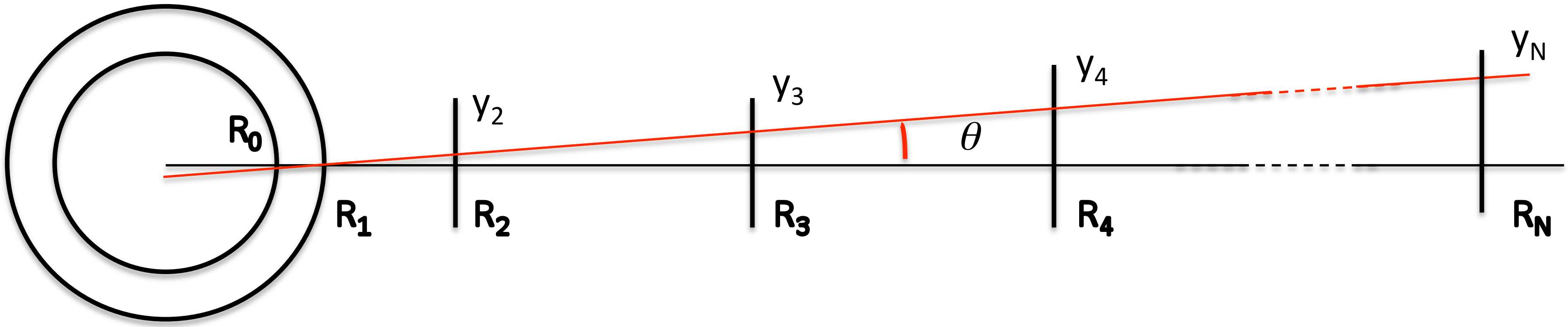
Calculating Tracker parameters

In view of upgrades to tracker, how much of performance can be estimated without running very lengthy simulations?

Quantities of interest: angular and momentum resolution, impact parameter resolution

Variables: no layers, locations, spatial resolution, material budget

Impact Parameter estimation



- IP determined by projection to beam from first measurement plane, R_1
  - including bending and multiple scattering at beam pipe

$$\sigma_{IP}^2 \approx \sigma_1^2 + R_0^2\sigma_0^2(\theta_{MS}) + R_1^2\sigma^2(\theta) + 0.25R_1^4\sigma^2(\rho) \qquad \rho = 1/R$$

  - actually an approximation: should properly include correlations
- Wish to know
  - angular and momentum resolution: $\sigma(\theta)$, $\sigma(\rho)$ & multiple scattering errors
- Assumes (good approximations)
  - angles are small
  - high enough momentum for curvature ρ to be small: ~ 1 GeV/c

Impact Parameter calculation

- Linear least squares problem for straight line

$$\chi^2 = \sum_i \frac{(y_i - R_i\theta)^2}{\sigma_i^2} \quad \text{with} \quad \frac{d\chi^2}{d\theta} = 0 \qquad \text{has solutions} \qquad \theta = \frac{\sum \frac{y_i R_{i1}}{\sigma_i^2}}{\sum \frac{R_{i1}^2}{\sigma_i^2}} \qquad \sigma^2(\theta) = \frac{1}{\sum \frac{R_{i1}^2}{\sigma_i^2}}$$

$$R_{i1} = R_i - R_1$$

- More general problem of fitting a circle (helix) to a series of points was solved - in the absence of multiple scattering - by
  - V. Karimäki CMS Note 1997/064 [NIM A410 (1998) 284] & NIM A305 (1991) 187
  - used for track fitting with UA1 wire chambers where MS is minor effect

$$\varepsilon_i \approx \tfrac{1}{2}\rho r_i^2 - (1+\rho d) r_i \sin(\phi - \phi_i) + \tfrac{1}{2}\rho d^2 + d$$

ρ = curvature = ±1/R where R is radius of curvature
ϕ = direction of propagation at point of closest approach
d = distance of closest approach to the origin

  - general prescription for estimating MS effect is to add spatial errors in quadrature

Errors from Karimäki calculation

- Covariance matrix: W and its inverse – when no correlations present -

$$W_{kl} = \sum_i w_i \frac{\partial \varepsilon_i}{\partial \alpha_k} \frac{\partial \varepsilon_i}{\partial \alpha_k} \qquad w_i = \sigma_i^{-2}$$

$$\rho = \alpha_1 \qquad \phi = \alpha_2 \qquad d = \alpha_3$$

- Solved explicitly to good approximation

$$\begin{cases} \sigma_{\rho\rho} & = & C\,[4\sigma_{xx} - 4\rho^2(\langle x^2\rangle^2 - \langle x\rangle\langle xr^2\rangle + \frac{1}{4}\rho^2\langle xr^2\rangle^2)] \\ \sigma_{\rho\phi} & = & C\,[2\sigma_{xr^2} - \rho^2(2\langle x^2\rangle\langle xr^2\rangle - \langle r^2\rangle\langle xr^2\rangle - \langle x\rangle\langle r^4\rangle + \frac{1}{2}\rho^2\langle r^4\rangle\langle xr^2\rangle)] \\ \sigma_{\phi\phi} & = & C\,[\sigma_{r^2r^2} + \rho^2\langle r^4\rangle(\langle y^2\rangle - \frac{1}{4}\rho^2\langle r^4\rangle)] \\ \sigma_{\rho d} & = & C\,[2\langle x\rangle\langle xr^2\rangle - 2\langle x^2\rangle\langle r^2\rangle - \rho^2(\langle xr^2\rangle^2 - \langle x^2\rangle\langle r^4\rangle)] \\ \sigma_{\phi d} & = & C\,[\langle x\rangle\langle r^4\rangle - \langle r^2\rangle\langle xr^2\rangle] \\ \sigma_{dd} & = & C\,[\langle x^2\rangle\langle r^4\rangle - \langle xr^2\rangle^2] \end{cases}$$

$$\sigma_{ab} = \langle ab\rangle - \langle a\rangle\langle b\rangle \qquad \langle u\rangle = \sum w_i u_i / \sum w_i$$

see papers for details

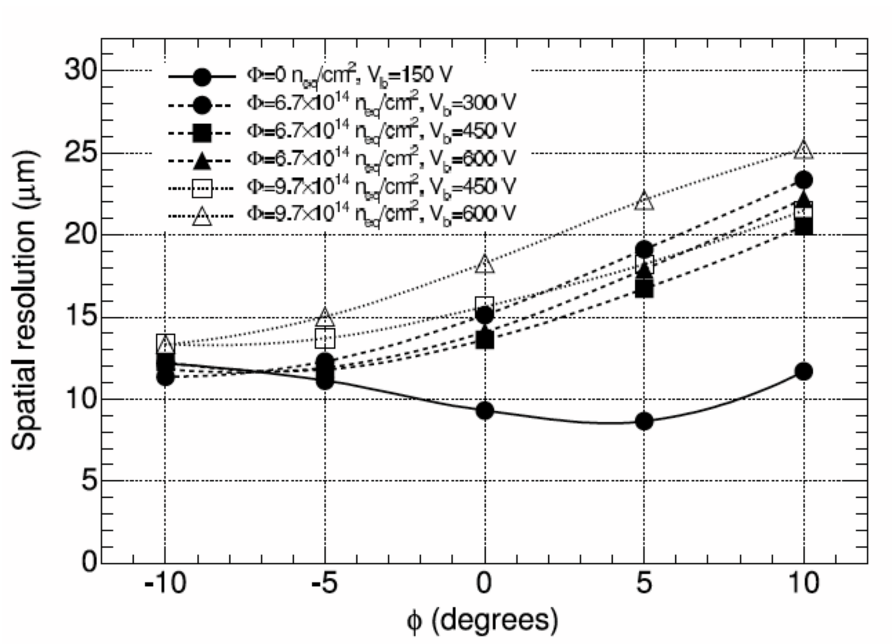
Measurement error

- σ ≈ 10μm (pixels)

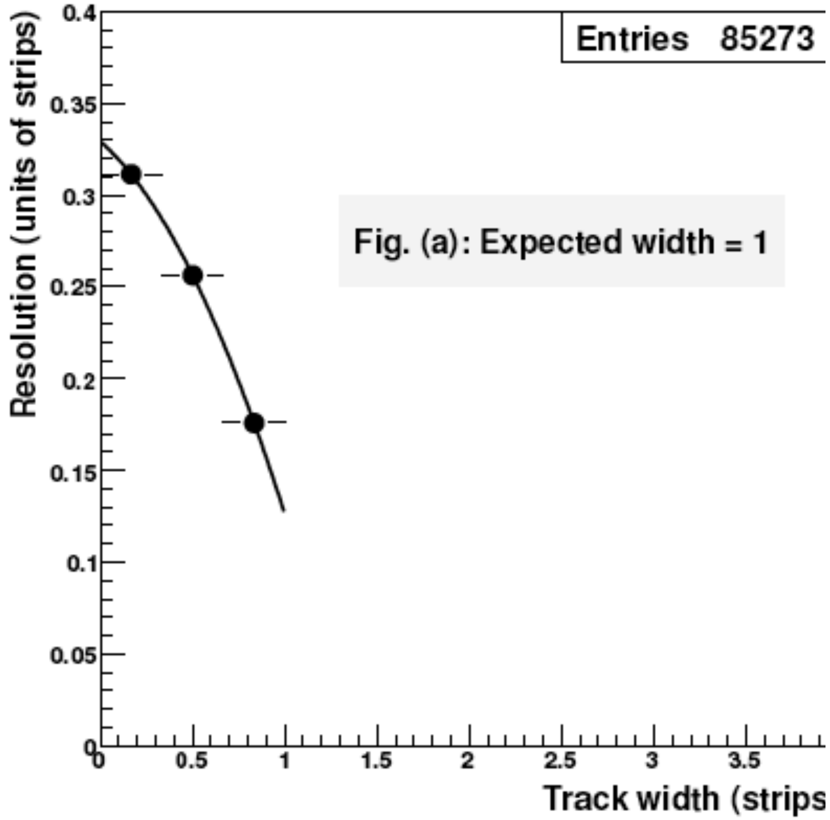
σ ≈ 0.30*pitch (strips)

better approximation for high p

at low p MS dominates

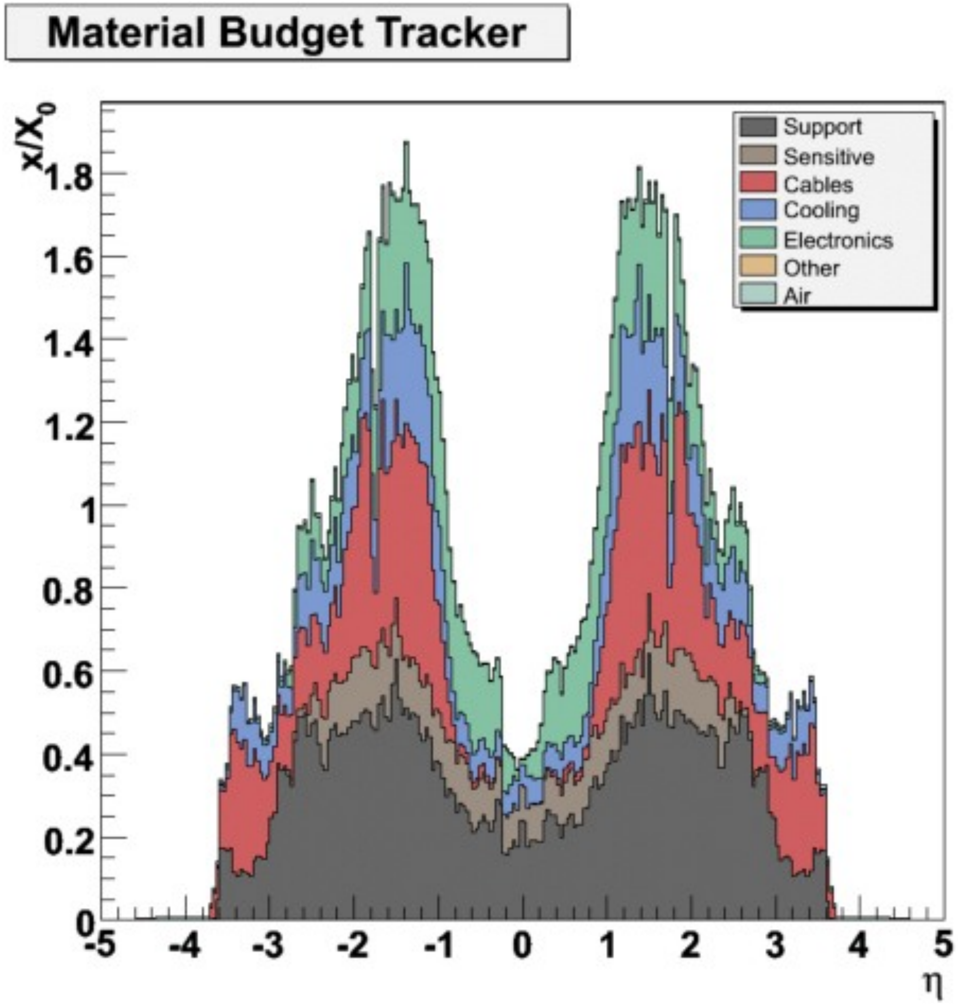


. 1.2. Expected position resolution of the CMS pixel barrel in the $r - \phi$ plane as a
he track angle of incidence.[2] The resolution degrades with increasing fluence and bi
ause of the reduced charge sharing.

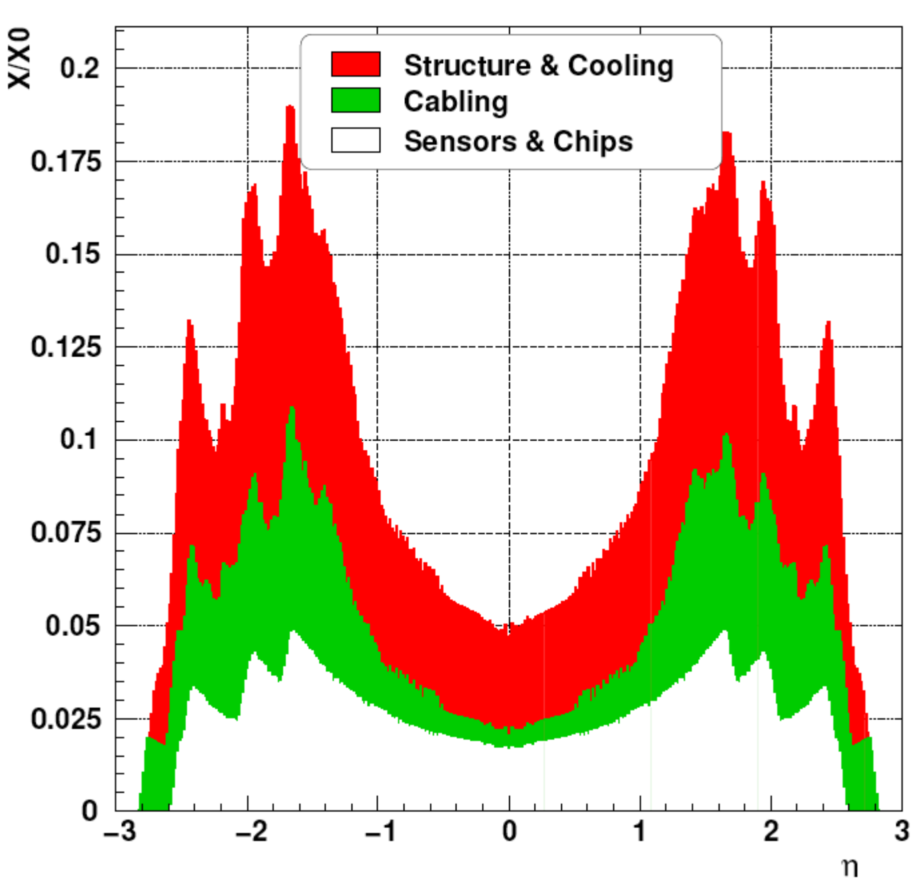


igure 6.6: Strip tracker cluster position r

Material in present system



Pixels only



Simplified model

- Present system: 3 pixel + 10 outer strip
- Each pixel layer assumed identical
  - $t_{eff} \approx$ 1.6mm Si equivalent => $t_{eff}/X_0 \approx 0.017$ per layer
  - Total $t_{eff}/X_0 \approx 0.051$ @ $\eta = 0$
- Outer tracker: two types of layer
  - $t_{eff} \approx$ 2.4mm Si equivalent (single) => $t_{eff}/X_0 \approx 0.026$ per layer
  - $t_{eff} \approx$ 4.84mm Si equivalent (stereo) => $t_{eff}/X_0 \approx 0.051$ per layer
  - Total $t_{eff}/X_0 \approx 0.359$ @ $\eta = 0$
- Beam pipe
  - 800µm Be beam pipe => $t_{eff}/X_0 \approx 0.0027$
  - only affects projection back to primary vertex

Errors assigned for fitting direction

- Combination of measurement + multiple scatter at earlier plane
  - multiple scattering dominates $$\theta_0 = \frac{13.6 MeV}{\beta c p} z \sqrt{x/X_0}\left[1 + 0.038 \ln(x/X_0)\right]$$

							p = 1 GeV/c	
i	R		t_{eff}	X_0	t_{eff}/X_0	σ_{meas}	$\sigma(\theta_{MS})$	σ_i
	[cm]		[cm]	[cm]		[μm]	mrad	[μm]
1	2.9	Be	0.08	35.28	0.0023		0.50	
2	4.4	Si	0.16	9.36	0.017	10	1.53	12
3	7.3	Si	0.16	9.36	0.017	10	1.53	51
4	10.2	Si	0.16	9.36	0.017	10	1.53	102
5	25.5	Si	0.46	9.36	0.051	24	2.72	501
6	34.7	Si	0.46	9.36	0.051	24	2.72	789
7	43.9	Si	0.23	9.36	0.025	36	1.89	1143
..	..		..	..	..	..	..	..
14	108.3	Si	0.23	9.36	0.026	36	2.15	7073

small corrections for incident angle

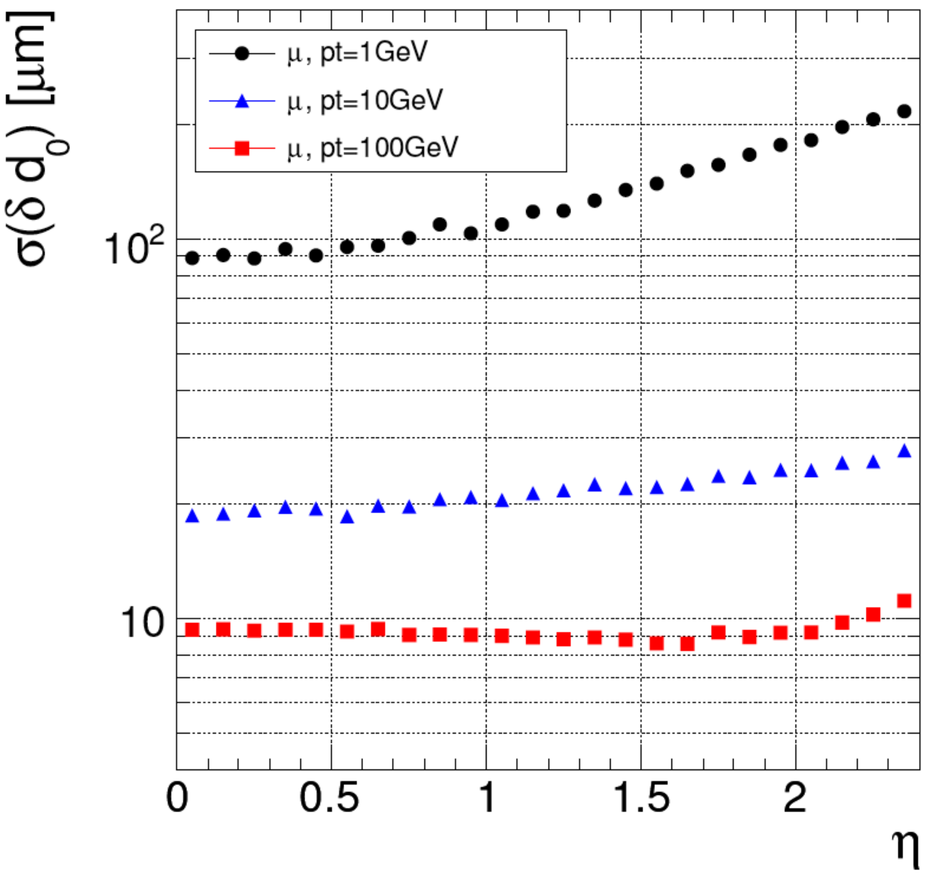
$t_{eff} = t_{layer}/\cos(\alpha)$

Note growth of σ with R

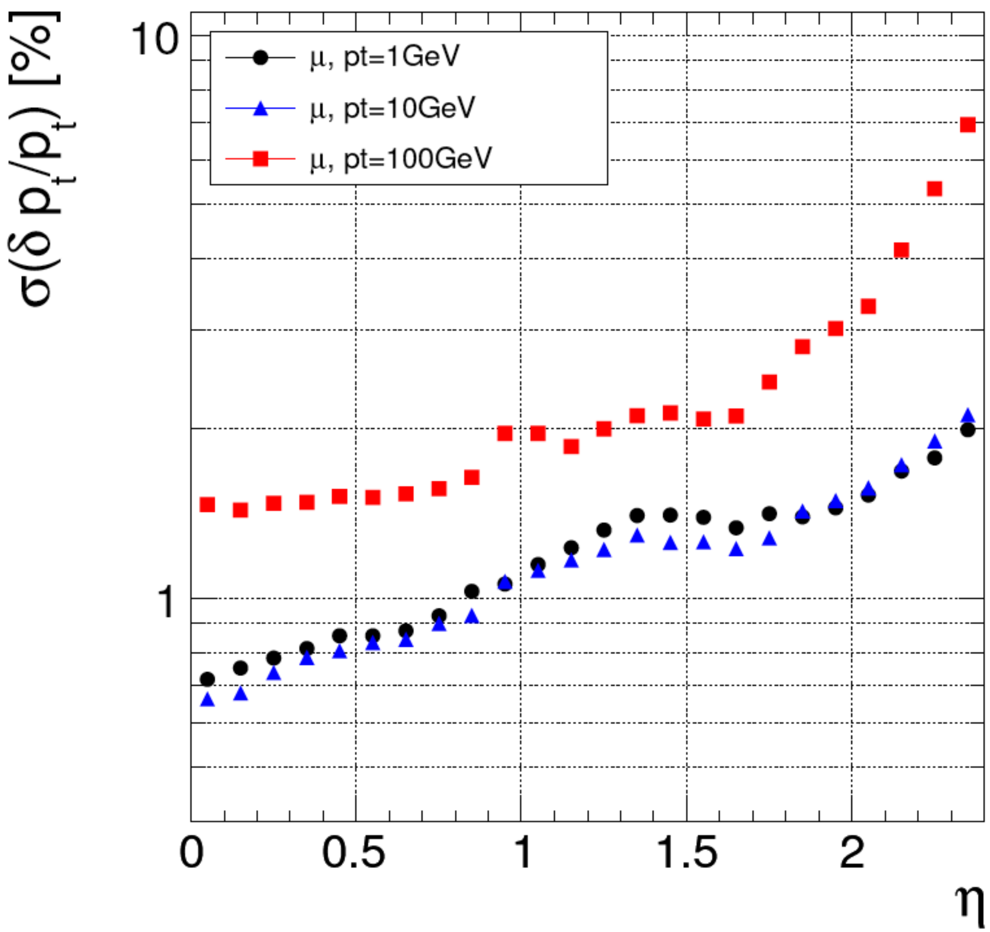
Example results from **full** simulation

From CMS detector paper & Physics TDR

Transverse impact parameter

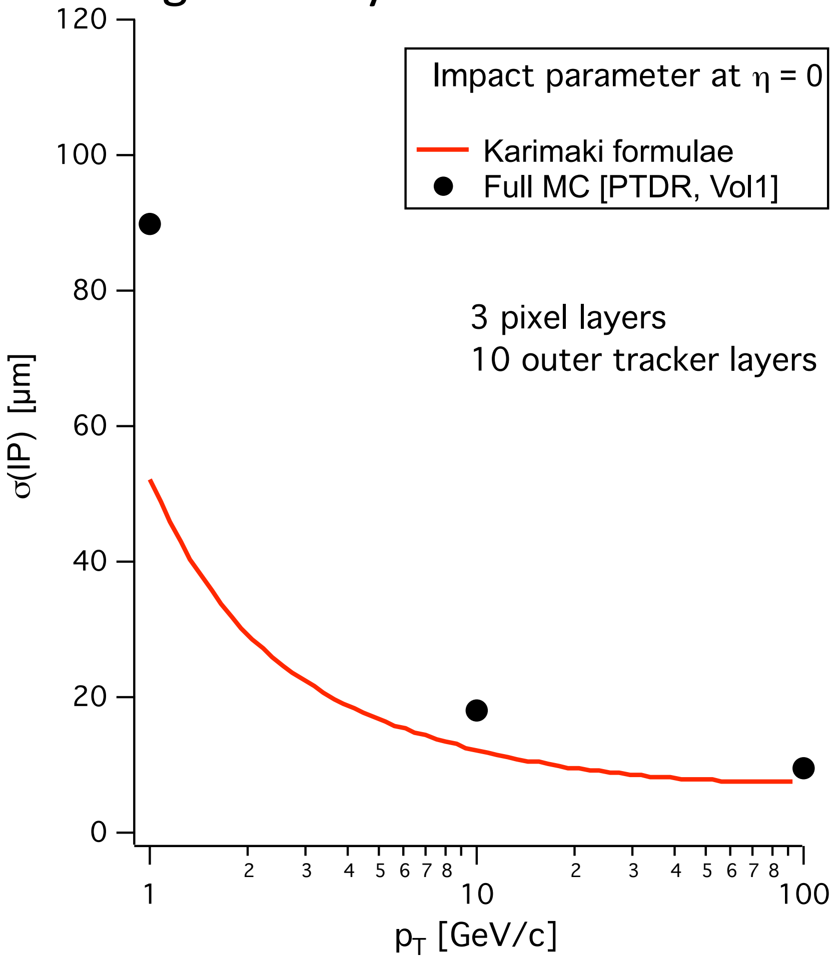


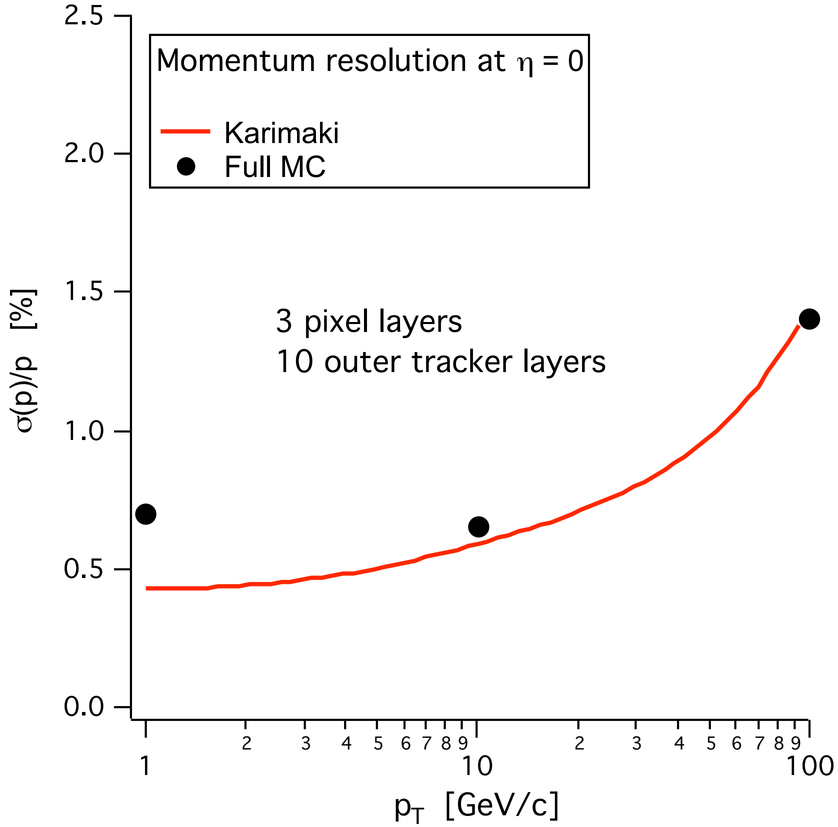
Momentum resolution



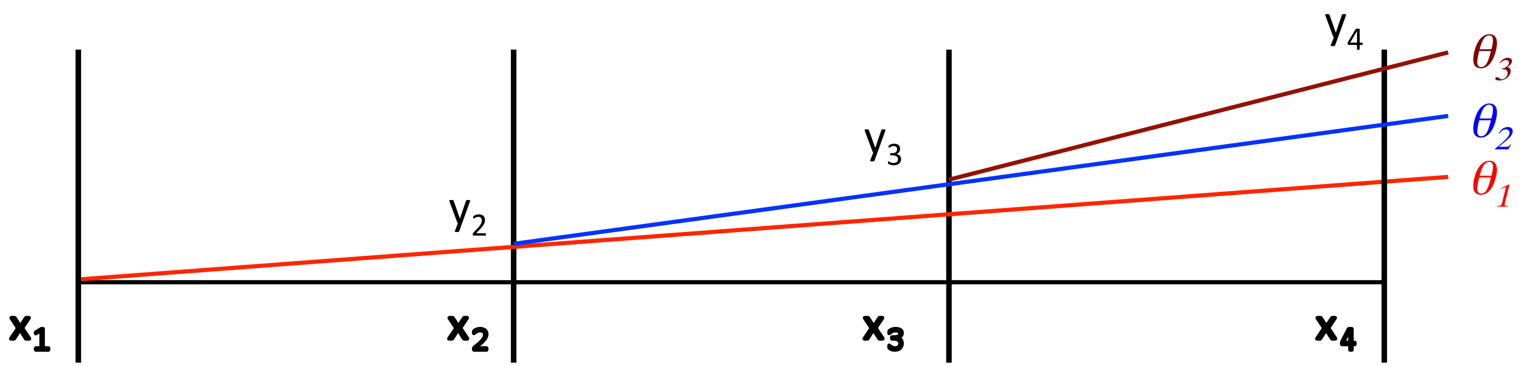
Results at $\eta = 0$

Calculation using Karimäki formulae correctly reproduces results where measurement errors dominate, but not in multiple scattering region. Why?





Correlations from multiple scattering



- $C_{11} = <y_1 y_1> = \sigma_1^2$ σ_1 = intrinsic measurement error
- $C_{22} = <y_2 y_2> = <x_{21}\theta_1 x_{21}\theta_1> = x_{21}^2<\theta_1^2>$ (+ σ_2^2 if not negligible, etc)
- $C_{33} = <y_3 y_3> = <(x_{31}\theta_1 + x_{32}\theta_2)(x_{31}\theta_1 + x_{32}\theta_2)> = x_{31}^2<\theta_1^2> + x_{32}^2<\theta_2^2>$
- $C_{21} = <y_2 y_1> = 0$
- $C_{23} = <y_2 y_3> = <x_{21}\theta_1(x_{31}\theta_1 + x_{32}\theta_2)> = x_{21}x_{31}<\theta_1^2>$

  etc
- Does inclusion of off-diagonal terms affect result?

Simple example

- 3 equally spaced, identical planes, Δx = δ, multiple scattering dominates
- Straight line case. Minimise $\chi^2 = \sum_{i,j} \varepsilon_i C_{ij}^{-1} \varepsilon_j$ $\qquad x_1 = 0,\ x_2 = \delta,\ x_3 = 2\delta$

Covariance matrix C = $\begin{bmatrix} \sigma_1^2 & C_{12} & C_{13} \\ C_{12} & \sigma_2^2 & C_{23} \\ C_{13} & C_{23} & \sigma_3^2 \end{bmatrix}$ $\qquad \sigma^2(\theta) = \sum_{i,j} \frac{\partial\theta}{\partial y_i} C_{ij}^{-1} \frac{\partial\theta}{\partial y_j} = \frac{1}{\sum x_i x_j C_{ij}^{-1}}$

- without correlations: C & C^{-1}

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \delta^2\langle\theta^2\rangle & 0 \\ 0 & 0 & 5\delta^2\langle\theta^2\rangle \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_1^2} & 0 & 0 \\ 0 & \frac{1}{\delta^2\langle\theta^2\rangle} & 0 \\ 0 & 0 & \frac{1}{5\delta^2\langle\theta^2\rangle} \end{bmatrix}$$

$$\sigma^2(\theta) = \frac{5}{9}\langle\theta^2\rangle$$

with correlations: C & C^{-1}

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \delta^2\langle\theta^2\rangle & 2\delta^2\langle\theta^2\rangle \\ 0 & 2\delta^2\langle\theta^2\rangle & 5\delta^2\langle\theta^2\rangle \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_1^2} & 0 & 0 \\ 0 & \frac{5}{\delta^2\langle\theta^2\rangle} & \frac{-2}{\delta^2\langle\theta^2\rangle} \\ 0 & \frac{-2}{\delta^2\langle\theta^2\rangle} & \frac{1}{\delta^2\langle\theta^2\rangle} \end{bmatrix}$$

$$\sigma^2(\theta) = \langle\theta^2\rangle$$

at least proves that correlations can't be ignored

Errors with correlation terms

- To calculate covariance matrix W and inverse in full, replace

$$W_{kl} = \sum_i w_i \frac{\partial \varepsilon_i}{\partial \alpha_k} \frac{\partial \varepsilon_i}{\partial \alpha_l} \qquad \text{with} \qquad W_{kl} = \sum_{i,j} \frac{\partial \varepsilon_i}{\partial \alpha_k} C_{ij}^{-1} \frac{\partial \varepsilon_j}{\partial \alpha_l}$$

Terms needed already provided by Karimäki:

$$\frac{\partial \varepsilon_i}{\partial \rho} = \tfrac{1}{2} r_i^2 \qquad \frac{\partial \varepsilon_i}{\partial \phi} = -x_i \qquad \frac{\partial \varepsilon_i}{\partial d} = 1 + \rho y_i$$

- The solutions are the diagonal terms of W^{-1}

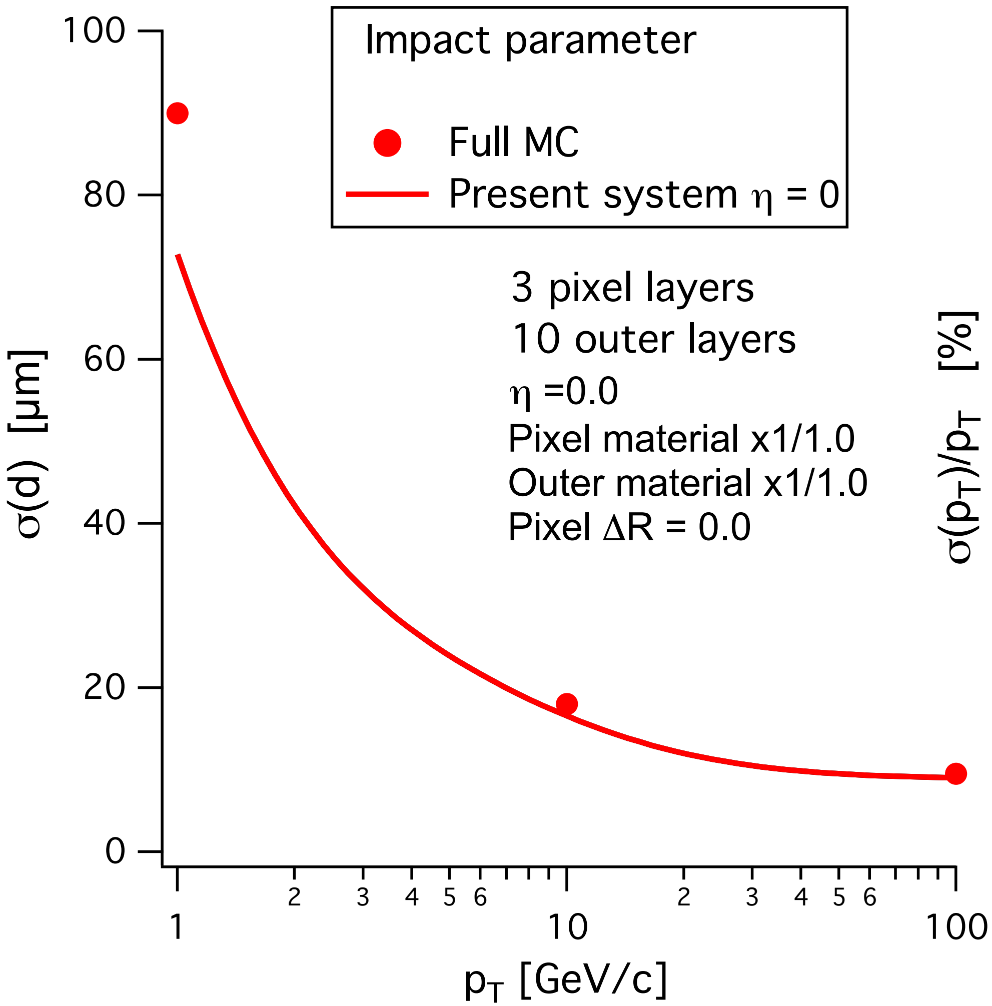
$$\sigma^2(\rho) = W_{11}^{-1} \qquad \sigma^2(\phi) = W_{22}^{-1} \qquad \sigma^2(d) = W_{33}^{-1}$$

- with the origin chosen to be on the beam line
  - The beam pipe is a dummy measurement plane to correctly include its off-diagonal contributions
  - This changed some of the results I have shown previously

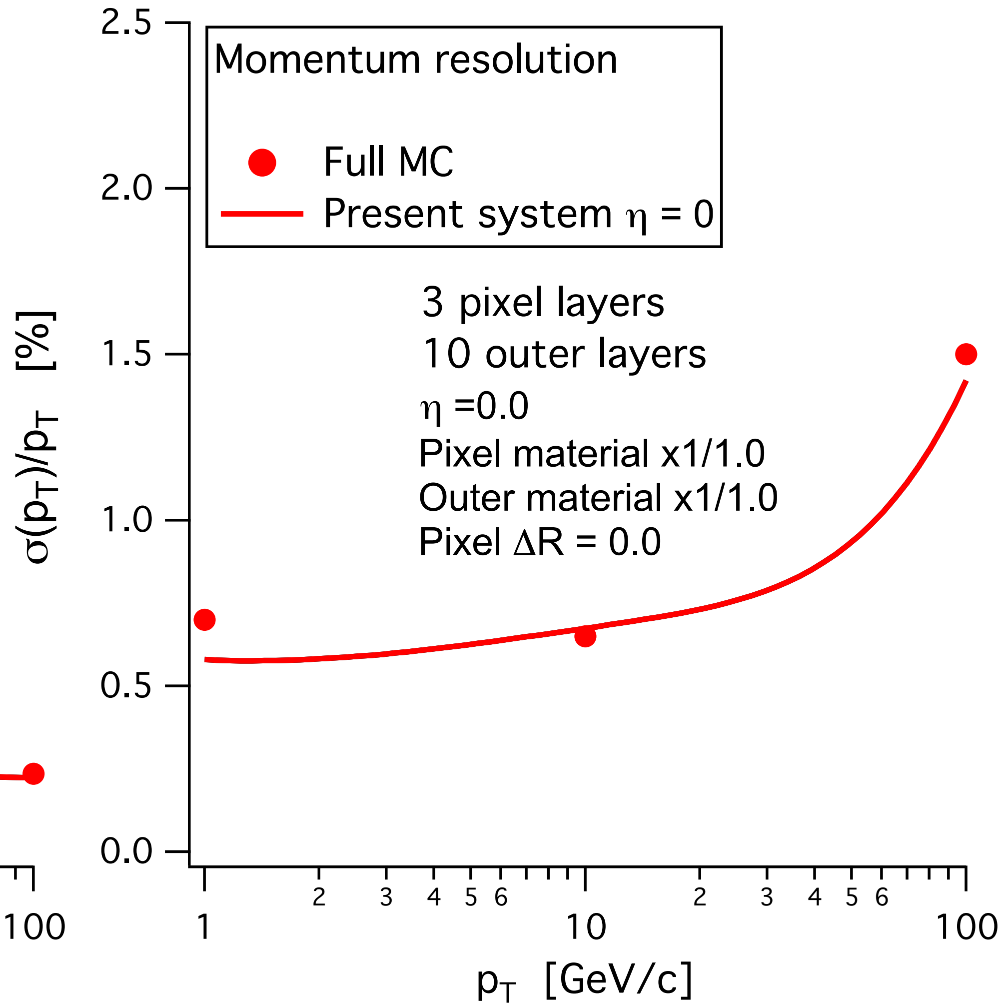
Results from calculation

$\eta = 0$

Transverse impact parameter

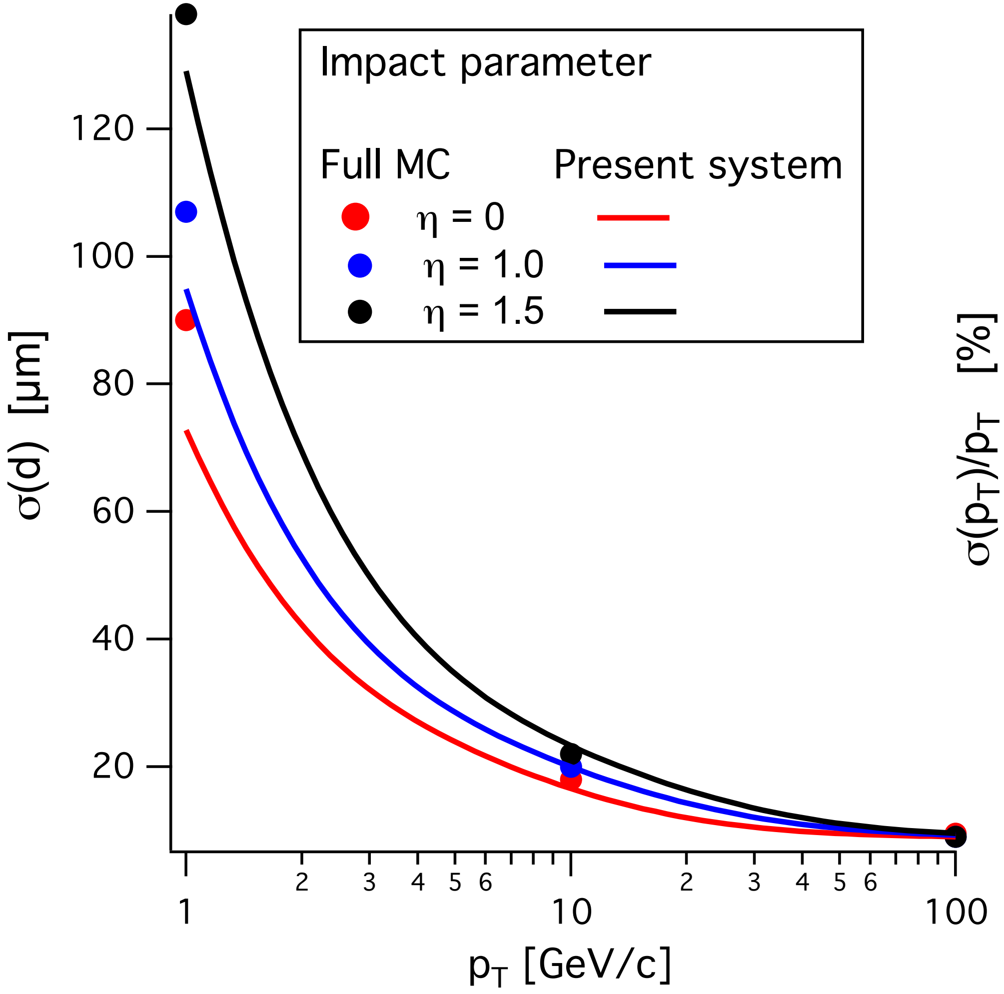


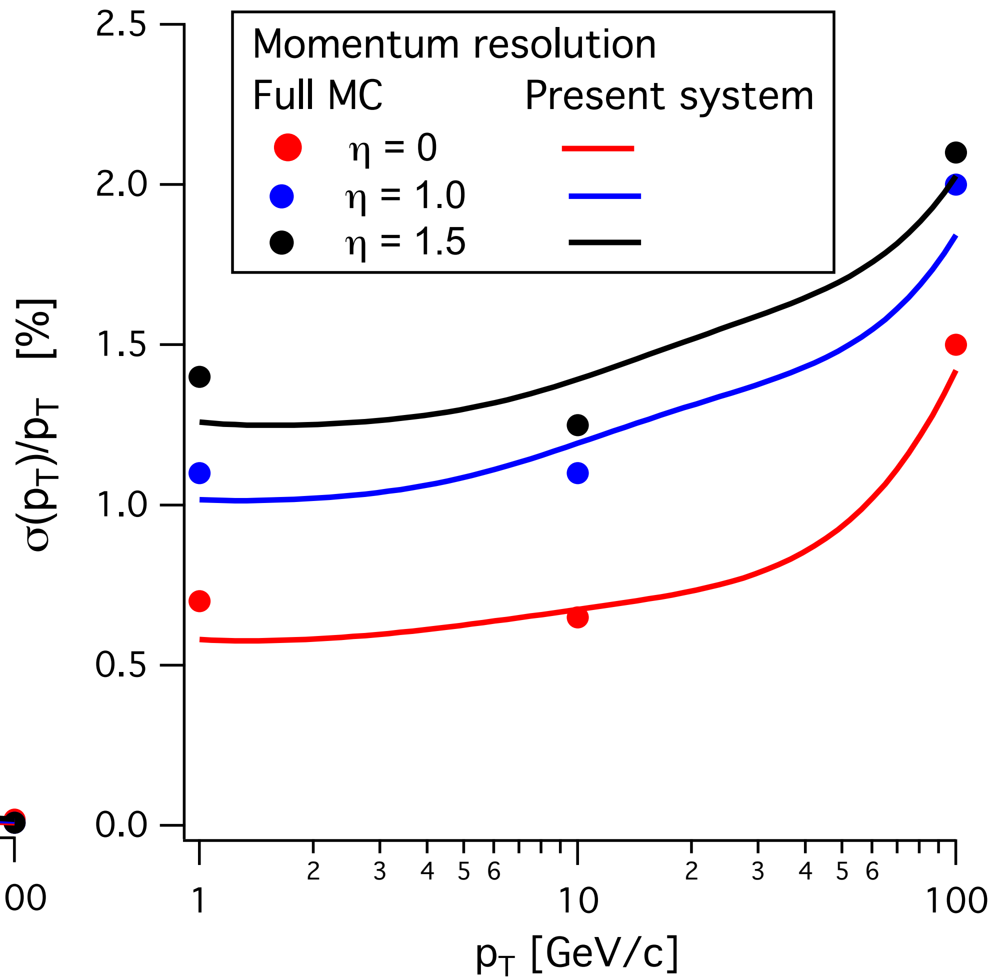
Momentum resolution



Results from calculation (ii)

Added numerical interpolation of material for $\eta > 0$



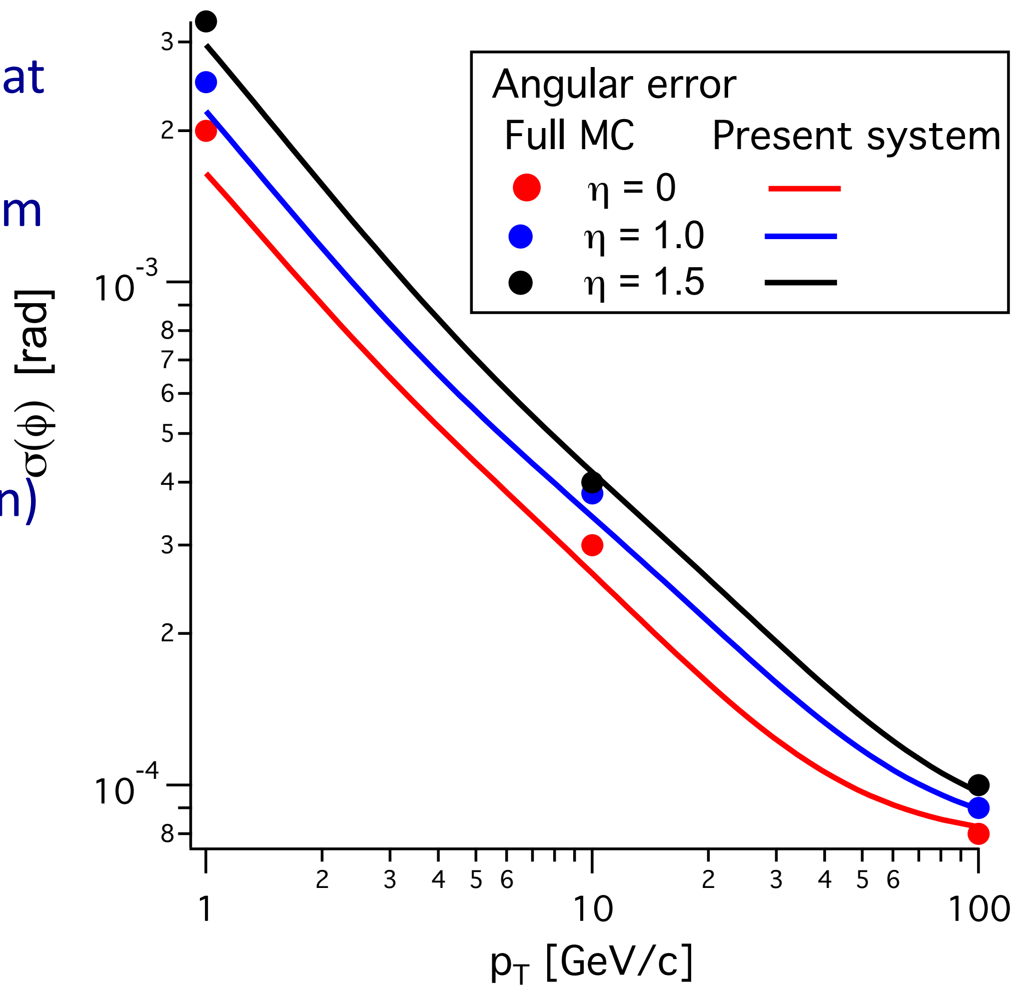


Results from calculation (iii)

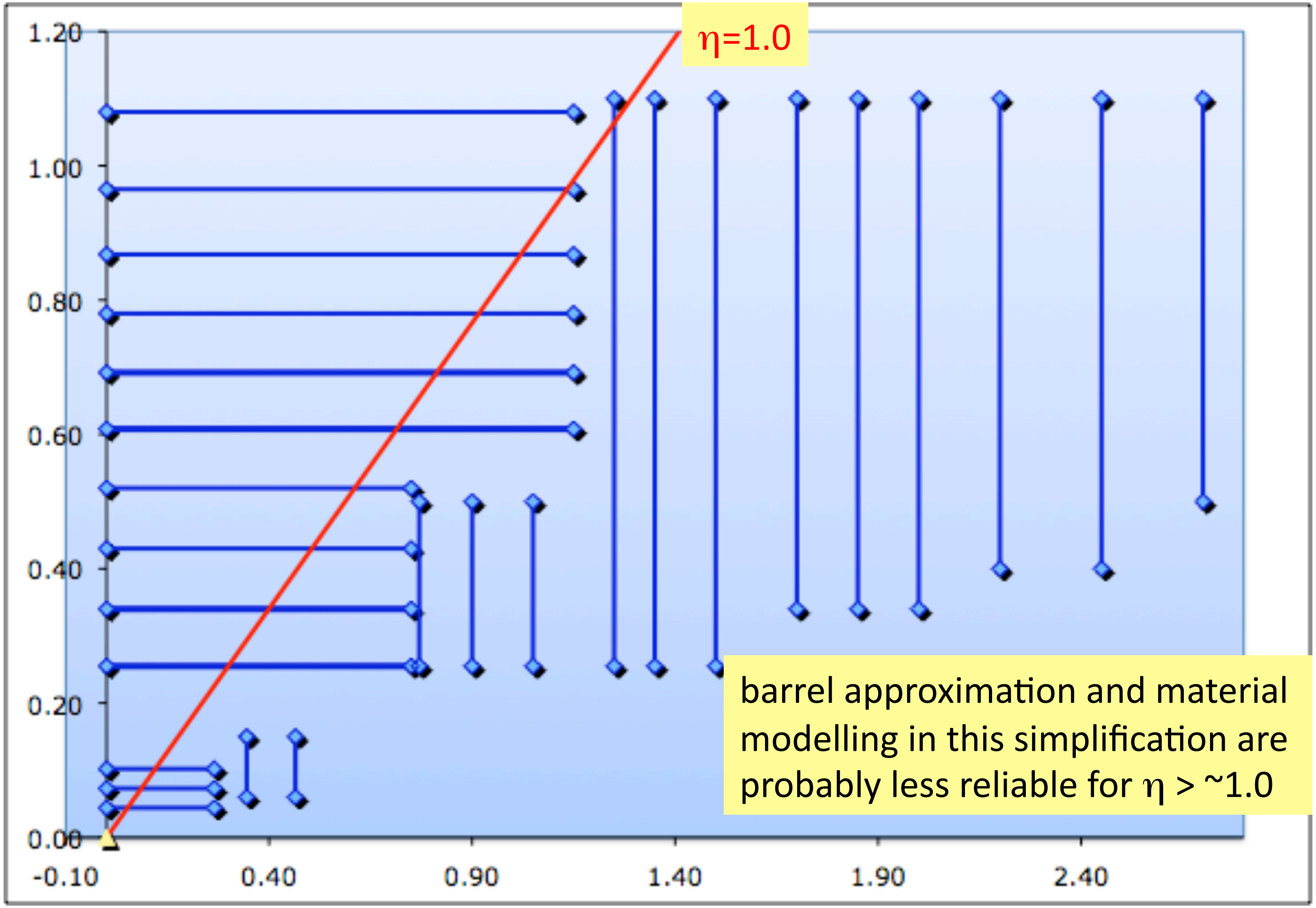
Angular resolution also calculated

NB small but noticeable sensitivity at low p to changes in pixel point resolution in layer 1, eg 10->15µm

However, this is at the limit of the achievable precision in the modelling (even in full simulation)



Scale view of Tracker



Impact error from pixels alone

Physics TDR Vol 1

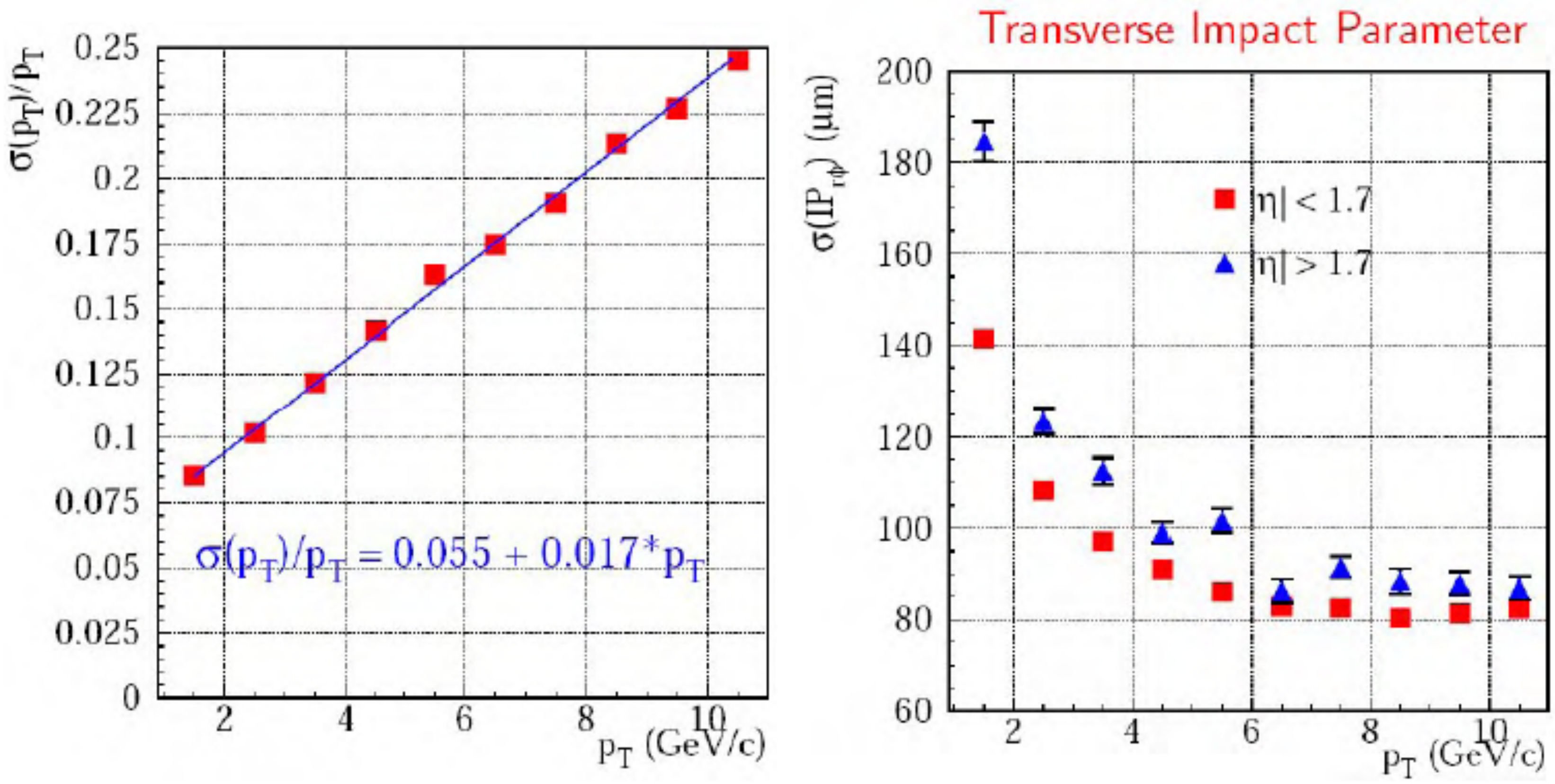
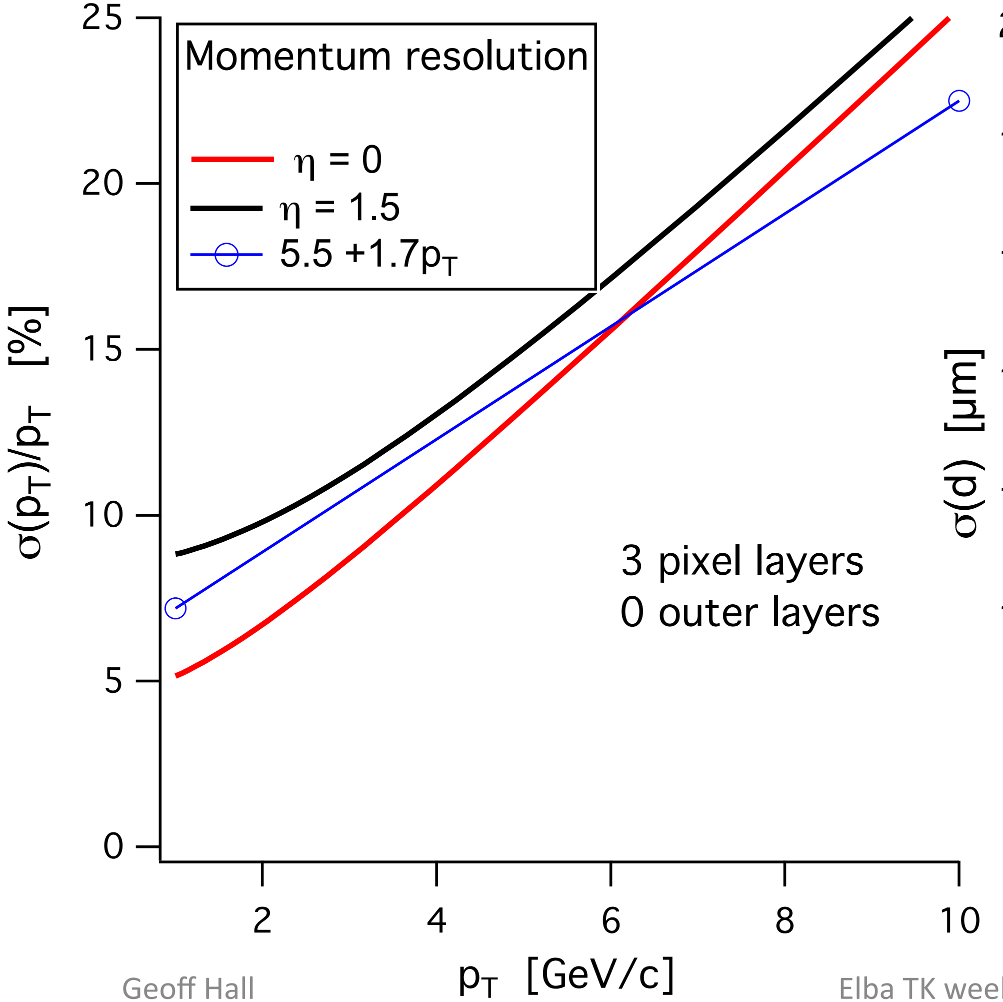
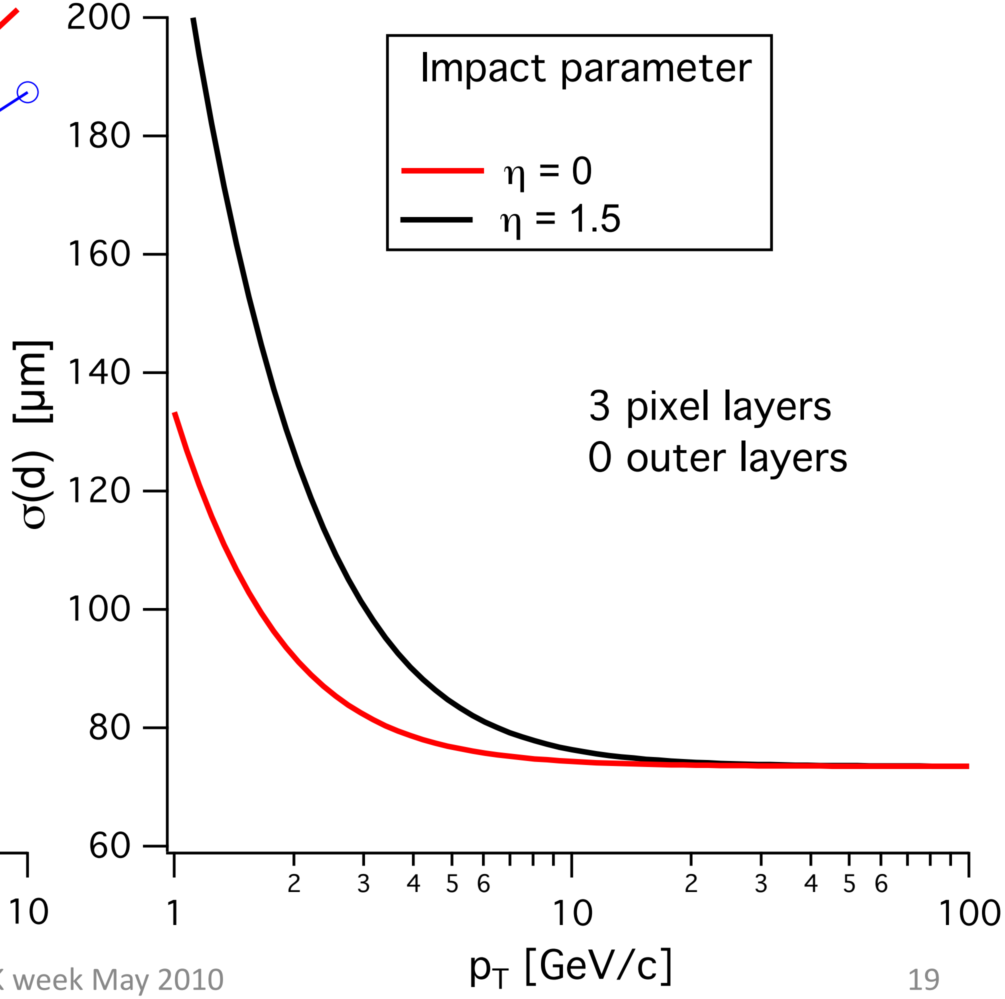


Figure 6.17: Transverse momentum (left) and transverse impact parameter (right) resolution for single muon tracks, created directly from hit triplets.

Present pixel stand alone

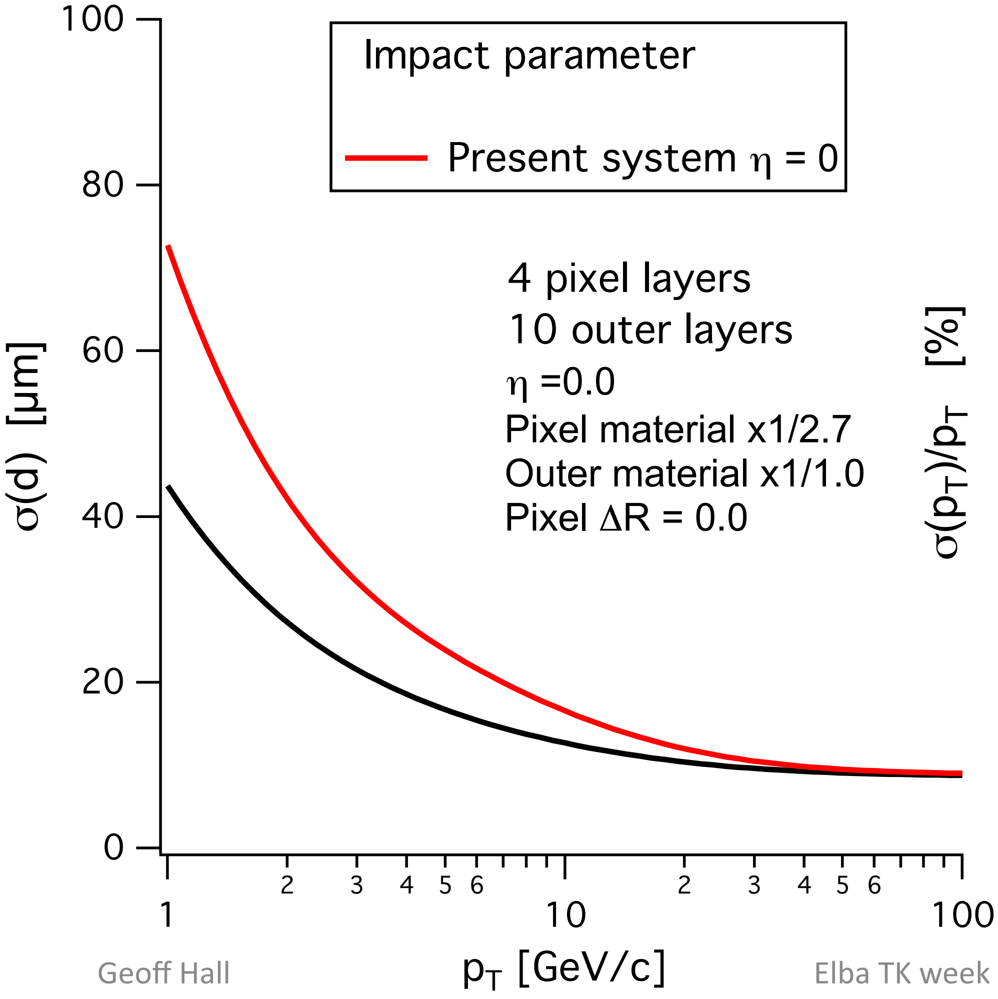
- Quite good agreement in both IP and momentum resolution
  - Proper matrix treatment removes discrepancy I thought due to hit merging

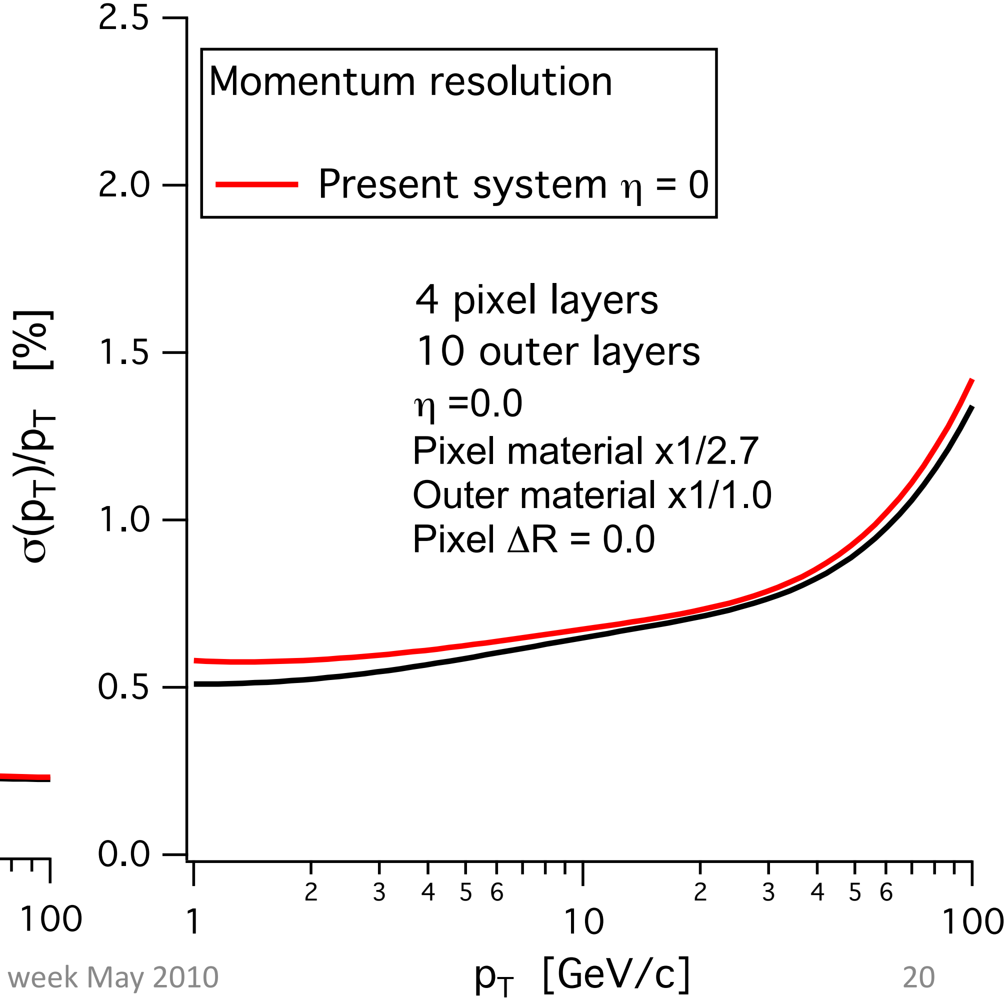




Preview of future

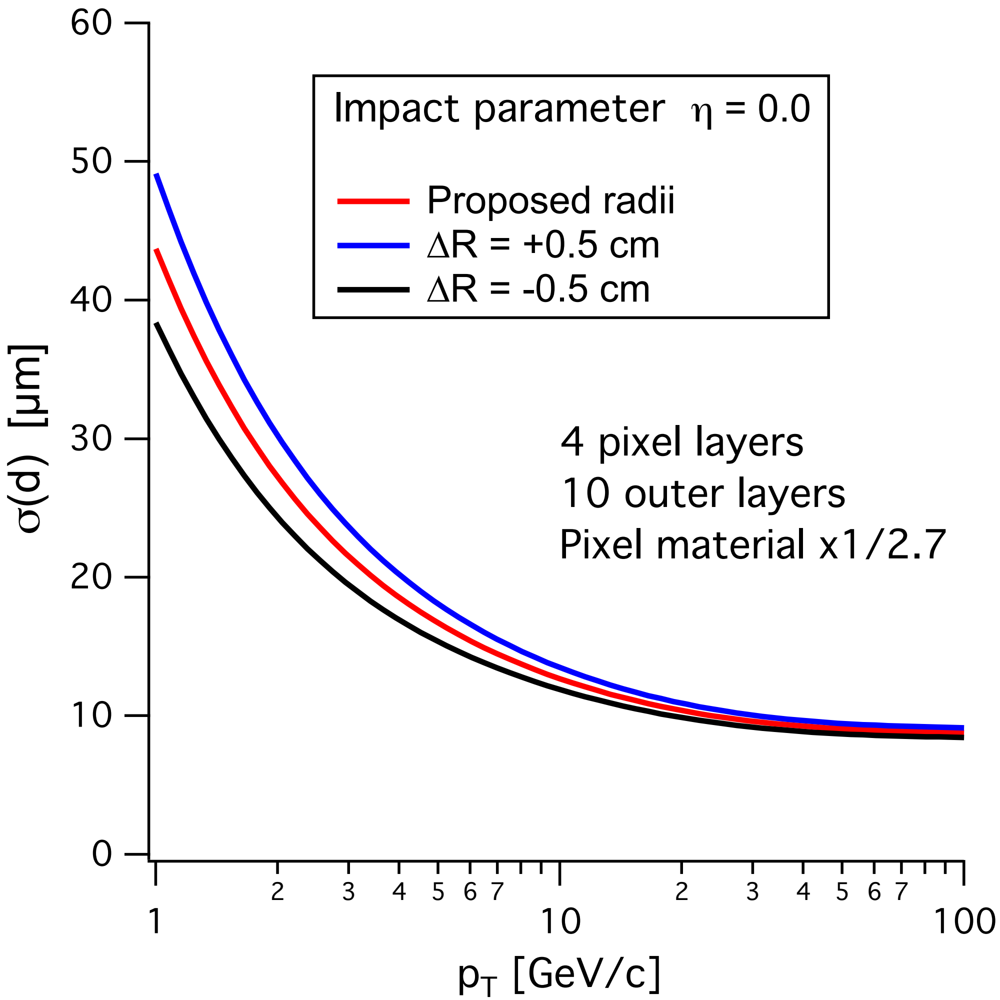
- Some indication of benefits of 4 layer system
  - reduced radii: 2.5 cm beam pipe, pixels: 3.9, 6.8, 10.9, 16.0 cm
  - pixel material reduced by factor 2.7 compared to present system





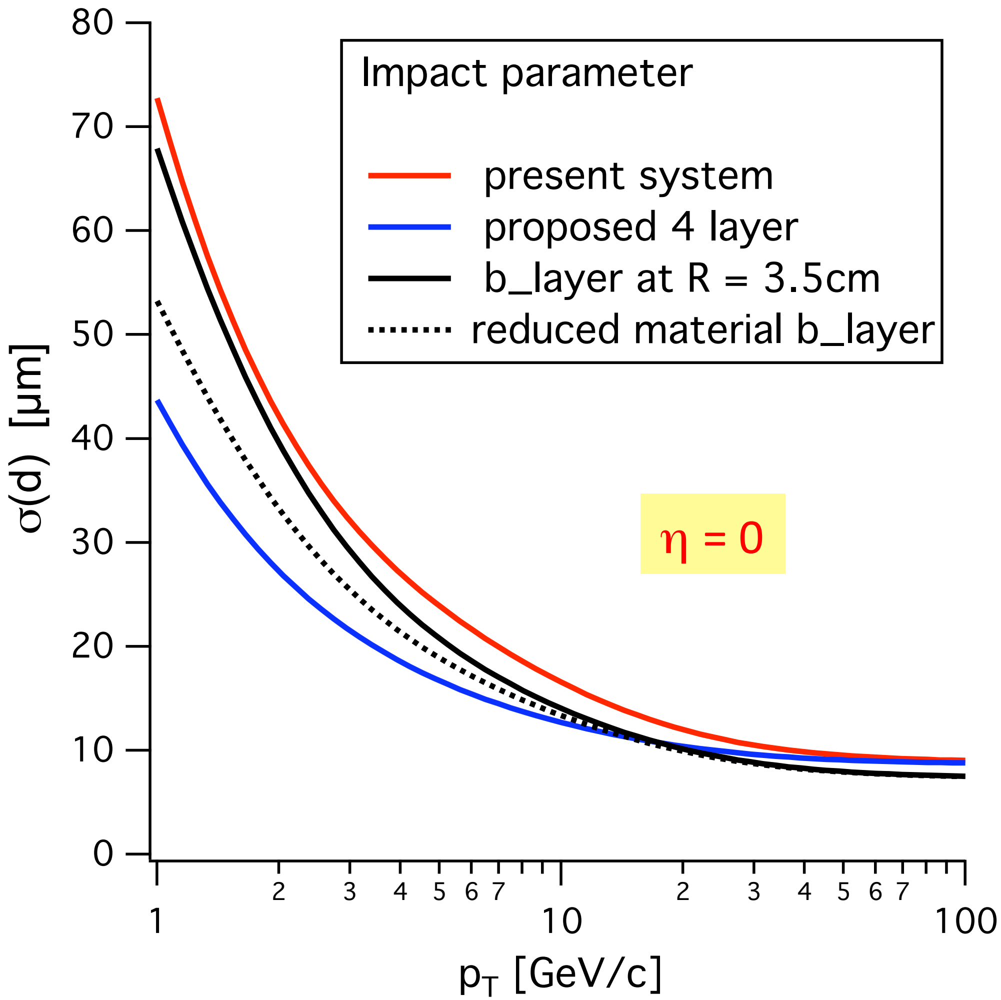
Effect of shrinking radially?

- All layers move in or out by fixed amount
- If material reduction is as expected, then further benefits from reducing radial dimensions appear to be limited



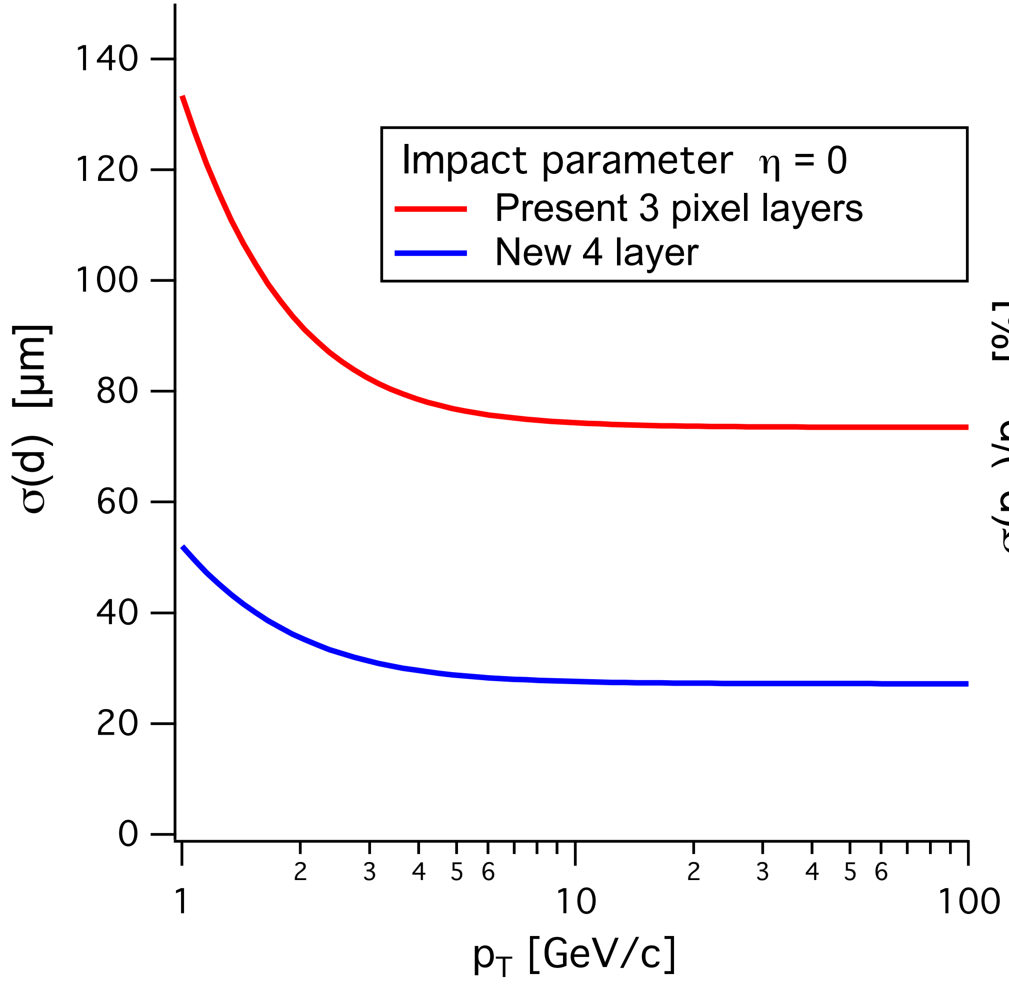
Use of b-layer

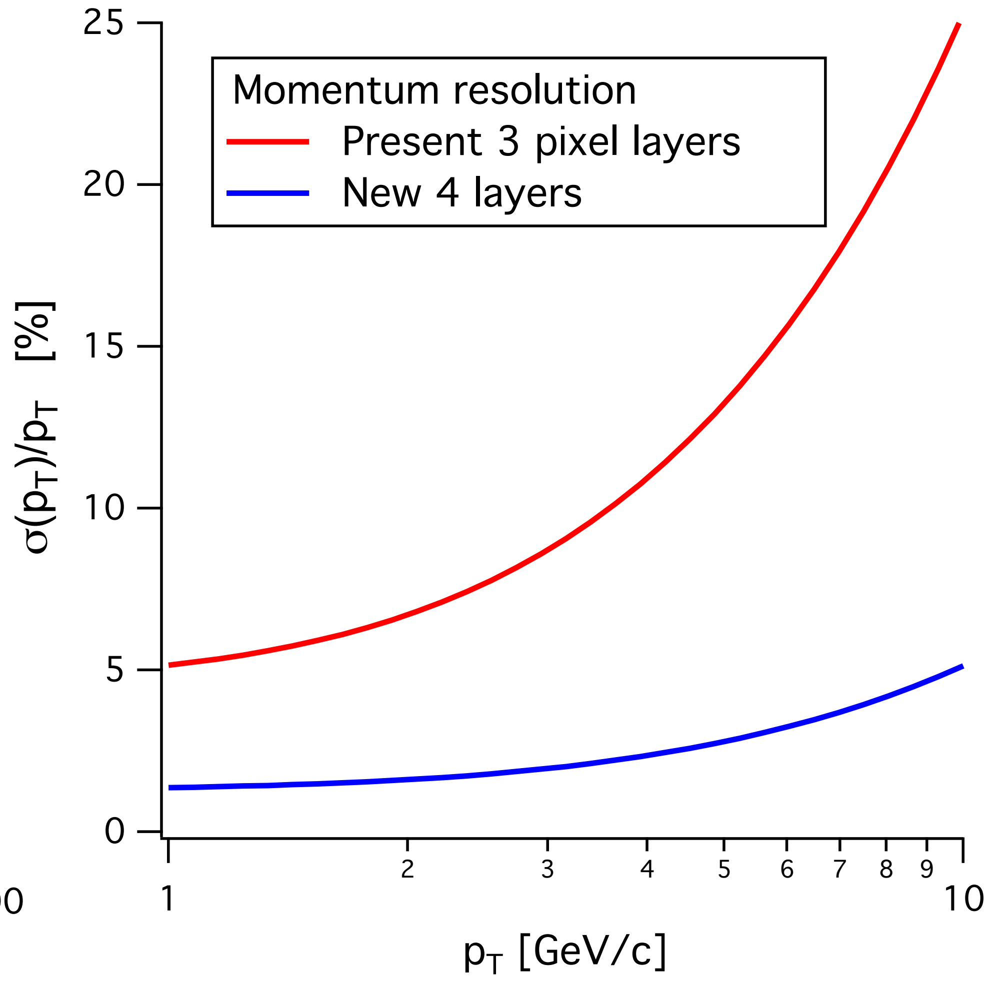
- Suppose we tried ATLAS approach?
- Assume b-layer could be inserted at R= 3.5cm
  - and other layers remain at:
  - R = 4.4, 7.3, 10.0
- Two variants of b-layer
  - with present material
  - material reduced by factor 2.7
  - very small improvement for higher p_T is because of 2 inner points at R = 3.5 & 4.4
  - disappears if pixel layer at 4.4 dies



Stand alone pixels

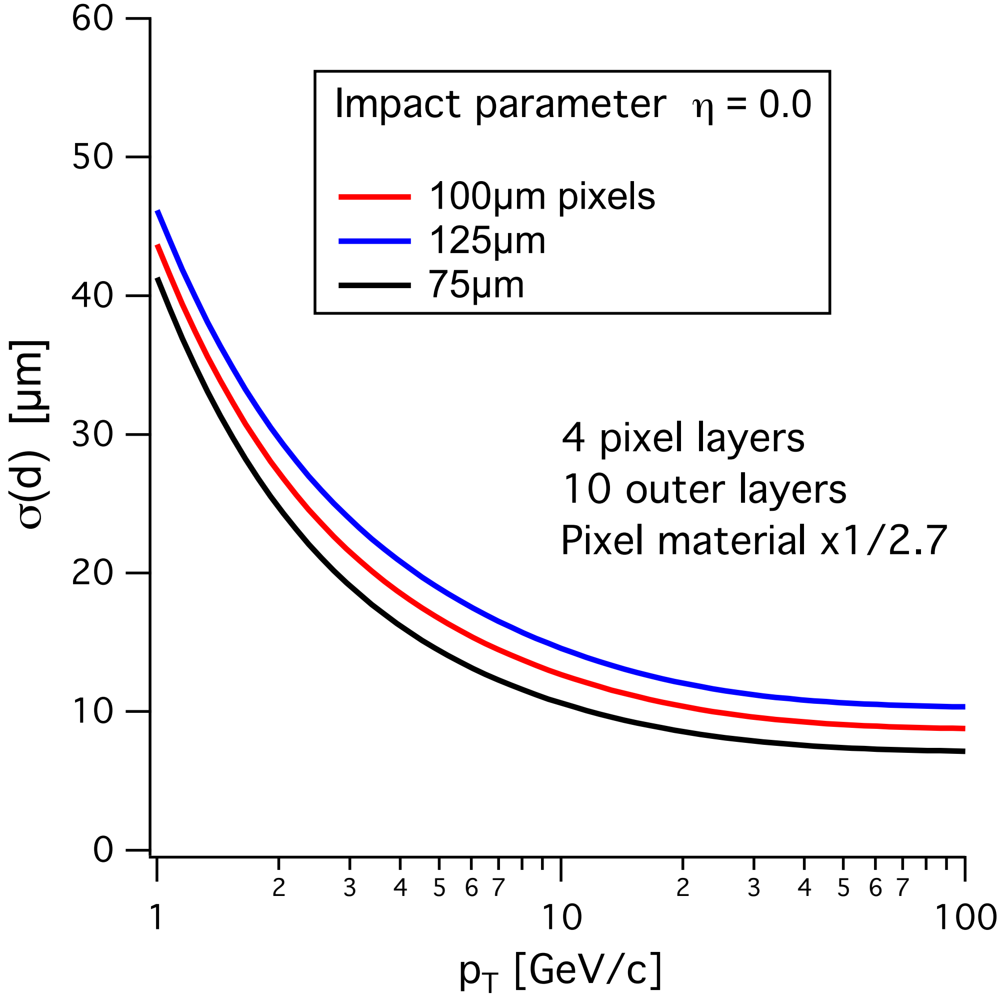
- Also benefit, if useful…





Smaller pixels?

- Assume r-ϕ resolution scales with pixel size
  - also indicates effect of degradation due to radiation damage
  - scope for further detailed evaluation if useful



Conclusions

- Full covariance matrix calculation of errors gives good insight into possible layout variations
  - basically, emulates real life or full Monte Carlo
  - essential where multiple scattering errors are important
  - allows very fast comparison between different options, including adding/ removing layers, changing material, etc
  - calculations can play a role in optimising and exploring tracker designs
  - can be added to Layout Tool for many more details to be evaluated
- Material degrades performance (you will be surprised to learn!)
  - in pixels degrades mainly impact parameter
  - in outer tracker affects mainly momentum resolution
  - innermost radial point is most important
- Many more interesting cases can be studied
  - and complements full simulations (of fewer options)

For reference: Covariance matrix W

$$W_{kl} = \sum_{i,j} \frac{\partial \varepsilon_i}{\partial \alpha_k} C_{ij}^{-1} \frac{\partial \varepsilon_j}{\partial \alpha_l}$$

$$\frac{\partial \varepsilon_i}{\partial \rho} = \tfrac{1}{2} r_i^2 \qquad \frac{\partial \varepsilon_i}{\partial \phi} = -x_i \qquad \frac{\partial \varepsilon_i}{\partial d} = 1 + \rho y_i$$

$$W_{11} = \sum_{i,j} \tfrac{1}{4} r_i^2 r_j^2 C_{ij}^{-1} \qquad W_{22} = \sum_{i,j} x_i x_j C_{ij}^{-1} \qquad W_{33} = \sum_{i,j} (1 + \rho y_i)(1 + \rho y_j) C_{ij}^{-1}$$

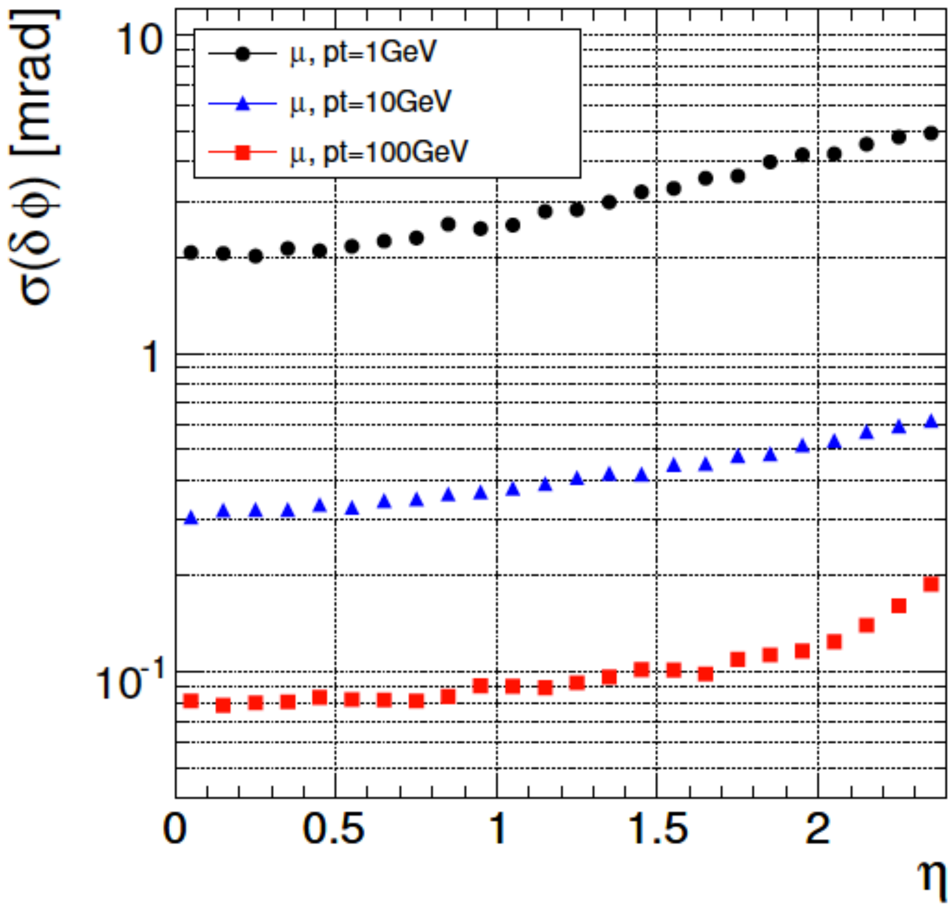
$$W_{12} = \sum_{i,j} -\tfrac{1}{2} r_i^2 x_j C_{ij}^{-1} \quad W_{13} = \sum_{i,j} \tfrac{1}{2} r_i^2 (1 + \rho y_j) C_{ij}^{-1} \quad W_{23} = \sum_{i,j} -x_i (1 + \rho y_j) C_{ij}^{-1}$$

$$\sigma^2(\rho) = W_{11}^{-1} \qquad \sigma^2(\phi) = W_{22}^{-1} \qquad \sigma^2(d) = W_{33}^{-1}$$

Impact error from full simulation

From CMS detector paper & Physics TDR v1

Angular resolution



Longitudinal impact error