populated by band-gap energy levels (traps)

  traps exchange charge with channel - ie. emit and capture electrons or holes

- Traps have lifetime to retain charge $h(t) \sim e^{-t/\tau}$

  Expect a range of traps with different time constants, distributed with $p(\tau)$

  in frequency domain $H(\omega) \sim 1/(1+j\omega\tau)$

  Deduce frequency spectrum by integrating over all values of τ

  $$w(f) \sim \int_0^\infty p(\tau)|H(\omega)|^2 d\tau$$

  If $p(\tau)$ = constant, ie all time constants equally probable

  $$w(f) \sim \int_0^\infty d\tau/(1 + \omega^2\tau^2)$$ [standard integral, put $\tan\theta = \omega\tau$]

  $$= A/f$$

  Many other real-life processes have $e^{-t/\tau}$ time distributions -

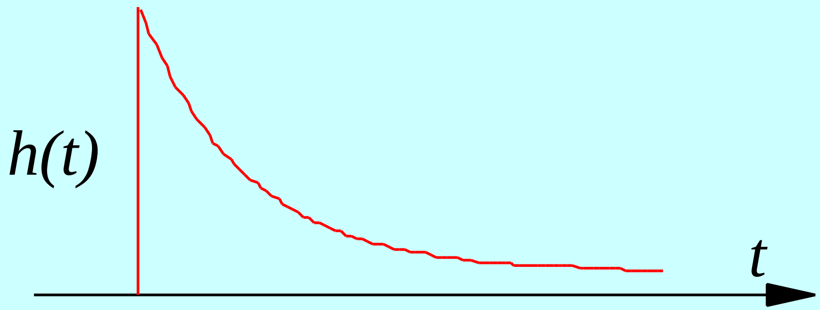
  typical of random, Poisson-type processes

Campbell's theorem - time domain

- Most amplifying systems designed to be linear

  $S(t) = S_1(t) + S_2(t) + S_3(t) + \ldots$

- Impulse response h(t) = response to δ

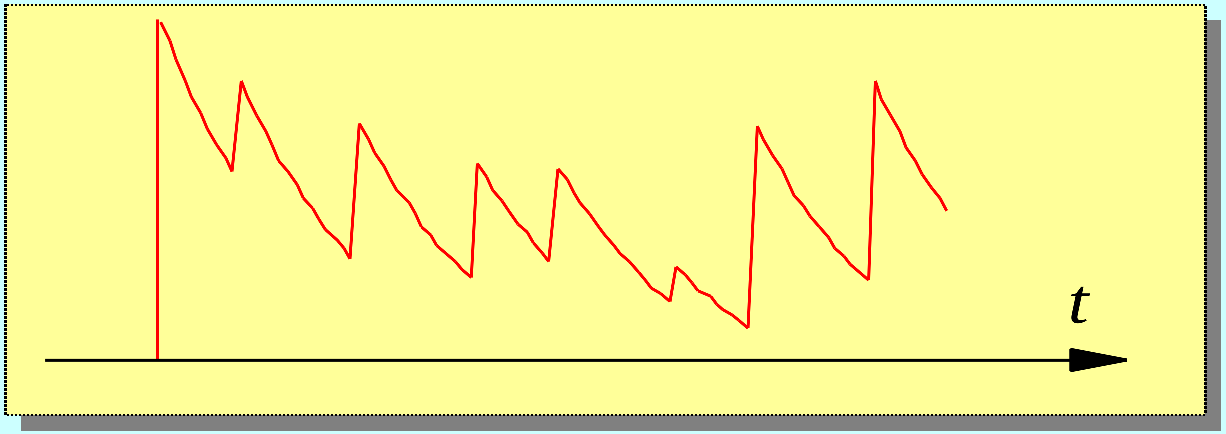


- In a linear system, if random impulses occur at rate n

average response $\langle v \rangle = n \int_{-\infty}^{t_{obs}} h(t)dt$ (A)

variance $\sigma^2 = n \int_{-\infty}^{t_{obs}} [h^2(t)]dt$ (B)

i.e. sum all pulses preceding time, t_{obs}, of observation



Campbell's theorem - frequency domain

- Recall relationship between impulse response h(t) and transfer function H(ω)

$H(\omega) = \int_{-\infty}^{\infty} h(t).e^{-j\omega t}\, dt$ $\qquad h(t) = \int_{-\infty}^{\infty} H(\omega)e^{j\omega t} df$ $\qquad H(\omega) = v_{out}(\omega) / v_{in}(\omega)$

- Rewrite (B) using Parseval's Theorem

$$\int_{-\infty}^{\infty} h^2(t)\, dt = \int_{-\infty}^{\infty} |H(\omega)|^2\, df = 2\int_0^{\infty} |H(\omega)|^2\, df$$

h(t) is real and thus
$H(-\omega) = H^*(\omega)$

so $\sigma^2 = n\int_{-\infty}^{\infty} h^2(t)\, dt = n\int_{-\infty}^{\infty} |H(\omega)|^2\, df$

This relates noise spectral densities at input and output:

$w_{out}(f) = w_{in}(f)\, |H(\omega)|^2$ can use theorem to calculate system response to noise

- eg. shot noise Consider impulse response to be impulse (ie unchanged!)

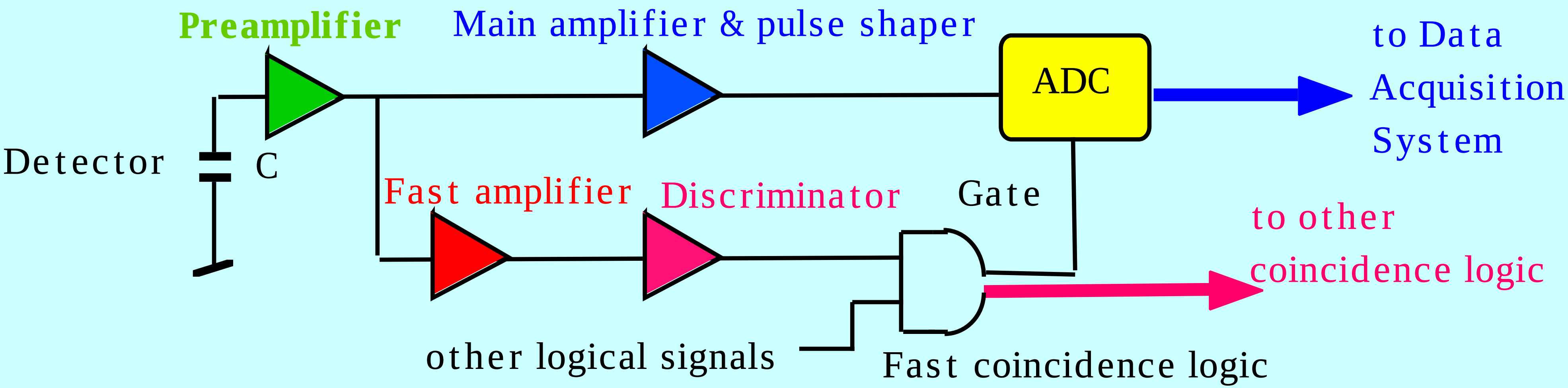
$h(t) = e\delta(t) \Rightarrow H(\omega) = 1.e$

$$\sigma^2 = n\, e^2 \int_{-\infty}^{\infty} \delta^2(t)\, dt = ne^2 \int_{-\infty}^{\infty} |H(\omega)|^2\, df = 2ne^2\Delta f$$

but $n = I/e \Rightarrow \sigma^2 = \underline{2eI\,\Delta f}$

Amplifier systems for spectroscopy

• typical application - precise measurements of x-ray or gamma-ray energies



• pre-amplifier *first stage of amplification*

• main amplifier - *adds gain and provides bandwidth limiting*

ADC - analogue to digital conversion - *signal amplitude to binary number*

• fast amplifier and logic -

start ADC ("gate") and flag interesting "events" to DAQ system

- most signals arrive randomly in time.

Other logic required to maximise chance of "good" event, eg second detector