# Sampling and digital processing

- Up to now all signals have been considered to be continuous in time

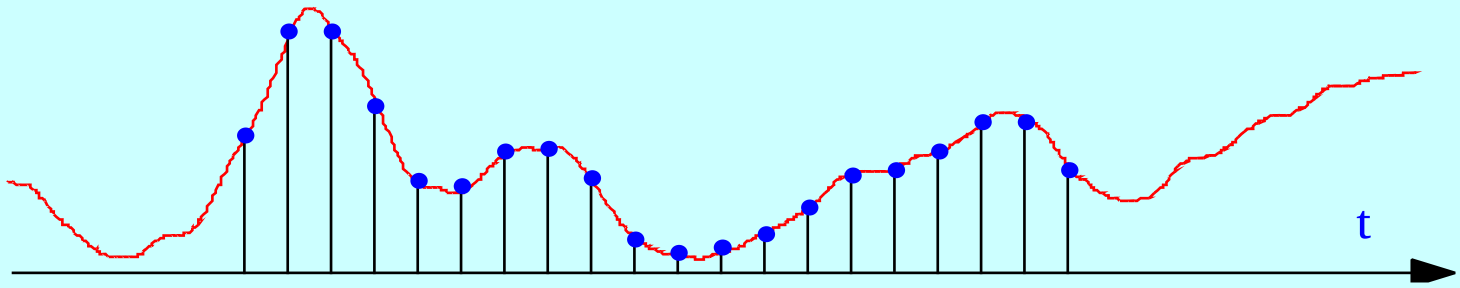
    but much modern digital electronics uses sampled signals

        a series of levels taken in coincidence with a clock pulse

        the amplitude can be represented by a digital value

- We can consider a sampled signal to be

    convolution of the continuous signal with discrete δ function




- Start with a puzzle..

    sample sine wave at fixed frequency (10kHz)

    vary frequency of sine wave from 250Hz to ~180kHz

    program calculates frequency from samples, and displays result

        is this what we expect?

# Sampling

- One of the most important applications is processing of sequences of sampled signals derived from continuous analogue waveforms

  under certain circumstances a continuous time signal can be completely represented by samples at points equally spaced in time

- Surprising?

  moving images (cinema/video)

  pointilliste paintings...

  not intuitively obvious - infinite no of signals need infinite no of samples

  why so important?

  dramatic advances in digital technology, now possible to:

  sample to convert continuous signal to discrete

  process - with discrete time system

  convert back to continuous

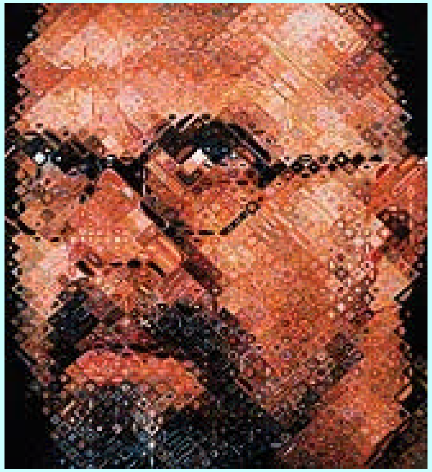
  digital audio is a good example

# Chuck Close

•US b 1940

"Phil"

Original size 9ft x 7ft



self portrait 1997

the images themselves (jpegs) are also examples of sampling

# Fourier transform of periodic functions

- The Fourier series expands in terms of natural frequencies of the system

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jk\omega_0 t}dt \qquad \omega_0 = \frac{2\pi}{T}$$

outside interval $\Delta t = T$, **function repeats**

- What is the Fourier series for $\delta(t)$?

$$a_k = \frac{1}{T} \quad \text{so} \quad \delta(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{jk\omega_0 t}$$

- Consider a signal $x(t)$ with FT $X(\omega)$ which is a unit impulse in **frequency**

$$X(\omega) = \delta(\omega - k\omega_0) \quad \text{then} \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\delta(\omega - \omega_0)e^{j\omega t}dt = \frac{1}{2\pi}e^{j\omega_0 t}$$

- therefore

$$X(\omega) = \sum_{k=-\infty}^{\infty}\frac{2\pi}{T}\delta(\omega - k\omega_0) \qquad x(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{jk\omega_0 t} = \delta(t)$$

- ie the Fourier transform of a periodic series of impulses in time
- is a series of (magnitude $2\pi/T$) impulses in frequency at harmonics $k\omega_0$

# Impulse train sampling

- We can think of a sampled waveform as a sum of different amplitude impulses

$$s(t) = x(t)p(t) \quad \text{where } p(t) = \sum_{k=-\infty}^{\infty}\delta(t - kT)$$

- This is a multiplication in time, so a convolution in frequency

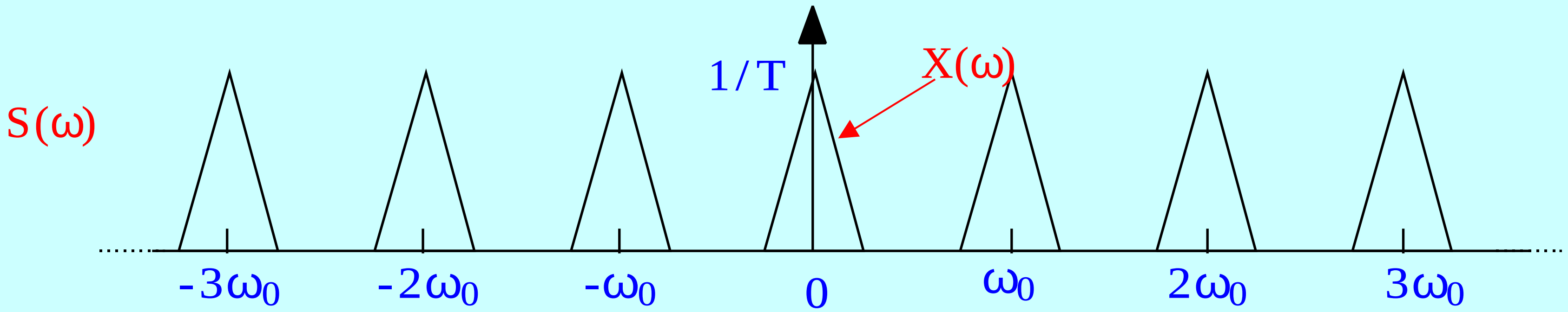
$$S(\omega) = (1/2\pi)\int_{-\infty}^{\infty} X(\upsilon).P(\upsilon - \omega).d\upsilon \qquad \text{and} \quad P(\omega) = (2\pi/T)\sum_{k=-\infty}^{\infty}\delta(\omega - k\omega_0)$$

$$S(\omega) = (1/T)\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty} X(\upsilon).\,\delta(\upsilon - \omega + k\omega_0)\,.d\upsilon$$

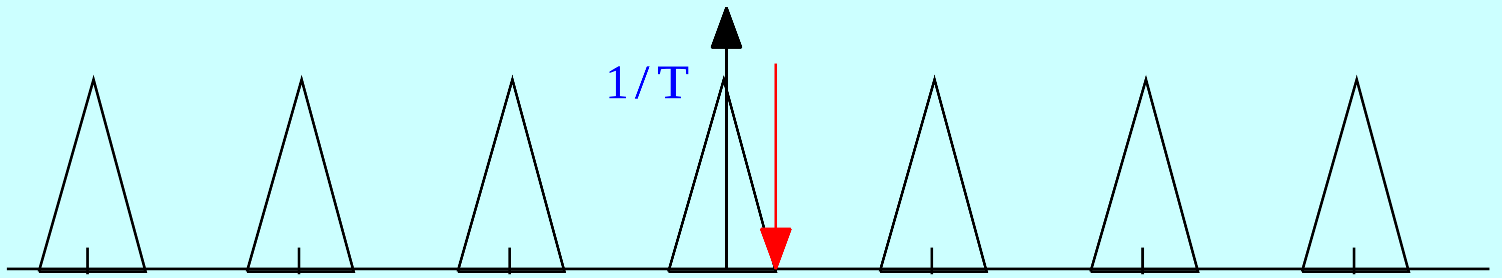
$$= (1/T)\sum_{k=-\infty}^{\infty} X(\omega - k\omega_0)$$

- The FT of the sampled waveform is a series of equally spaced replicas of X(ω)

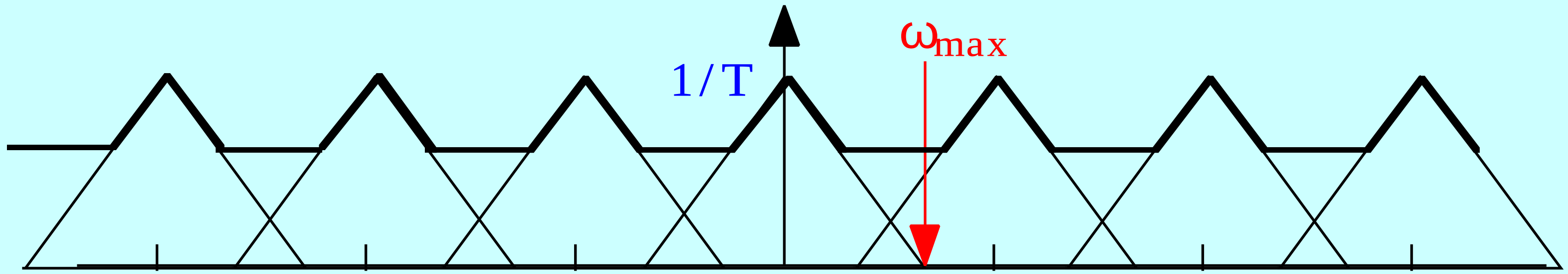
ie the FT of the signal x(t) [ scaled by 1/T]



# Sampling Theorem (Nyquist)




Signal can be recovered with a low- or band-pass filter




Higher harmonics will be sampled

- From this we can conclude

A continuous analogue function x(t) which has a **limited** Fourier spectrum

ie $X(\omega) = 0$ for $\omega > \omega_{max}$

is uniquely described from its values at uniformly spaced time instants $\Delta t$

$\Delta t = 2\pi/\omega_s$ and $\omega_s \geq 2\,\omega_{max}$ **Nyquist rate**

# Explanation of ...

- ..earlier puzzle

  should now be clearer

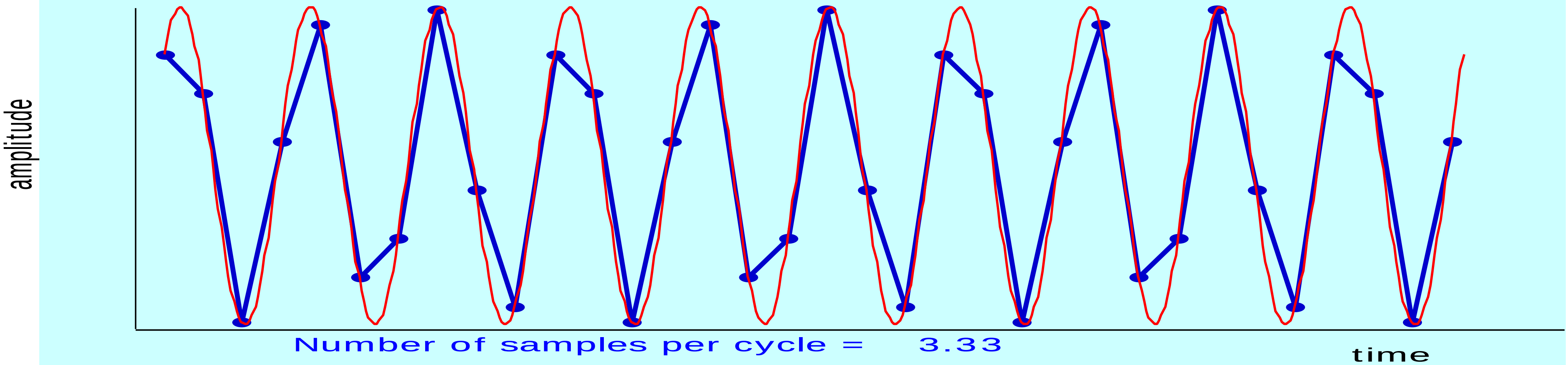
- What happens if we undersample?

  ie take fewer samples than required by Nyquist for maximum frequency component in signal

- Let's take a look...

# Examples

- Oversampling




- Undersampling