

# Jargon, names & concepts

- Linear systems *will be a frequent assumption*  
input signal =  $f(t)$  output =  $g(t)$   
expect output to vary with input as  $Af(t) \rightarrow Ag(t)$

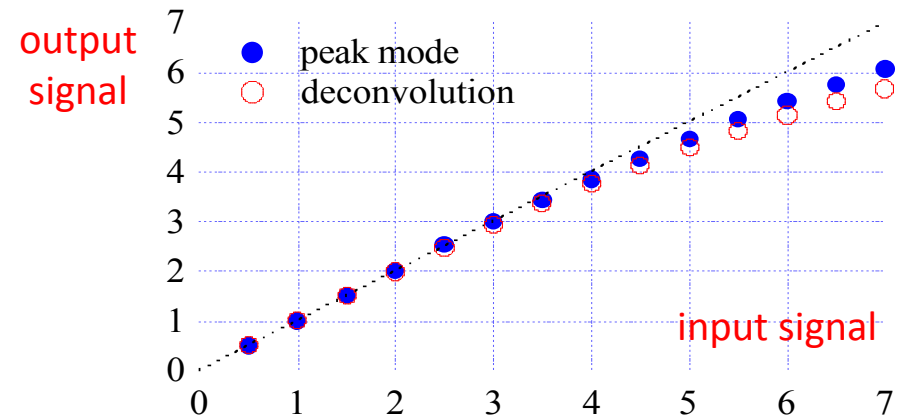
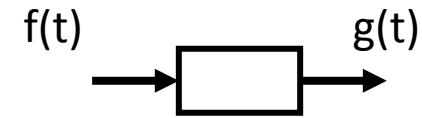
not always the case,  
eg amplifiers frequently exhibit saturation  
*can arise for several reasons*

- Superposition

important principle in many areas of physics & mathematical physics

If  $f_1(t) \rightarrow g_1(t)$  and  $f_2(t) \rightarrow g_2(t)$

then  $af_1(t) + bf_2(t) \rightarrow ag_1(t) + bg_2(t)$



# Dynamic range

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- In most systems there will be a **smallest measurable signal**  
if noise is present, most likely to be related to the smallest signal distinguishable from noise **3 x rms noise?** **5 x rms noise?**  
or the measurement quantisation unit (digitisation)
- and a **largest measurable signal**  
most likely set by apparatus or instrument, eg saturation
- **Dynamic range** = ratio of largest to smallest signal  
often expressed in dB or bits  
eg 8 bits = dynamic range is  $256_{10} = 48\text{dB}$  (if signal is voltage)
- decibels (dB)  
signal magnitudes cover wide range so frequently prefer logarithmic scale  
Number of dB =  $10\log_{10}(P_2/P_1)$   
often measuring voltages in system:  $\text{dB} = 20 \log_{10}(V_2/V_1)$
- Not an absolute unit and sometimes encounter variants  
dBm: dB with  $P_{\text{in}} = 1\text{mW}$

# Precision

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- many measurements involve detection of a particle or radiation quantum (photon)  
simple presence or absence is sometimes sufficient, i.e. binary (0 or 1)  
binary measurements are convenient for modern digital electronics
- alternative measurements are of amplitude, e.g. energy
- why do we need such (analogue) observations?
  - they carry information, which may be valuableprimary measurement may be energy  
*eg medical imaging using gammas or high energy x-rays, astro-particle physics*  
extra information to improve data quality  
*removes experimental background, eg Compton scattered photons mistaken for real signal*  
optical communications - pressure to increase “bandwidth” – e.g. number of telephone calls carried per optical fibre  
*wavelength division multiplexing - several “colours” or wavelengths in same fibre simultaneously*
- what is the ultimate limit to precision?

# Statistical limit to energy measurement

- Assume no limit from anything other than sensor  
*often not realistic assumption, but best possible case*

$$N_{\text{quanta observed}} = E/\varepsilon$$

= energy deposited by radiation

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energy required to generate quantum of measurement

examples

*semiconductor: energy for electron-hole pair ~ few eV*

*gaseous ionisation detector: energy for electron-ion ~ few x 10 eV*

*scintillation sensor: energy per photon of scintillation light ~ 100 eV*

- Basic Poisson statistics

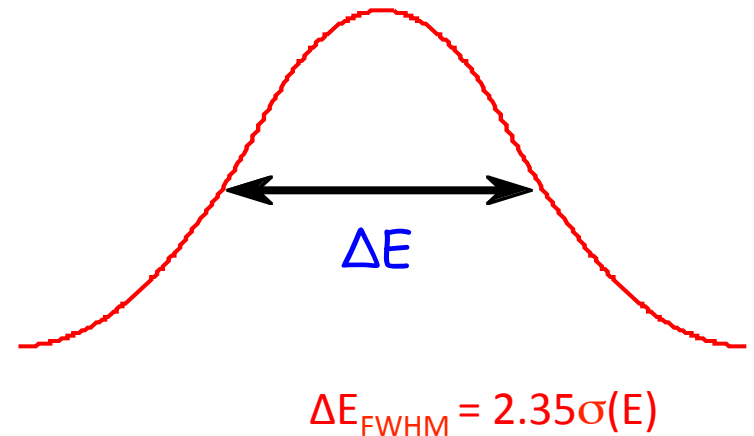
$$E_{\text{meas}} \sim N_q$$

$$\sigma^2(N_q) = N_q$$

$$\sigma(E)/E = \sigma(N_q)/N_q = 1/\sqrt{N_q}$$

*expect gaussian distribution of  $N_q$  for large  $N_q$*

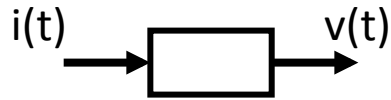
advantage in sensor with small  $\varepsilon$



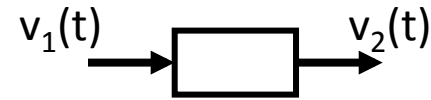
# Filters

- Device or components to transform electrical signals from one form to another

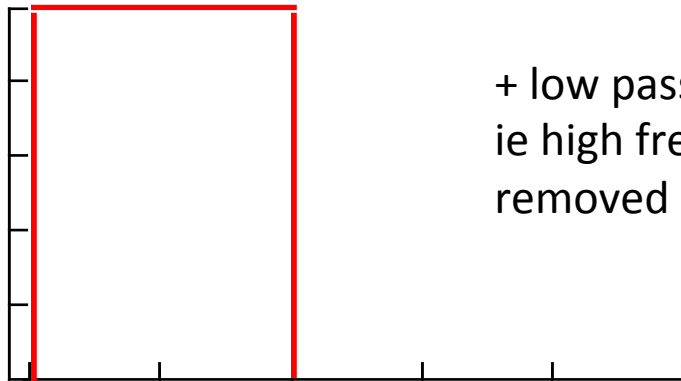
eg



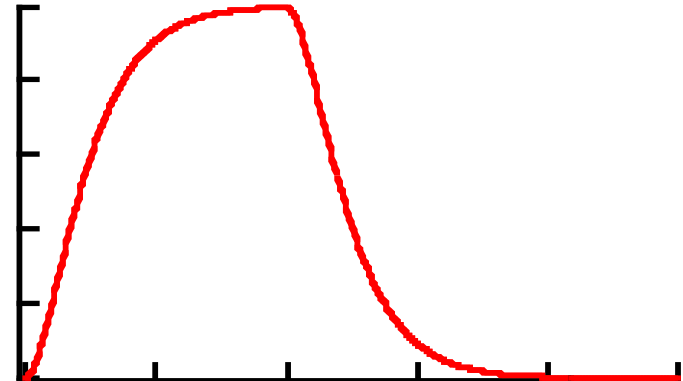
or



usually when doing so, amplitudes of frequencies in output are different from those input,  
*ie spectral content is changed or some frequencies filtered out*



+ low pass filter  
ie high frequencies  
removed



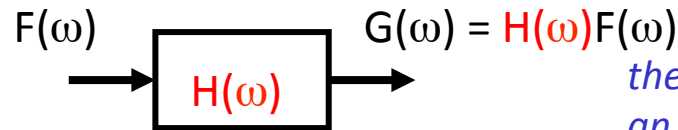
frequently want to analyse signals and systems in terms of frequency content, as well as behaviour in time

# Transfer Function

- Inputs to, and outputs from, system considered as sum of components, with each component a single frequency

$$F(\omega) = A(\omega) e^{j\omega t}$$

$$\omega = 2\pi f$$



*the Fourier or Laplace transform is an important tool*

$H(\omega)$  is transfer function of system =  $G(\omega)/F(\omega)$

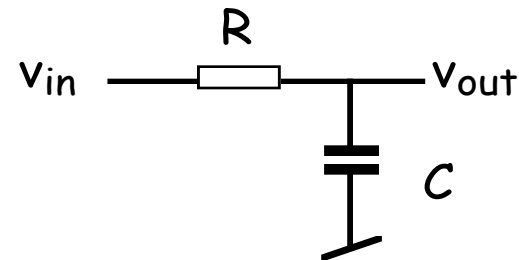
generally complex so introduces both **phase** and **amplitude** changes

- High pass filter

$$H(\omega) = R/(R + 1/j\omega C) = j\omega\tau/(1 + j\omega\tau)$$

$$\tau = RC$$

response to voltage **step**  $\sim e^{-t/\tau}$



- Low pass filter

$$H(\omega) = (1/j\omega C)/(R + 1/j\omega C) = 1/(1 + j\omega\tau)$$

response to voltage **step**  $\sim 1 - e^{-t/\tau}$

rise time: usually define as 10-90%

