Model for Pion Production in Proton-Nucleus Interactions

N. V. Mokhov and S. I. Striganov⁺

Fermi National Accelerator Laboratory, Batavia, IL 60510¹ ⁺Now at Institute for High-Energy Physics, Protvino, Moscow region, Russia²

Abstract. A new phenomenological model has been developed to describe pion production in high-energy proton-nucleus interactions. Special attention is paid to low-momentum pions (0.1< p <2 GeV/c) for intermediate proton momenta 5< p_0 <30 GeV/c. It is shown that the model predictions are in an excellent agreement with data in the entire kinematic region. Comparisons to other models are also presented. The model is embedded into the MARS13 code.

INTRODUCTION

Reliable prediction of pion yield in hadron-nucleus (hA) collisions is vital in numerous applications, particularly in the planning of future experiments and accelerators. The newest examples include a $\mu^+\mu^-$ collider project [1] and neutrino experiments at Fermilab Main Injector [2] and Booster [3]. There are a few models capable of generating pions in $pA \to \pi^{\pm}X$ reactions, e. g., [4-9]. Theoretical calculations based on the intranuclear cascade model are reliable at proton momenta $p_0 < 5 \text{ GeV/c}$, but drastically overestimate hadron yield at higher energies. Microscopic models, such as DPMJET [5] (based on the dual topological unitarization approach) and FRITIOF [6] (based on the LUND model) were developed mainly for high energies $\gtrsim 50$ GeV/c. As it is shown in [1], there is an uncertainty up to a factor of 5 in the pion yield at p < 1 GeV/c on heavy nuclei for proton momenta $5 < p_0 < 30 \text{ GeV/c}$ - the region that is especially interesting for the $\mu^+\mu^-$ collider project. On the other hand, there are many data on inclusive charged pion production in hA collisions obtained over the last three decades. Based on those data and our original model [4.9], we develop a phenomenological model for a reliable description of inclusive pion production in the entire kinematic range for pA collisions at 5 GeV/c $< p_0 < 10$ TeV/c.

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PHENOMENOLOGICAL MODEL

Many reliable data and parameterizations exist on pion yield in *pp*-collisions. We can compensate for the lack of data for *pA* reactions by using the following form (see, e. g., [4,9]) for the double differential cross section of the $pA \to \pi^{\pm}X$ reaction:

$$\frac{d^2 \sigma^{pA \to \pi^{\pm} X}}{dp d\Omega} = R^{pA \to \pi^{\pm} X} (A, p_0, p, p_{\perp}) \frac{d^2 \sigma^{pp \to \pi^{\pm} X}}{dp d\Omega}, \tag{1}$$

where p and p_{\perp} are the total and transverse momenta of π^{\pm} , and A is an atomic mass of the target nucleus. The function $R^{pA \to \pi^{\pm}X}$, measured with much higher precision than the absolute yields, is almost independent of p_{\perp} and its dependence on p_0 and p is much weaker than for the differential cross-section itself. Because of rather different properties of pion production on nuclei in the forward $(x_F \gtrsim 0)$ and backward $(x_F \lesssim 0)$ hemispheres, where x_F is the Feynman's longitudinal variable, we treat these two regions differently.

R at $x_F \gtrsim 0.05$. In this region we assume $R^{pA \to \pi^{\pm}X} \sim A^{\alpha}$. The power α is almost independent of the pion sign. The following parameterization was proposed in [10] for $p_0 \geq 70$ GeV/c:

$$\alpha_g = 0.8 - 0.75 \cdot x_F + 0.45 \cdot x_F^3 / |x_F| + 0.1 \cdot p_\perp^2.$$
⁽²⁾

Fig. 1(a) shows our compilation of data [11–15] on α for π^- -production. It turns out that (2) describes data very well at $p_0 \geq 24$ GeV/c and can be successfully used at lower momenta ($5 \leq p_0 \leq 24$ GeV/c) if it is replaced with (see Fig. 1(a)):

$$\alpha = \alpha_q - 0.0087 \cdot (24 - p_0). \tag{3}$$

The $R^{pA \to \pi^{\pm}X} \sim A^{\alpha}$ form doesn't extrapolate well to A=1 because of the difference ence in the π -yield in proton-proton and proton-neutron collisions. This difference can be taken into account if one uses the following form for $R^{pA \to \pi^{\pm}X}$ [10]:

$$R^{pA \to \pi^{\pm}X} = (\frac{A}{2})^{\alpha} \cdot f(p_0, Y),$$
(4)

where $f(p_0, Y) = \frac{d\sigma}{dp}(pd \to \pi^{\pm})/\frac{d\sigma}{dp}(pp \to \pi^{\pm})$. It turns out that pion yields in pd and pp collisions are not very different, i. e. $f(p_0, Y) \approx 1$. Using FRITIOF results, we found that $f(p_0, Y)_{\pi^-} = 1 + 0.225/N_{\pi^-} - a_{\pi^-} \cdot Y_{cms}$, where N_{π^-} is mean π^- multiplicity in pp collisions and Y_{cms} is pion rapidity in the center-of-mass system (CMS). Data [16] show linear dependence of N_{π^-} on free energy $W = \frac{(\sqrt{s}-2\cdot m_p)^{0.75}}{\sqrt{s}^{0.25}}$, where \sqrt{s} is the CMS collision energy. Our fit to the data gives $N_{\pi^-} = 0.81 \cdot (W - 0.6)$. The other parameter $a_{\pi^-} = 0.16$ for $p_0 \leq 20$ GeV/c, and depends on energy for higher momenta as $a_{\pi^-} = -0.055 + 0.747/log(s)$. $f(p_0, Y)_{\pi}$ is forced to be 1 if it becomes less than 1. For π^+ production the approximation is

much simpler $f(p_0, Y)_{\pi^+} = 0.85 + 0.005 \cdot p_0$ for $p_0 \le 30$ GeV/c and $f(p_0, Y)_{\pi^+} = 1$ for higher momenta.

R at $x_F \leq 0.05$. In this region, due to the lack of experimental data on α , we use the following expression for the function R in (1):

$$R^{pA \to \pi^{\pm} X} = \frac{dN/dY(pA)}{dN/dY(pp)}$$
(5)

The following scaling law was proposed in [17] for charged shower particle ($\beta > 0.7$) production in pA collisions at 20 – 400 GeV/c:

$$\frac{Y_0}{\langle N_s \rangle} \cdot \frac{dN}{d\eta} = f(A, \frac{\eta}{Y_0}),\tag{6}$$

where $\langle N_s \rangle$ is a mean multiplicity of shower particles, Y_0 is rapidity of primary proton and $\eta = -log(tan(\theta/2))$ is pseudorapidity of a secondary particle. We found that this approximation is in a reasonable agreement with data at $p_0 > 7.5$ GeV/c [18]. Unfortunately, η is not a convenient variable to describe forward pion production ($\theta \approx 0$). Our analysis of the $pA \rightarrow \pi^- X$ data [19,20] at $10 < p_0 < 100$ GeV/c shows that replacing η in (6) with rapidity Y

$$\frac{dN}{dY} = \frac{\langle N_{\pi} \rangle}{Y_0} \cdot F(A, \frac{Y}{Y_0}) \tag{7}$$

provides better description of the pion yield in the entire kinematic range. Here $\langle N_{\pi} \rangle$ is mean pion multiplicity, $Y_0 = log(\frac{E_0 + p_0}{m_p})$ is rapidity of incident proton and $Y = log(\frac{E_{\pi} + p_z}{m_{\perp}})$ is π rapidity, $m_{\perp} = \sqrt{p_{\perp}^2 + m_{\pi}^2}$. We choose the Gaussian form for the scaling function:

$$F(A, \frac{Y}{Y_0}) = c_1 \cdot exp(-(\frac{Y}{Y_0} - c_2)^2/c_3), \tag{8}$$

where for π^- : $c_1 = 1.149 \cdot A^{0.0479}$, $c_2 = 0.492 \cdot A^{-0.0565}$, and $c_3 = 0.214 \cdot A^{-0.121}$. Reliable rapidity distributions for π^+ at $x_F < 0$ are measured only for $p_0 \ge 100$ GeV/c. Assuming that the scaling (7)-(8) is valid for π^+ also, we found the following parameters from data [20]: $c_1=1.6$, $c_2 = 0.521 \cdot A^{-0.0416}$, and $c_3=0.12$. The data on dN/dY for $pp \rightarrow \pi^{\pm}X$ reaction is well described by a Gaussian [21]: $dN/dY = C_{pp} \cdot exp(-Y_{cn}^2/2\sigma^2)$, where $\sigma_{\pi^+} = 0.402 + 0.198 \cdot log(p_0)$ and $\sigma_{\pi^-} = 0.465 + 0.157 \cdot log(p_0)$. The normalization parameter in (5), combined of $< N_{\pi} >$, C_{pp} etc, is chosen to match the functions (4) and (5) at $x_F=0.05$.

 $pp \rightarrow \pi^{\pm} \mathbf{X}$. To describe the invariant cross section of charged pion production in pp collisions we use the form proposed in [22] that we modified at low [13.23] and high [4] p_{\perp} :

$$E\frac{d^3\sigma^{pp\to\pi^{\pm}X}}{dp^3} = A(1-\frac{p^*}{p_{max}^*})^B exp(-\frac{p^*}{C\sqrt{s}})V_1(p_{\perp})V_2(p_{\perp}),\tag{9}$$

TABLE 1. Parameters in formula (9).

	A	В	С	D	Е	F
π^+	60.1	1.9	0.18	0.3	12	2.7
π^{-}	51.2	2.6	0.17	0.3	12	2.7

where p^* and p^*_{max} are pion momentum and maximum momentum transfer in CMS and parameters are given in Table 1. The best description of the p_{\perp} dependence is obtained with:

$$egin{aligned} V_1(p_\perp) &= egin{cases} (1-D)exp(-Ep_\perp^2) + Dexp(-Fp_\perp^2), & p_\perp \leq 0.933 \; \mathrm{GeV/c}, \ 0.2625/(p_\perp^2+0.87)^4, & p_\perp > 0.933 \; \mathrm{GeV/c}, \ V_2(p_\perp) &= egin{cases} 0.7363 \, exp(0.875p_\perp), & p_\perp \leq 0.35 \; \mathrm{GeV/c}, \ 1, & p_\perp > 0.35 \; \mathrm{GeV/c}. \end{aligned} \end{aligned}$$

COMPARISON TO DATA

The model developed agrees very well with available data and reliable DPMJET-II [5] predictions at proton momenta 50 GeV/c< $p_0 <10$ TeV/c (as used in the original model [4,9]). In this section we compare pion yield predicted by the new model and other models with data available at lower proton momenta down to $p_0 \approx 5$ GeV/c on thin and thick nuclear targets. Fig. 1(b) shows comparison of calculated π^- rapidity distributions in pC and pTa collisions at $p_0=10$ GeV/c with



FIGURE 1. (a) Parameter α (2)-(3) calculated for $p_0=6.7$, 12.9, 19.2 and >24 GeV/c (from bottom up) in comparison with data [11-15]; (b) rapidity distributions of π^- in pC and pTa interactions at 10 GeV/c as calculated with FRITIOF, DPMJET and the model developed in this paper in comparison with data [19]; (c) $pCu \rightarrow \pi^- X$ at $p_0=17-19$ GeV/c for $\theta=12.5$, 30, 40, 50, 60 and 70 mrad (from top down), data from [13,25,26].



FIGURE 2. (a) $pCu \rightarrow \pi^- X$ ($\theta \approx 0$), data from [11,12,24–28]; (b) $pCu \rightarrow \pi^+ X$ ($\theta \approx 0$), data from [12,27–29]. Two curves presented in some plots correspond to higher (top) and lower (bottom) p_0 shown in those plots.

data [19]. One sees that our model gives much better results than the DPMJET and FRITIOF codes for soft pions at 0 < Y < 2. Production of energetic pions is also nicely described by our model (Fig. 1(b),(c)). Fig. 2 shows that the developed model gives a reliable description of pions generated at $\theta \approx 0$ in the very 'difficult' region of intermediate momenta. $5 < p_0 < 19$ GeV/c, for other models.



FIGURE 3. Pion spectra at $p_0=24$ GeV/c: (a) $pCu \rightarrow \pi^- X$ at 5 angles, data from [14.26]; (b) $pHg \rightarrow \pi^- X$ and (c) $pHg \rightarrow \pi^+ X$ according to several codes (MARS13(97) with the new model).



FIGURE 4. Double differential spectra from thick targets as calculated with GEANT-FLUKA and MARS13(97) and measured: (a) π^- from 6-in copper target at $p_0=6$ GeV/c, data [25]; (b) π^- and (c) π^+ from 10-cm lead target at $p_0=8$ GeV/c, data [30].

The 'mystery' with the soft pion production began with the analysis of the pHg reaction at 24 GeV/c [1]. As Fig. 3(a) shows, the model developed gives a good agreement with data (calculations for nuclei other than copper nuclei at this energy are not shown but agree very well with data), and gives guidance on the other codes in this considered case (see Fig. 3(b),(c)). Calculations with the MARS13(97) code (with the new model) of the pion double differential spectra with cascading in the thick copper and lead targets at $p_0=6$ and 8 GeV/c agree nicely with data [25,30] in the momentum region $0.1 GeV/c (so crucial for <math>\mu^+\mu^-$ collider applications) (see Fig. 4). At the same time, GEANT, even in the most appropriate FLUKA mode, certainly has some problem here.

CONCLUSION

This model development began while one of us (NM) pointed out that there is a great degree of uncertainty in central ($x_F \approx 0$) pion production on medium and heavy nuclei in the medium proton momentum range $5 < p_0 < 30$ GeV/c [1]. Successful benchmarking, performed with the MARS13(97) code with the new model embedded, against data in the wide kinematic range for nuclei ranging from hydrogen to lead, assure that we have now a tool for reliable prediction of pion yield for the proton momentum range estimated as 5 GeV/c $< p_0 < 10$ TeV/c.

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