Instrumentation Problem Sheet 6

(1) A delay element for digital pulses can be constructed using RC circuit and comparator. Devise a way to do this. A clock waveform (ie square digital pulses with equal ON and OFF times) has a period 4T. Using a delay element which produces a delay T, show that the XOR'ed output of the clock and delayed clock produces another clock waveform. What is it's period?

(2) Satisfy yourself that the following logic identities are true:

[notation is A' = NotA , + = OR, x = AND, AxB = AB]

A(B+C) = AB + AC

A + AB = A

A + BC = (A+B)(A+C)

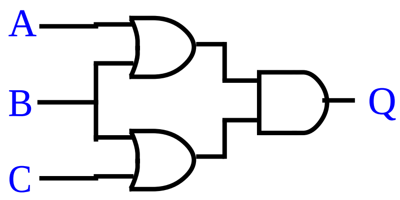
AA' = 0

and DeMorgan's theorems:

(A+B)' = A'.B'   and   (AB)' = A' + B'

(remember these are logic levels, or binary numbers).

(3) Find the the truth table for the logic diagram illustrated.



(4) Design a 3-bit Gray-to-binary decoder using 2 XOR gates. (Hint: start with the two high order bits. The lowest bit needs the result of the higher binary digit.) Show that this is easily extended to 4-bits (and beyond) by duplicating the lowest order bit logic.

(5) Use the Laplace transform to find a solution to the differential equation

$$x'' - a^2x = f(t)$$

(6) Two identical amplifiers with impulse response $f(t) = te^{-t}$ in the time domain are connected in series.

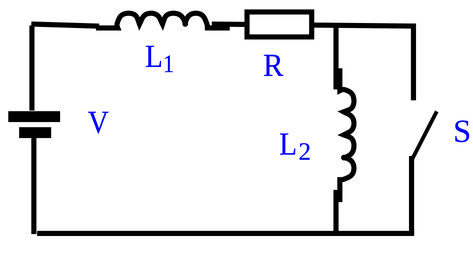
a) What is the transfer function of a single amplifier in isolation ?

b) What is the transfer function of the two amplifiers in series ?

c) Using partial fractions find the time domain output of the two amplifiers in series for a step function input x(t) = u(t) at a time t=0.

(7) A circuit containing a resistor and two inductors is constructed as shown.

Initially the switch S is closed (shorting out inductor L2). The switch is opened at a time t=0, increasing the effective inductance of the circuit from L_1 to $(L_1 + L_2)$.



a) Write down a simple expression for the steady state current that flows through the circuit at a time $t<0$ before the opening of the switch.

b) Write down a simple expression for the steady state current in the limit $t \rightarrow \infty$ after the opening of the switch.

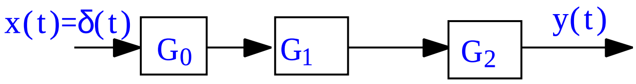
c) Write down a differential equation relating the current and voltage in the circuit for times $t > 0$. Use the Laplace transform to solve this equation and derive an expression for the time domain behaviour for all times $t > 0$.

(8) The inductance L_2 of the previous problem is replaced with a capacitor C. Show that the current flowing for $t > 0$ can be written as

$$i(t) = (V/L)[Ae^{at} + Be^{bt}] + C$$

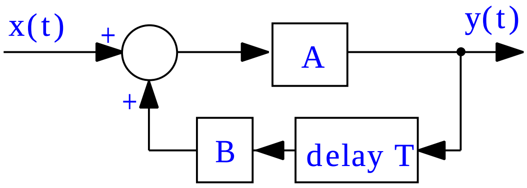
$L = L_1$. What are the values of A, B, C, a and b? Is the system stable?

(9) Use the Laplace transform and partial fractions to find the system impulse response of the system in the figure.



G_0 is a unity gain integrator, G_1 is a high pass filter with $\tau_1 = 1/2$, and G_2 is a low pass filter with $\tau_2 = 1/3$.

(10) The system in the figure represents an audio system where x(t) is the input at the microphone and y(t) is the output at the loudspeakers. A represents amplification following the microphone.



The feedback path represents a signal returned to the input as a consequence of the microphone sensing the sound from the speakers. B is an attenuation factor and there is also a delay. Write down an equation to represent the system response in the time domain. Derive the corresponding equation in the s-domain using the time shift property of the Laplace transform. Hence show under what conditions stability can be guaranteed.

(11) Find the z-transform of the filter which has an impulse response $(t/\tau)\exp(-t/\tau)$. Hence find the inverting filter which is needed to recover an impulse, and the weights which should be applied to consecutive samples. (There should be just 3).