

NONUNITARY NEUTRINO OSCILLATIONS

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1. INTRODUCTION

Beyond the standard model physics can:

- change the states of producing and detecting neutrinos,

$$|v_P\rangle = \frac{1}{N} \sum_i A(1 + A \rightarrow v_i + B) |v_i\rangle \text{ where}$$
$$N = \sqrt{\sum_{i=1}^3 |A(1 + A \rightarrow v_i + B)|^2}$$

- give some new interaction for neutrino propagating inside matter

$$H^{\text{eff}} = H^{\text{SM}} + H^{\text{NP}}$$

$$H^{\text{NP}} = \frac{A_e}{2E} \begin{pmatrix} \epsilon_{ee} & \epsilon_{e\mu} e^{i\chi_{e\mu}} & \epsilon_{e\tau} e^{i\chi_{e\tau}} \\ \epsilon_{e\mu} e^{-i\chi_{e\mu}} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} e^{i\chi_{\mu\tau}} \\ \epsilon_{e\tau} e^{-i\chi_{e\tau}} & \epsilon_{\mu\tau} e^{-i\chi_{\mu\tau}} & \epsilon_{\tau\tau} \end{pmatrix}$$

We would like to investigate the new physics effects which can give non-unitary mixing between light neutrinos.

EXAMPLE

The light and heavy neutrinos mix together, and their mixing is described by the $3 + n_R$ unitary matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_e \\ N_\mu \\ N_\tau \end{pmatrix} = \begin{pmatrix} \mathbf{U}_{\alpha i} & \mathbf{V}_{\alpha I} \\ \mathbf{V}'_{A i} & \mathbf{U}'_{A I} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ N_I \\ N_{II} \\ N_{III} \end{pmatrix}$$

- Bilenky, Giunti;
- Grossmann;
- Gonzalez-Garcia,
Grossman, Gusso,
Nir;
- Czakon, Gluza,
Z.M.
- Beckman, Gluza,
Holeczek, Syska,
Z.M.

Three flavour + sterile neutrinos

Matrix $U = (3+n_s) \times (3+n_s)$

$$U_\nu = \begin{pmatrix} U & V \\ V' & U' \end{pmatrix}$$

Matrix $U' = (n_R \times n_R)$

Heavy neutrinos

In this lecture we will not consider specific models which can produce non-decoupling for light neutrino.

$$|\nu_i\rangle \equiv |i\rangle \quad i = 1, 2, 3$$

Eigenmass states in vacuum

$$|\nu_\alpha\rangle \equiv |\alpha\rangle \quad \alpha = e, \mu, \tau$$

Flavour eigenstates. States for production and detection process

$$|\nu_{\bar{i}}\rangle \equiv |\bar{i}\rangle \quad \bar{i} = \bar{1}, \bar{2}, \bar{3}$$

Eigenmass states in matter

All bases are orthonormal, so:

$$|\bar{i}\rangle = \sum_{k=1}^3 \tilde{W}_{\bar{i}k} |k\rangle, \quad |k\rangle = \sum_{\bar{i}=\bar{1}}^{\bar{3}} \tilde{W}_{\bar{i}k}^* |\bar{i}\rangle$$

$$|\alpha\rangle = \sum_{i=1}^3 U_{\alpha i}^* |i\rangle, \quad |i\rangle = \sum_{\alpha=e,\mu,\tau} U_{\alpha i} |\alpha\rangle,$$

$$\mathbf{W}\tilde{\mathbf{W}} = \mathbf{U}$$

$$|\alpha\rangle = \sum_{\bar{i}=\bar{1}}^{\bar{3}} W_{\alpha\bar{i}}^* |\bar{i}\rangle, \quad |\bar{i}\rangle = \sum_{\alpha=e,\mu,\tau} W_{\alpha\bar{i}} |\alpha\rangle$$

Neutrino states in general case

$$|\nu_i\rangle \equiv |i\rangle$$

$$i = 1, 2, 3, \dots, n$$

$$|\nu_{\bar{i}}\rangle \equiv |\bar{i}\rangle$$

$$\bar{i} = \bar{1}, \bar{2}, \bar{3}, \dots, \bar{n}$$

$$|\nu_\alpha\rangle \equiv |\alpha\rangle$$

$$\alpha = e, \mu, \tau, \dots, N_e, N_\mu, N_\tau.$$

$$|\nu_\alpha\rangle = \sum_{i=1}^n (U_\nu^*)_{\alpha i} |\nu_i\rangle = \sum_{i=1}^{3+n_s} U_{\alpha i}^* |\nu_i\rangle + \sum_{i=3+n_s+1}^n V_{\alpha i}^* |\nu_i\rangle$$

Experimentally produced neutrino flavour states:

$$|\tilde{\nu}_\alpha\rangle = \frac{1}{\lambda_\alpha} \sum_{i=1}^{3+n_s} U_{\alpha i}^* |\nu_i\rangle \equiv \sum_{i=1}^{3+n_s} \tilde{U}_{\alpha i}^* |\nu_i\rangle$$

$$\alpha = e, \mu, \tau$$

$$\tilde{U}_{\alpha i} = \lambda_\alpha^{-1} U_{\alpha i}$$

This mixing matrix is not unitary

$$\tilde{U}\tilde{U}^\dagger \neq \mathbf{I}$$

$$\tilde{U}^\dagger\tilde{U} \neq \mathbf{I}$$

$$\langle \tilde{\nu}_\alpha | \tilde{\nu}_\beta \rangle \neq 1 \text{ for } \alpha \neq \beta;$$

$$\langle \tilde{\nu}_\alpha | \tilde{\nu}_\alpha \rangle = 1$$

with

$$\lambda_\alpha = \sqrt{1 - (VV^\dagger)_{\alpha\alpha}} \ .$$

As elements of matrices V are small it is convenient to parameterize

$$U = Y \cdot U ; \quad \tilde{U} = \Lambda Y U; \quad \Lambda = \text{diag}(1/\lambda_e, 1/\lambda_\mu, 1/\lambda_\tau)$$

where U is the standard 3x3 unitary matrix parameterized in the way

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \ .$$

If we parameterize

$$Y = I - \delta Y,$$

then from unitary condition of the full mixing matrix

$$YY^+ = I - VV^+$$

we get

$$\delta Y = \frac{1}{2} \begin{pmatrix} c_{ee} & 0 & 0 \\ 2c_{\mu e} & c_{\mu\mu} & 0 \\ 2c_{\tau e} & 2c_{\tau\mu} & c_{\tau\tau} \end{pmatrix},$$

where

$$c_{\alpha\beta} = (VV^+)_{\alpha\beta}$$

From low energy physics
bounds on $c_{\alpha\beta}$ exist

From charge lepton processes there are bounds on the c parameters

$$(\Gamma_Z)_{\text{invisible}} \rightarrow N_\nu = \frac{\Gamma_{\text{invisible}}}{\Gamma_{\nu\bar{\nu}}} = 2.9840 \pm 0.0082$$

$$\mu \rightarrow e \nu_e \nu_\mu$$

$$\tau \rightarrow \mu \nu_\mu \nu_\tau$$

$$\mu \rightarrow e \gamma$$

$$\pi \rightarrow e \nu_e, \quad \pi \rightarrow \mu \nu_\mu,$$

$$\mu \rightarrow e \nu_e \nu_\mu,$$

$$\tau \rightarrow \mu \nu_\mu \nu_\tau,$$

K_{13} decay,

β decay,

$$c_{ee}^2 \leq 0.0054;$$

$$c_{\mu\mu}^2 \leq 0.0094;$$

$$c_{\tau\tau}^2 \leq 0.016;$$

$$|c_{e\mu}|^2 \leq 0.0001;$$

$$|c_{\mu\tau}|^2 \leq 0.01.$$

- Langacker, London;
- Nardi, Roulet, Tommasini;
- Bornaboim, Bernabeu, Jorskok, Tommasini;
- Bergman, Kagan;
- Ilana, Rieman;

2. PROPAGATION OF NEUTRINO STATES IN MATTER,

Based on: B.Bekman, et. al.,
Phys. Rev.,D66(2002)093004.

Everything is much better to consider in the neutrino mass states

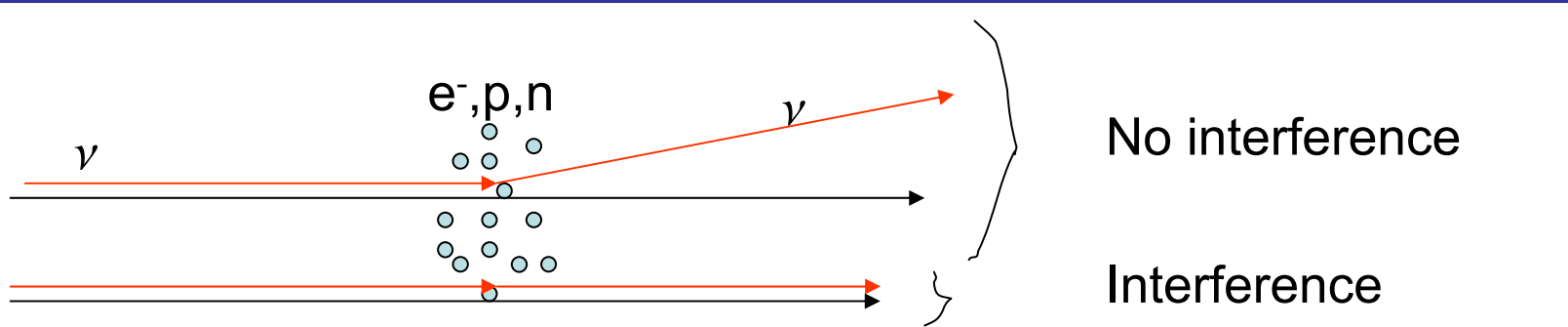
Let us assume that our neutrino interaction has general form:

$$n = 3 + n_s + n_R$$

$$L_{CC} = \frac{e}{2\sqrt{2}\sin\theta_W} \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^n \bar{\psi}_\alpha \gamma^\mu (1-\gamma_5) (U_\nu)_{\alpha i} N_i W_\mu^- + h.c.;$$

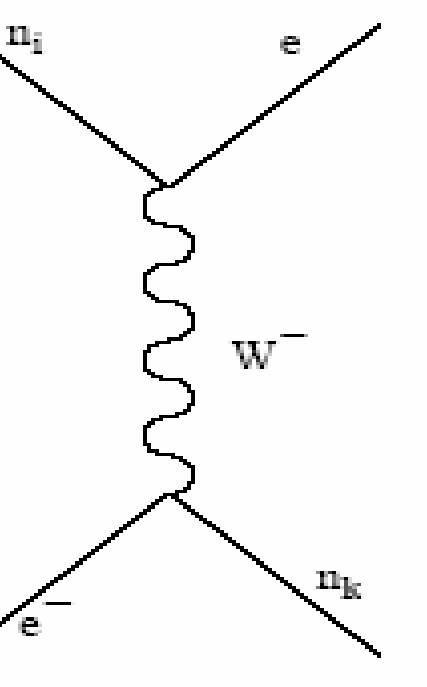
$$L_{NC} = \frac{e}{4\sin\theta_W \cos\theta_W} \sum_{i,j=1}^n \bar{N}_i \gamma^\mu (1-\gamma_5) \Omega_{ij} N_j Z_\mu, \quad \text{where}$$

$$\Omega_{ij} = \sum_{\alpha=e,\mu,\tau} (U_\nu)_{\alpha i}^* (U_\nu)_{\alpha j}$$

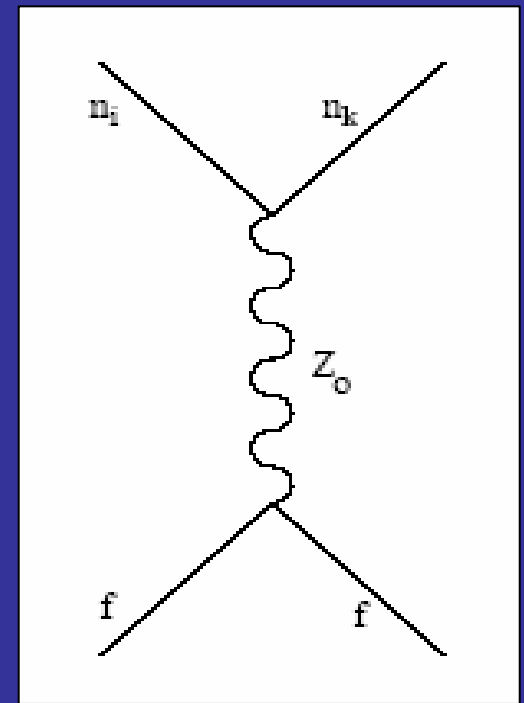
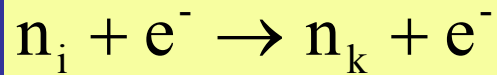


Neutrino interaction are described by the L_{CC} and L_{NC} Lagrangians

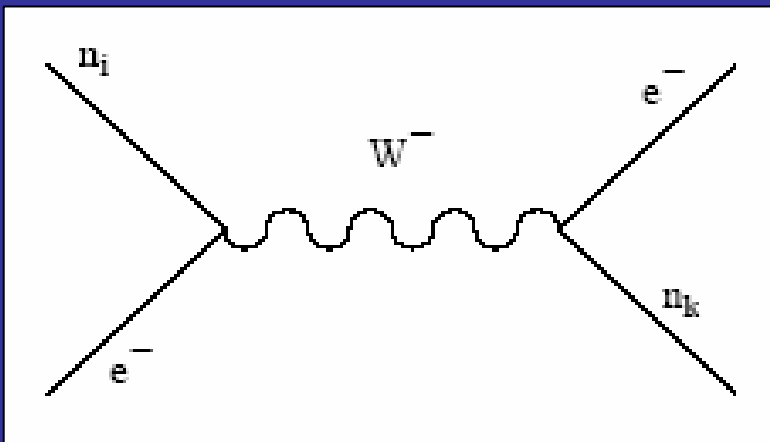
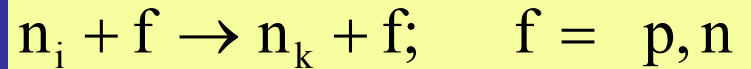
**COHERENT NEUTRINO SCATTERING IS DESCRIBED
BY THREE TYPES OF FEYNMAN DIAGRAMS**



All three diagrams contribute to neutrino - electron scattering:



Only this diagram contributes to neutrino- nucleon scattering:



For $q^2 \ll M_W^2, M_Z^2$, the effective interaction of light neutrinos with background particles can be written in the form:

$$H_{\text{int}}^f(\mathbf{x}) = \frac{G_F}{\sqrt{2}} \sum_{i,k=1}^3 \sum_{a=V,A} [\bar{n}_i \Gamma_a n_k] [\bar{\psi}_f \Gamma^a (g_{fa}^{ki} + \bar{g}_{fa}^{ki} \gamma_5) \psi_f]$$

where

$$\begin{aligned} g_{eV}^{ki} &= -\bar{g}_{eA}^{ki} = U_{ek}^* U_{ei} + \rho \Omega_{ki} \left(-\frac{1}{2} + 2 \sin^2 \Theta_W \right), \\ \bar{g}_{eV}^{ki} &= -g_{eA}^{ki} = -U_{ek}^* U_{ei} + \frac{1}{2} \rho \Omega_{ki}, \\ g_{fV}^{ki} &= -\bar{g}_{fA}^{ki} = \rho \Omega_{ki} (T_{3f} - 2Q_f \sin^2 \Theta_W), \\ \bar{g}_{fV}^{ki} &= -g_{fA}^{ki} = -\rho \Omega_{ki} T_{3f}, \end{aligned}$$

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \Theta_W}, T_{3p} = -T_{3n} = 1/2, Q_p = 1, Q_n = 0.$$

Amplitudes for forward scattered fermions f (their momenta and spins are untouched) is defined as:

$$(M_a^f)_{ki} = \langle f, \vec{p}, \vec{\lambda} | \bar{\Psi}_f \Gamma_a (g_{fa}^{ki} + \bar{g}_{fa}^{ki} \gamma_5) \Psi_f | f, \vec{p}, \vec{\lambda} \rangle.$$

Let $\rho_f(\vec{p}, \vec{\lambda})$ describes a distributions of momentum (p) and spins (λ) of the background particles f normalized in such a way that N_f , defined as

$$N_f \equiv \sum_{\vec{\lambda}} \int \frac{d^3 p}{(2\pi)^3} \rho_f(\vec{p}, \vec{\lambda})$$

is the number of fermions f in a unit volume ($V=1$).

Let us define the average amplitude for fermions (f) forward scattering:

$$(V_a^f)_{ki} = \frac{G_F}{\sqrt{2}} \sum_{\vec{\lambda}} \int \frac{d^3 p}{(2\pi)^3} \rho_f(\vec{p}, \vec{\lambda}) (M_a^f)_{ki}.$$

And the effective sum over all fermions:

$$V_{ki} = \sum_f V_{ki}^f = \sum_f \sum_a \Gamma^a (V_a^f)_{ki}$$

Then the global effect of matter – light neutrino interaction can be described by the Hamiltonian

$$H_{\text{int}}(\mathbf{x}) = \sum_{i,k=1}^3 \bar{\mathbf{n}}_k V_{ki} \mathbf{n}_i$$

The amplitudes M^f can be calculated ($u^+u = 1$):

$$\left(M_V^f\right)_{ki}^\mu = - \left(M_A^f\right)_{ki}^\mu = g_{fV}^{ki} \left(\frac{p^\mu}{E_f}\right) + m_f \bar{g}_{fV}^{ki} \left(\frac{S_f^\mu}{E_f}\right)$$

where E_f , m_f and $S_f^\mu = \frac{1}{m_f} \left(\vec{p}\vec{\lambda}, \vec{\lambda}m_f + \frac{\vec{p}(\vec{p}\vec{\lambda})}{m_f + E_f} \right)$ are energy, mass and spin four – vector of the f fermion.

So finally, for the effective potential V we obtain:

$$V_{ki}^f = \left(A_f^\mu \right)_{ki} \gamma_\mu P_L$$

where

$$\left(A_f^\mu \right)_{ki} = \sqrt{2} G_F N_f \left[g_{fV}^{ki} \left\langle \frac{p_f^\mu}{E_f} \right\rangle + m_f \bar{g}_{fV}^{ki} \left\langle \frac{S_f^\mu}{E_f} \right\rangle \right]$$

and the symbol $\langle z \rangle$ is defined as:

$$\langle z \rangle = \frac{1}{N_f} \sum_{\vec{\lambda}} \int \frac{d^3 p}{(2\pi)^3} \rho_f(\vec{p}, \vec{\lambda}) z(\vec{p}, \vec{\lambda})$$

For the (k,i) matrix of the element of the Hamiltonian (in the eigenmass base) we obtain:

$$H_{ki}^{int} = \langle \nu_k | \int_{V=1} d^3x H_{int}(x) | \nu_i \rangle = \begin{cases} A_{ki}^\mu \bar{u}_k \gamma_\mu P_L u_i & \text{for Dirac neutrinos,} \\ -(A_{ki}^\mu)^* \bar{u}_k \gamma_\mu P_R u_i & \text{for Dirac antineutrinos,} \\ A_{ki}^\mu \bar{u}_k \gamma_\mu P_L u_i - (A_{ki}^\mu)^* \bar{u}_k \gamma_\mu P_R u_i & \text{for Majorana neutrinos,} \end{cases}$$

$$A_{k\vec{z}}^\mu = \sum_f (A_f^\mu)_{k\vec{z}}$$

For relativistic neutrinos:

$$\bar{u}_k \gamma_\mu P_L u_i |_{\lambda=-1} = \bar{u}_k \gamma_\mu P_R u_i |_{\lambda=+1} = \left(1, -\frac{\vec{k}}{|\vec{k}|} \right)$$

$$\bar{u}_k \gamma_\mu P_L u_i |_{\lambda=+1} = \bar{u}_k \gamma_\mu P_R u_i |_{\lambda=-1} = (0, \vec{0}),$$

We obtain the final, general Hamiltonian for neutrino interaction with background fermions:

$$H_{ki}^{int} = \begin{cases} A_{ki}^0 - \frac{\vec{k}}{|\vec{k}|} \vec{A}_{ki} & \text{for Dirac and Majorana neutrinos with } \lambda = -1, \\ -(A_{ki}^0)^* + \frac{\vec{k}}{|\vec{k}|} (\vec{A}_{ki})^* & \text{for Dirac antineutrinos and Majorana neutrinos with } \lambda = +1. \end{cases}$$

We see that our Hamiltonian has the properties:

$$H_{particle}^{int} = -[H_{antiparticle}^{int}]^*.$$

Valid for V-A
neutrino
interaction

So the most general Hamiltonian for light relativistic neutrinos which interact in the V-A way, propagating in any background medium has the form:

$$H_{ki}^{int} = A_{ki}^0 - \frac{\vec{k}}{|\vec{k}|} \vec{A}_{ki}$$

$$A_{ki}^\mu = \sqrt{2}G_F \sum_{f=e,p,n} N_f \left(g_{fV}^{ki} \left\langle \frac{\mathbf{p}_f^\mu}{E_f} \right\rangle + m_f \bar{g}_{fV}^{ki} \left\langle \frac{S_f^\mu}{E_f} \right\rangle \right)$$

NORMAL MATTER

unpolarized:

$$\langle \vec{\lambda}_f \rangle = 0$$

isotropic:

$$\langle \vec{p}_f \rangle = 0$$

electrically neutral:

$$N_e = N_p \neq N_n$$

We arrive to the simple Hamiltonian which has $(3 + n_s) \times (3 + n_s)$ dimension

$$H_{ki}^{\text{int}} = \sqrt{2}G_F [N_e U_{ek}^* U_{ei} - \frac{1}{2} \rho N_n \Omega_{ki}],$$

Which in the standard case ($n_s = n_R = 0$) reproduce (in the flavour base) the well known Hamiltonian ($\alpha, \beta = e, \mu, \tau$):

$$H_{\alpha\beta}^{\text{int}} = \sqrt{2}G_F N_e \delta_{e\beta} \delta_{e\alpha} \Rightarrow \sqrt{2}G_F N_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$


If a single sterile neutrino is present ($n_s=1$, $\alpha, \beta = e, \mu, \tau, s$):

$$H_{\alpha\beta} = \left(U \frac{\Delta m^2}{2E} U^+ \right)_{\alpha\beta} + \sqrt{2} G_F [N_e \delta_{\alpha e} \delta_{\beta e} + \frac{1}{2} N_n \delta_{\alpha s} \delta_{\beta s}];$$

WE WILL CONSIDER CASES WHEN IS AT LEAST ONE NON-DECOUPLING HEAVY NEUTRINO;

$$n_R \geq 1$$

The energy and momentum conservation do not allow heavy neutrinos to be produced or detected

$$H^{\text{int}} \Rightarrow \begin{matrix} & \begin{matrix} 3+n_s & n_R \end{matrix} \\ \begin{pmatrix} H^{\text{int}} & 0 \\ 0 & 0 \end{pmatrix} \end{matrix}$$


If no sterile neutrinos are present ($n_s=0$) but $n_R = 1,2 ..$

$$H_{\alpha\beta} = \left(U \frac{\Delta m^2}{2E} U^+ \right)_{\alpha\beta} + \sqrt{2} G_F [N_e \{ \delta_{\alpha e} \delta_{\beta e} - c_{\alpha\beta} (\delta_{\alpha e} + \delta_{\beta e}) + c_{e\alpha} c_{e\beta} \} \\ + \frac{1}{2} N_n \{ 2c_{\alpha\beta} - \sum_{\gamma=e,\mu,\tau} c_{\alpha\gamma} c_{\gamma\beta} \}];$$

If a single sterile neutrino ($n_s = 1$) and some heavy neutrinos exist ($n = 4 + n_R$)

$$H_{\alpha\beta} = \left(U \frac{\Delta m^2}{2E} U^+ \right)_{\alpha\beta} + \sqrt{2} G_F [N_e \{ \delta_{\alpha e} \delta_{\beta e} - c_{\alpha\beta} (\delta_{\alpha e} + \delta_{\beta e}) + c_{e\alpha} c_{e\beta} \} \\ + \frac{1}{2} N_n \{ \delta_{\alpha s} \delta_{\beta s} + c_{\alpha\beta} (2 - \delta_{\alpha s} - \delta_{\beta s}) - \sum_{\gamma=e,\mu,\tau} c_{\alpha\gamma} c_{\gamma\beta} \}];$$

And generally in the full flavour base:

$$\mathbf{H}_{\alpha\beta} \Rightarrow \begin{pmatrix} \mathbf{U} \mathbf{H}^{\text{eff}} \mathbf{U}^+ & \mathbf{U} \mathbf{H}^{\text{eff}} \mathbf{V}'^+ \\ \mathbf{V}' \mathbf{H}^{\text{eff}} \mathbf{U}^+ & \mathbf{V}' \mathbf{H}^{\text{eff}} \mathbf{V}'^+ \end{pmatrix}_{\alpha\beta}$$

Even we consider the problem of light neutrino propagation only, in the flavour basis we have to deal with $(3 + n_s + n_R)$ dimensional matrices

No heavy mass eigenstates can experimentally be produced \rightarrow the properly normalized light neutrino states produced in real experiment:

$$|\tilde{\nu}_\alpha\rangle = \frac{1}{\lambda_\alpha} \sum_{i=1}^{3+n_s} U_{\alpha i}^* |\nu_i\rangle \equiv \sum_{i=1}^{3+n_s} \tilde{U}_{\alpha i}^* |\nu_i\rangle$$

Usual notation of flavour neutrino loose its meaning.
Neutrino created with electron can produce muon and tau.

Propagation equation in the experimentally accessible states:

$$i \frac{d}{dt} \langle \tilde{\nu}_\alpha | \psi(t) \rangle = \sum_\beta \tilde{H}_{\alpha\beta} \langle \tilde{\nu}_\beta | \psi(t) \rangle;$$

where the Hamiltonian is not hermitian:

$$\tilde{H} = \tilde{U} H \tilde{U}^{-1} = \frac{1}{2E} \tilde{U} \begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ 0 & \delta m_{21}^2 & 0 & 0 & \dots \\ 0 & 0 & \delta m_3^2 & 0 & \dots \\ 0 & 0 & 0 & \delta m_{41}^2 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix} \tilde{U}^{-1} + \sqrt{2} G_F \tilde{U} \tilde{U}^\dagger \lambda^2 \begin{pmatrix} (N_e - \frac{N_n}{2}) & 0 & 0 & 0 & \dots \\ 0 & -\frac{N_n}{2} & & 0 & \dots \\ 0 & 0 & -\frac{N_n}{2} & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix};$$

If we assume that the initial neutrino states is:

$$|\psi(0)\rangle = |\tilde{\nu}_\alpha(0)\rangle$$



$$\langle \tilde{\nu}_\beta | \tilde{\nu}_\alpha(0) \rangle = (\lambda_\alpha \lambda_\beta)^{-1} (\delta_{\alpha\beta} - c_{\alpha\beta});$$

The initial condition:

In the eigenmass basis the propagation equation:

$$i \frac{d}{dt} \langle \nu_k | \tilde{\nu}_\alpha(t) \rangle = \sum_{i=1}^{3+n_s} H_{ki} \langle \nu_i | \tilde{\nu}_\alpha(t) \rangle;$$

is simple, Hamiltonian is hermitian:

$$H = \frac{1}{2E} \begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ 0 & \delta m_{21}^2 & 0 & 0 & \dots \\ 0 & 0 & \delta m_3^2 & 0 & \dots \\ 0 & 0 & 0 & \delta m_{41}^2 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix} + \sqrt{2} G_F U^+ \begin{pmatrix} (N_e - \frac{N_n}{2}) & 0 & 0 & 0 & \dots \\ 0 & -\frac{N_n}{2} & 0 & 0 & \dots \\ 0 & 0 & -\frac{N_n}{2} & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix} U$$

The initial condition:

$$\langle \nu_k | \tilde{\nu}_\alpha(0) \rangle = \lambda_{\alpha}^{-1} U_{\alpha k}^*$$

Nonorthogonality of neutrino states has some impact on theoretically calculated amplitudes:

$$A(\nu_\alpha X \rightarrow l_\beta^- Y) = \lambda_\alpha^{-1} \sum_{i=1}^{3+n_s} U_{\alpha i}^* A(\nu_i X \rightarrow l_\beta^- Y) \cong \lambda_\alpha^{-1} \sum_{i=1}^{3+n_s} U_{\alpha i}^* U_{\beta i} A(\nu_{m=0} X \rightarrow l_\beta^- Y)$$

and cross sections:

$$\sigma(\nu_\alpha X \rightarrow l_\beta^- Y) = \lambda_\alpha^{-2} \left| \delta_{\alpha\beta} - c_{\alpha\beta} \right|^2 \sigma^{SM}(\nu_\beta X \rightarrow l_\beta^- Y)$$

3. OSCILLATION OF LIGHT NEUTRINO

Oscillation of 3 light (no sterile) with at least one heavy will be considered:

For one heavy neutrino:

$$V = \begin{pmatrix} \varepsilon_e \\ e^{-i\chi_\mu} \varepsilon_\mu \\ e^{-i\chi_\tau} \varepsilon_\tau \end{pmatrix};$$

$$V' = (V'_{R1}, V'_{R2}, V'_{R3})$$

$$V'_{Ri} = -U_{ei} \varepsilon_e - U_{\mu i} e^{i\chi_\mu} \varepsilon_\mu - U_{\tau i} e^{i\chi_\tau} \varepsilon_\tau$$

Then full neutrino interaction Hamiltonian in the eigenmass basis:

$$H_{ki}^{\text{int}} = \frac{1}{2E} (m_i^2 \delta_{ik} + \sqrt{2} G_F E [N_e U_{ek}^* U_{ei} - \frac{1}{2} N_n \Omega_{ki}]),$$

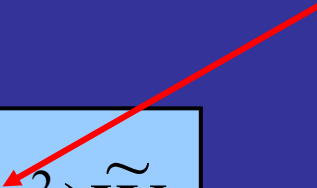
With the Ω matrix:

$$\Omega_{ik} = \delta_{ik} - V_{Rk}^* V'_{Ri}, \quad i, k = 1, 2, 3.$$

For neutrino propagation in a **uniform density medium** equation of motion can be solved analytically.

Effective Hamiltonian can be diagonalized:

Effective neutrino mass in matter



$$H^{\text{eff}} = \frac{1}{2E} \tilde{W}^+ \text{diag}(m_i^2) \tilde{W};$$

and the propagation equation takes the form:

$$i \frac{d}{dt} \Psi^\alpha(t) = \frac{1}{2E} \tilde{W}^+ \text{diag}(m_i^2) \tilde{W} \Psi^\alpha(t);$$

where:

$$\Psi^\alpha(t) = \begin{pmatrix} \langle \nu_1 | \tilde{\nu}_\alpha(t) \rangle \\ \langle \nu_2 | \tilde{\nu}_\alpha(t) \rangle \\ \langle \nu_3 | \tilde{\nu}_\alpha(t) \rangle \end{pmatrix}$$

The equation of motion together with the initial condition give:

$$\Psi_k^\alpha(t) = \sum_i (\tilde{W}^+)_{ki} e^{-i\frac{\tilde{m}_i^2}{2E}t} (\tilde{W}\tilde{U}^+)_{i\alpha}$$

Giving the amplitude for the $\nu_\alpha \longrightarrow \nu_\beta$ oscillation after travelling a distance L:

$$A_{\alpha \rightarrow \beta}(L) = \langle \tilde{\nu}_\beta | \tilde{\nu}_\alpha(L=t) \rangle = \sum_{i=1}^3 \bar{W}_{\beta i} \bar{W}_{\alpha i}^* e^{-i\frac{\tilde{m}_i^2}{2E}L}$$

Where the new tensors are defined by:

$$\bar{W}_{\alpha i} = (\tilde{U}\tilde{W}^+)_{\alpha i} = \lambda_\alpha^{-1} \sum_k U_{\alpha k} \tilde{W}_{ik}^* \equiv \lambda_\alpha^{-1} W_{\alpha i}$$

Now we can calculate the transition probability:

$$P_{\alpha \rightarrow \beta}(\mathbf{L}) = \left| A_{\alpha \rightarrow \beta}(\mathbf{L}) \right|^2 ;$$

And we have:

$$P_{\alpha \rightarrow \beta}(\mathbf{L}) = \frac{1}{\lambda_{\alpha}^2 \lambda_{\beta}^2} \left\{ (\delta_{\alpha\beta} - |c_{\alpha\beta}|)^2 - 4 \sum_{i>k} R_{\alpha\beta}^{ik} \sin^2 \Delta_{ik} + 8 I_{\alpha\beta}^{12} \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} + 2 [A_{\alpha\beta}^{(1)} \sin 2\Delta_{31} + A_{\alpha\beta}^{(2)} \sin 2\Delta_{32}] \right\}$$

Where:

$$R_{\alpha\beta}^{ik} = \text{Re}[W_{\alpha i} W_{\beta k} W_{\alpha k}^* W_{\beta i}^*]$$

$$I_{\alpha\beta}^{ik} = \text{Im}[W_{\alpha i} W_{\beta k} W_{\alpha k}^* W_{\beta i}^*]$$

$$A_{\alpha\beta}^{(i)}(c_{\alpha\beta}) = I_{\alpha\beta}^{ik} = \text{Im}[W_{\alpha i}^* W_{\beta k} c_{\alpha\beta}^*]$$

$$\Delta_{ik} = 1.267 \frac{(\tilde{m}_i^2 - \tilde{m}_k^2)[\text{eV}^2]L[\text{km}]}{E[\text{GeV}]}$$

The transition probability for Dirac antineutrino or Majorana neutrino with $\lambda=+1$ can be obtained after the replacement:

$$P_{\bar{\alpha} \rightarrow \bar{\beta}}(L) = P_{\alpha \rightarrow \beta}(L; U \rightarrow U^*, G_F \rightarrow -G_F)$$

We can find now the CP and T asymmetries:

$$\begin{aligned} \Delta P_{\alpha \rightarrow \beta}^{\text{CP}} &\equiv P_{\alpha \rightarrow \beta} - P_{\bar{\alpha} \rightarrow \bar{\beta}} = \\ &- 2 \frac{1}{\lambda_{\alpha}^2 \lambda_{\beta}^2} \left\{ 2 \sum_{i>k} [R_{\alpha\beta}^{ik}(G_F) \sin^2 \Delta_{ik}(G_F) - R_{\alpha\beta}^{ik}(-G_F) \sin^2 \Delta_{ik}(-G_F)] \right. \\ &- 4 [I_{\alpha\beta}^{12}(G_F) \sin \Delta_{21}(G_F) \sin \Delta_{31}(G_F) \sin \Delta_{32}(G_F) + \\ &\quad I_{\alpha\beta}^{12}(-G_F) \sin \Delta_{21}(-G_F) \sin \Delta_{31}(-G_F) \sin \Delta_{32}(-G_F)] - \\ &\quad [A_{\alpha\beta}^{(1)}(G_F) \sin 2 \Delta_{31}(G_F) + A_{\alpha\beta}^{(2)}(G_F) \sin 2 \Delta_{32}(G_F) + \\ &\quad \left. A_{\alpha\beta}^{(1)}(-G_F) \sin 2 \Delta_{31}(-G_F) + A_{\alpha\beta}^{(2)}(-G_F) \sin 2 \Delta_{32}(-G_F)] \right\} \end{aligned}$$

$$\Delta P_{\alpha \rightarrow \beta}^T \equiv P_{\alpha \rightarrow \beta} - P_{\beta \rightarrow \alpha} = 4 \frac{1}{\lambda_{\alpha}^2 \lambda_{\beta}^2} \left[4 I_{\alpha\beta}^{12} \sin\Delta_{21} \sin\Delta_{31} \sin\Delta_{32} + \right. \\ \left. A_{\alpha\beta}^{(1)} \sin 2\Delta_{31} + A_{\alpha\beta}^{(2)} \sin 2\Delta_{32} \right]$$

Nonunitarity of mixing matrix produce to type of effect:

- R and I tensors depend on ϵ and χ parameters ,
- Additional new terms proportional to A appear.

Seen in numerical analysis

More spectacular, survive in vacuum

4. NON - UNITARY EFFECTS IN NEUTRINO OSCILLATION

In vacuum $G_F \rightarrow 0$

$$\Delta P_{\alpha \rightarrow \beta}^{\text{CP}}(\text{vacuum}) = \Delta P_{\alpha \rightarrow \beta}^{\text{T}}(\text{vacuum})$$

For $\alpha = \beta$ $A_{\alpha\alpha}^{(i)} = 0$

$$\Delta P_{\alpha \rightarrow \alpha}^{\text{T}}(\text{matter}) = \Delta P_{\alpha \rightarrow \alpha}^{\text{T}}(\text{vacuum}) = \Delta P_{\alpha \rightarrow \alpha}^{\text{CP}}(\text{vacuum}) = 0$$

For $(\alpha \neq \beta)$, CP asymmetry effect is larger:

$$\Delta P_{\alpha \rightarrow \alpha}^{\text{CP}}(\text{matter})$$

$$\Delta_{\text{LBL}} \approx \Delta_{31} \cong \Delta_{32} \cong 0(1), \quad \Delta_{21} \cong 0$$

For unitary oscillation in vacuum, the CP and T asymmetries for LBL disappear, in the non-unitary case:

$$\Delta P_{\alpha \rightarrow \beta}^{\text{T}}(\text{vacuum}) = \Delta P_{\alpha \rightarrow \beta}^{\text{CP}}(\text{vacuum}) \neq 0$$

$$\begin{aligned} \Delta P_{\alpha \rightarrow \beta}^{\text{T}}(\text{vacuum}) = & 4 \frac{1}{\lambda_{\alpha}^2 \lambda_{\beta}^2} (A_{\alpha\beta}^{(1)} + A_{\alpha\beta}^{(2)}) \sin 2\Delta_{\text{LBL}} = \\ & - 4 \frac{1}{\lambda_{\alpha}^2 \lambda_{\beta}^2} \text{Im}(W_{\alpha 3}^* W_{\beta 3} c_{\alpha\beta}^*) \sin 2\Delta_{\text{LBL}} \end{aligned}$$

CP and T asymmetries appear even for two flavour neutrino transitions

For unitary 3 flavour transitions, moduli of all Jarlskog invariants are equal and all T and CP asymmetries in vacuum are equal, eg.

$$\Delta P_{e \rightarrow \mu}^T = \Delta P_{\mu \rightarrow \tau}^T = \Delta P_{\tau \rightarrow e}^T$$

In unitary case, if any elements of the mixing matrix is small (vanish) then above asymmetries are also small (vanish), eg.

$$\Delta P_{\alpha \rightarrow \beta}^T = \Delta P_{\alpha \rightarrow \beta}^{\text{CP}} \approx \sin^2(2\theta_{13})$$

In the non-unitary case the Jarlskog invariants are not equal, and:

$$\begin{aligned} I_{e\mu}^{\text{ik}} &= I_{\mu\tau}^{\text{ik}} + \text{Im}[W_{\mu i} W_{\mu k}^* (\tilde{W} V'^T V'^* \tilde{W}^T)_{\text{ik}}] = \\ &I_{\tau e}^{\text{ik}} - \text{Im}[W_{ei} W_{ek}^* (\tilde{W} V'^T V'^* \tilde{W}^T)_{\text{ik}}] \end{aligned}$$

And

$$I_{\alpha\beta}^{12} = I_{\alpha\beta}^{23} - \text{Im}[W_{\alpha 2}^* W_{\beta 2} \mathbf{c}_{\alpha\beta}^*] = \\ - I_{\alpha\beta}^{13} + \text{Im}[W_{\alpha 1}^* W_{\beta 1} \mathbf{c}_{\alpha\beta}^*]$$

The above relations and terms proportional to A tensors imply:

$$\Delta P_{e \rightarrow \mu}^T \neq \Delta P_{\mu \rightarrow \tau}^T \neq \Delta P_{\tau \rightarrow e}^T$$

We have calculated the differences ΔP and asymmetries defined by:

$$A_{\mu \rightarrow \tau}^{\text{CP}} = \frac{\Delta P_{\mu \rightarrow \tau}^{\text{CP}}}{P_{\mu \rightarrow \tau} + P_{\bar{\mu} \rightarrow \bar{\tau}}}$$

$$A_{\mu \rightarrow \tau}^T = \frac{\Delta P_{\mu \rightarrow \tau}^T}{P_{\mu \rightarrow \tau} + P_{\tau \rightarrow \mu}}$$

P
A
R
A
M
E
T
E
R
S

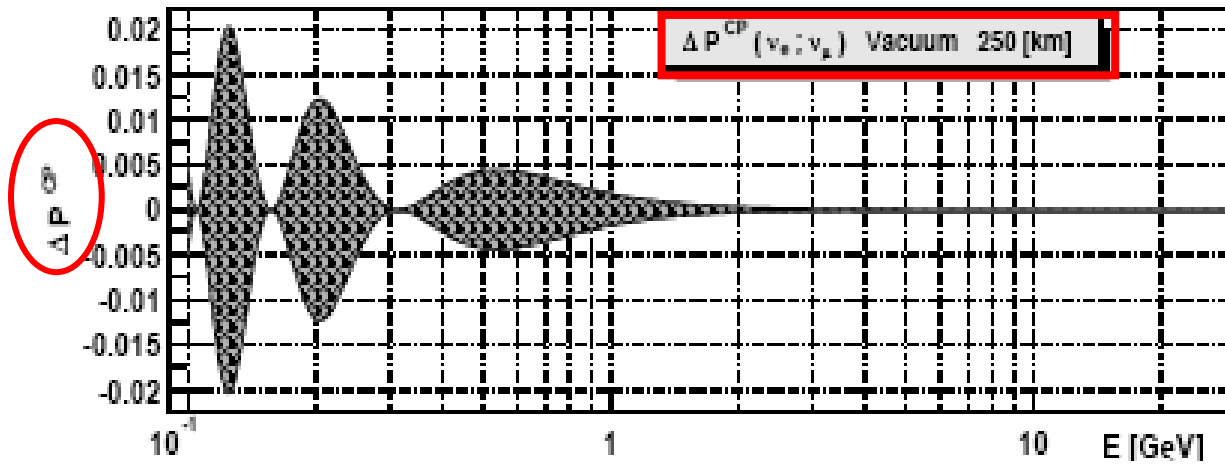
- Standard mass square differences and mixing angles,
- $L = 295 \text{ km}$ and $L = 732 \text{ km}$,
- Energy $E = 0.1 - 30 \text{ GeV}$,
- medium density $\rho = \text{const} = 2.6 \text{ g/cm}^3$,
- $Y_e = 0.494$
- The ϵ parameters:

$$\text{A) } \epsilon_e \sim 0.001, \epsilon_\mu = \epsilon_\tau \sim 0.1;$$

$$\text{B) } \epsilon_e = \epsilon_\mu \sim 0.01, \epsilon_\tau \sim 0.1$$

$$A_e [\text{eV}^2] = 2\sqrt{2}G_F N_e E = 7.63 \times 10^{-5} \left[\frac{\rho}{\text{g/cm}^3} \right] \left[\frac{Y_e}{0.5} \right] \left[\frac{E}{\text{GeV}} \right]$$

$$A_n [\text{eV}^2] = 2\sqrt{2}G_F N_n E = 7.63 \times 10^{-5} \left[\frac{\rho}{\text{g/cm}^3} \right] [1 - Y_e] \left[\frac{E}{\text{GeV}} \right]$$



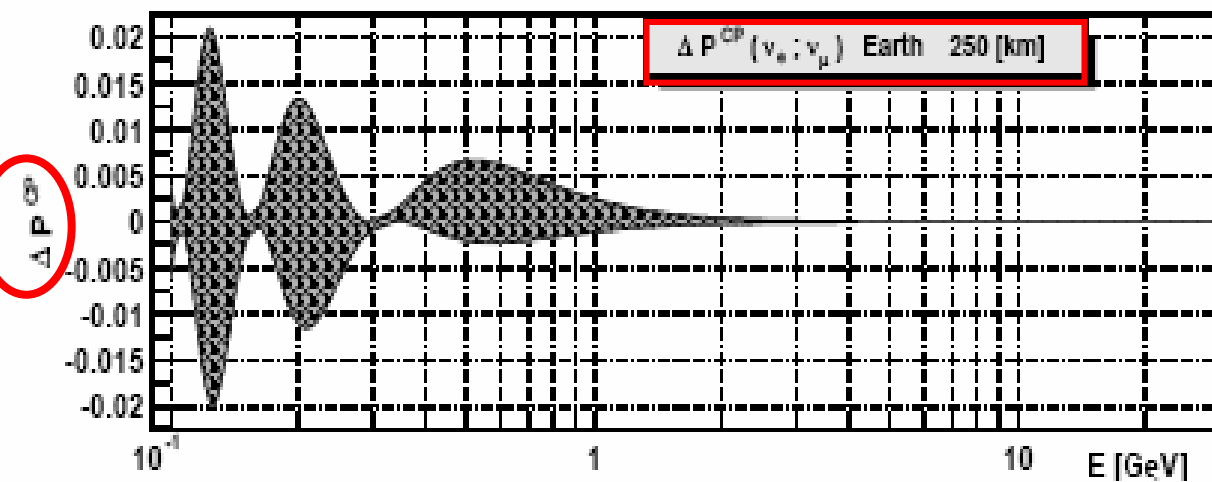
Hatched region

$$0 < \delta_{\text{CP}} < 2\pi$$

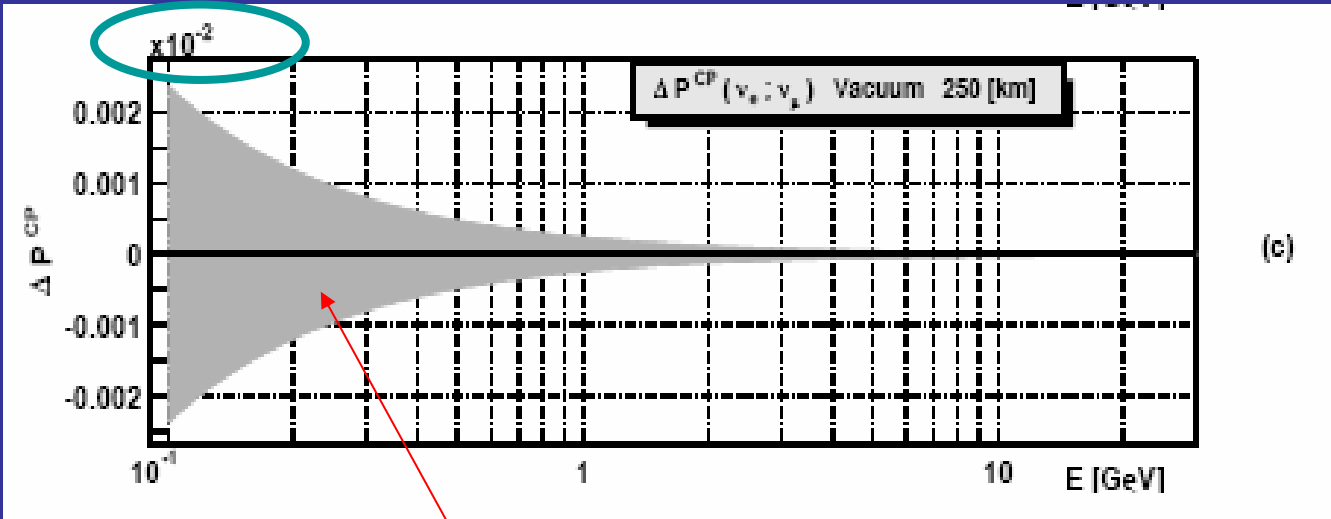
Shaded region

$$0 < \chi_i < 2\pi$$

$$\text{Tan}^2(2\theta_{13}) = 0.005$$

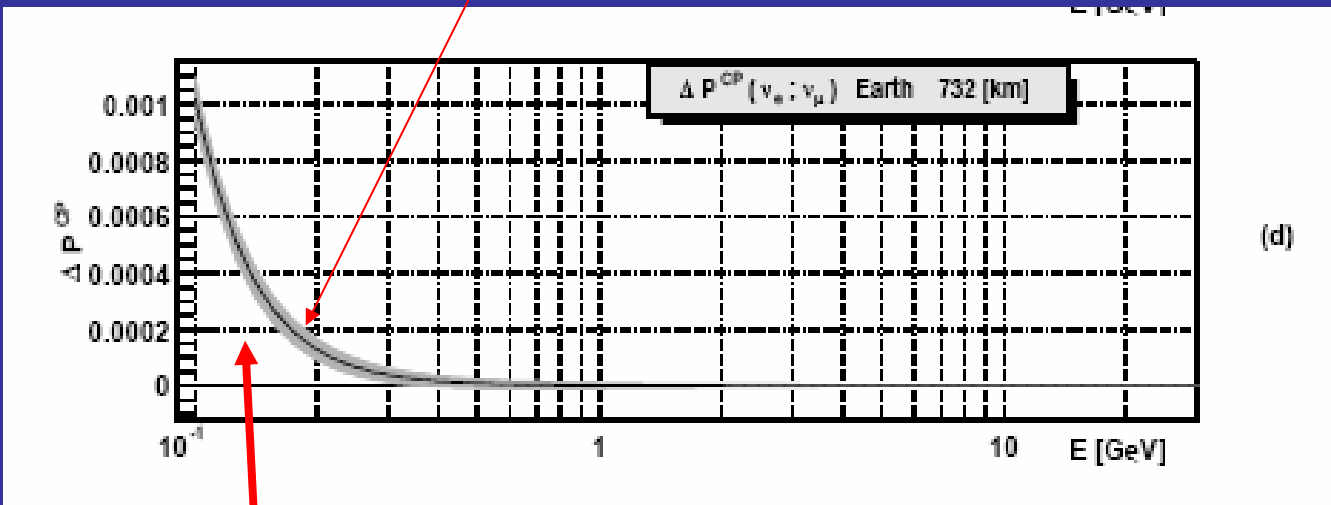


$$\Theta_{13} = 0$$



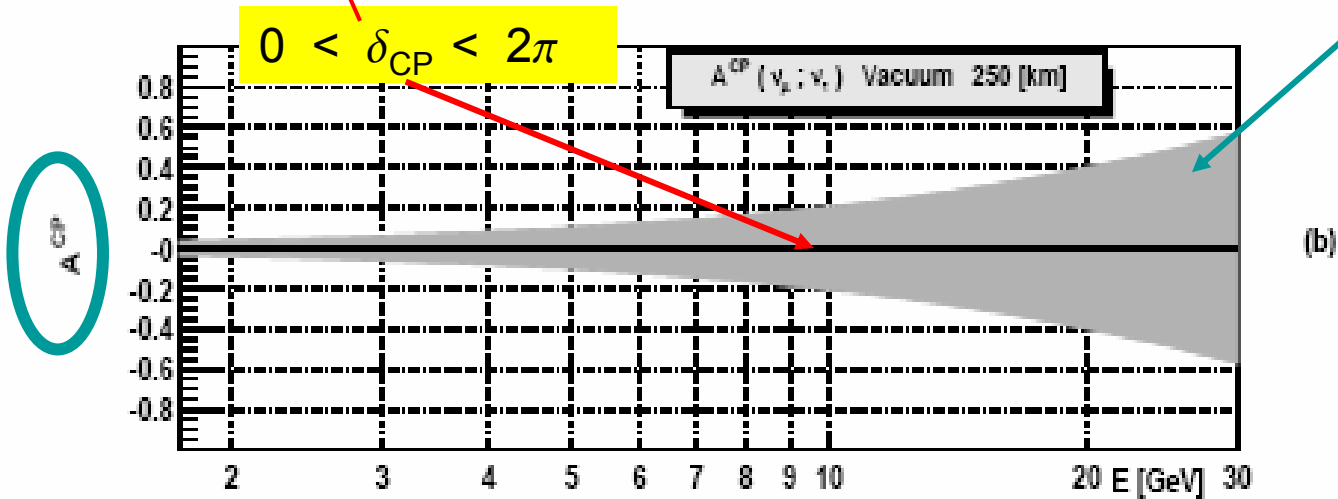
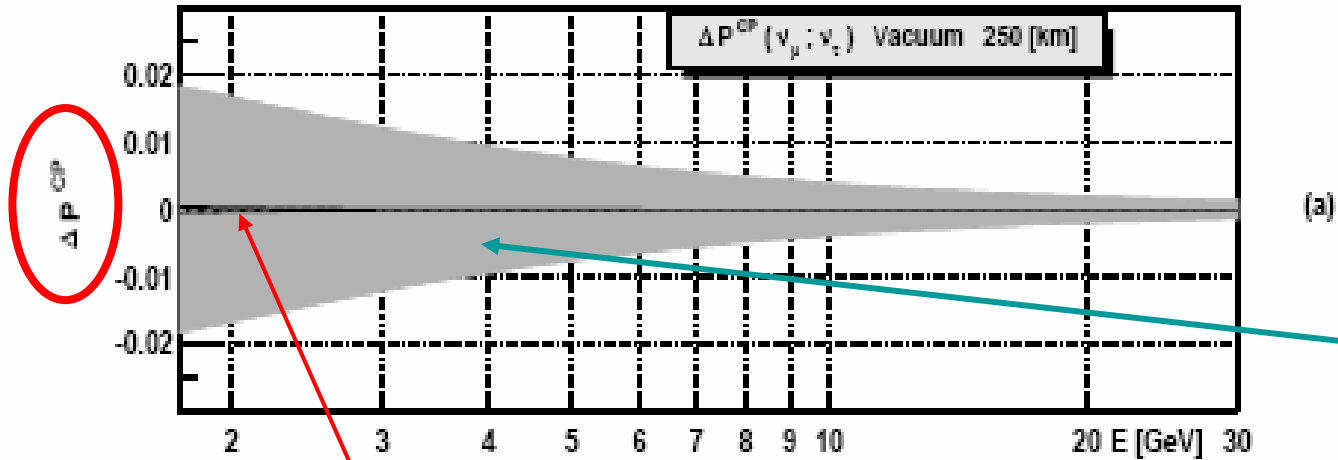
Shaded region - effect of nonunitarity

Shaded region:
 $0 < \chi_i < 2\pi$



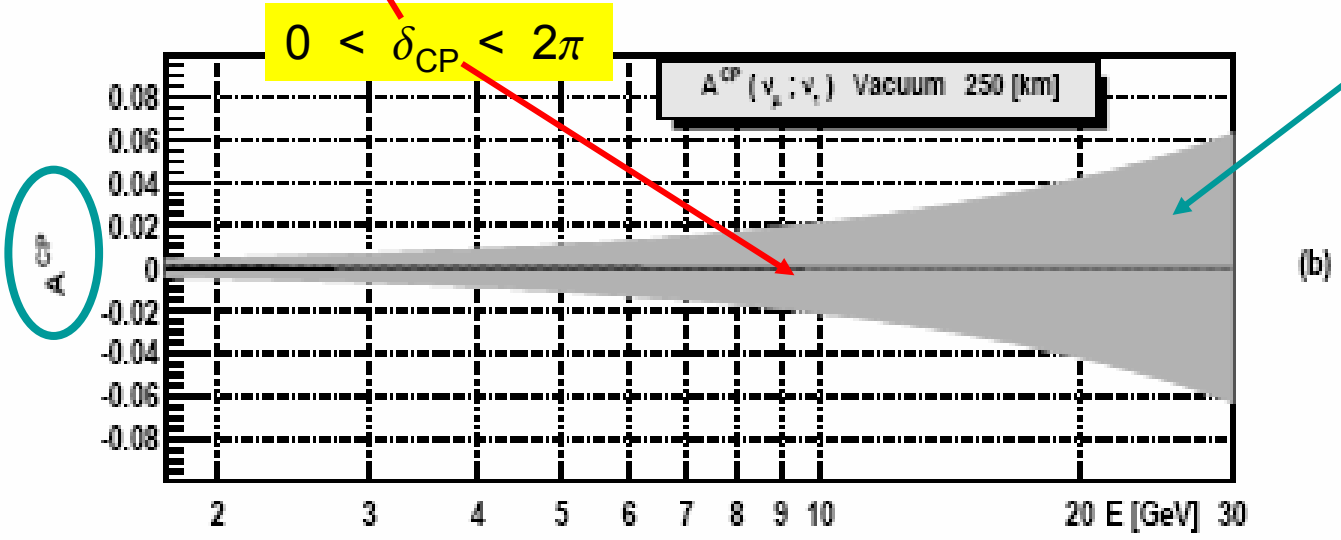
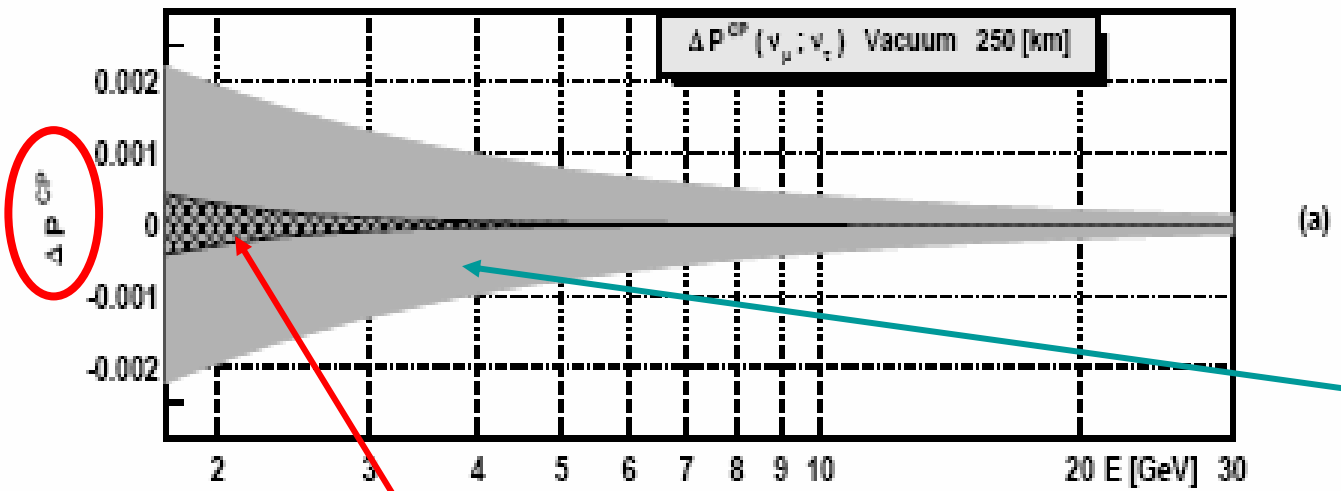
Earth effect: $(\Delta P \neq 0)$ even for unitary neutrino oscillation

A) $\epsilon_e \sim 0.001$,
 $\epsilon_\mu = \epsilon_\tau \sim 0.1$;



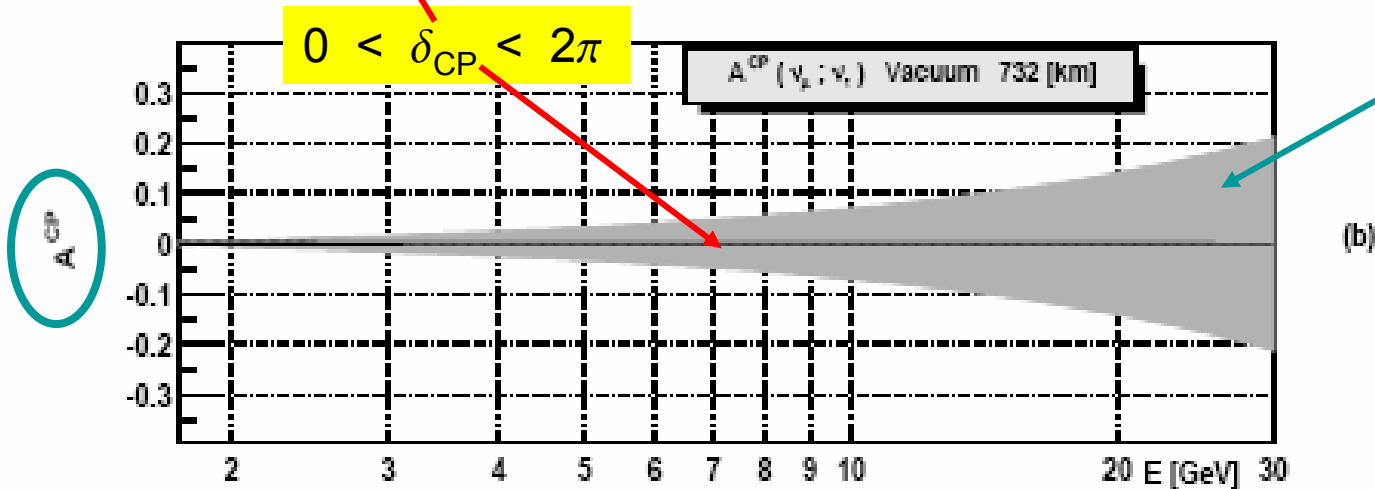
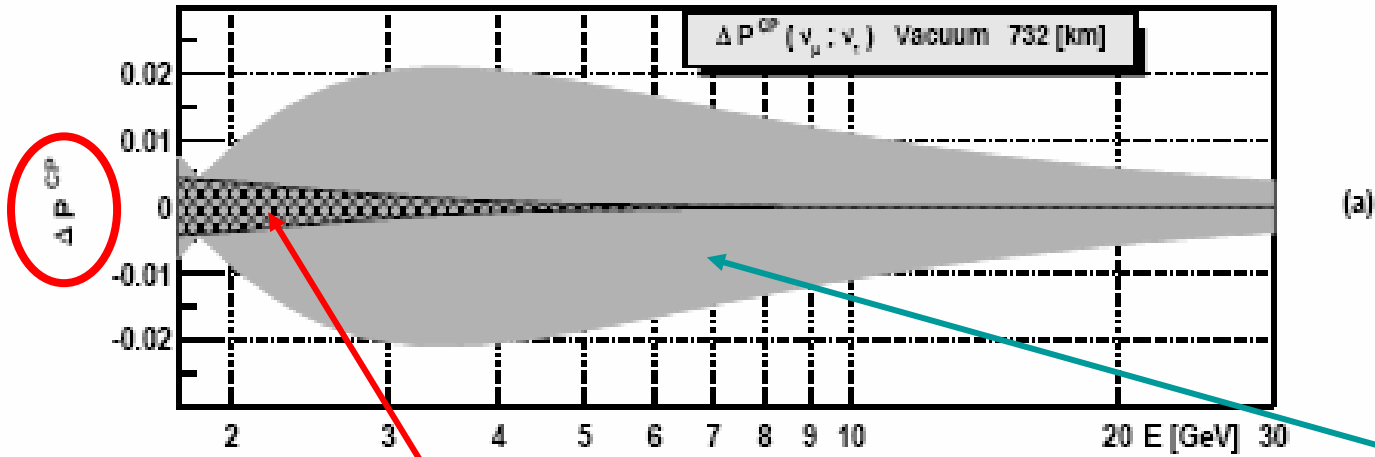
$\mu \rightarrow \tau$ transition in vacuum for 250 km

B) $\epsilon_e = \epsilon_\mu \sim 0.01,$
 $\epsilon_\tau \sim 0.1$



$\mu \rightarrow \tau$ transition in vacuum for 250 km

A) $\epsilon_e \sim 0.001$,
 $\epsilon_\mu = \epsilon_\tau \sim 0.1$;



$\mu \rightarrow \tau$ transition in vacuum for 732 km

4. RESONANCE PHENOMENA,

For unitary neutrino oscillations, the effective mixing angle sometimes has resonance behaviour:

$$\sin 2\theta_{\text{eff}} = \frac{\sin 2\theta_{12}}{\sqrt{\left(\frac{2\sqrt{2}G_F|\vec{k}|N_e}{\Delta m_{21}^2} - \cos 2\theta_{12}\right)^2 + \sin^2 2\theta_{12}}}$$

And we see, that for:

$$\frac{2\sqrt{2}G_F|\vec{k}|N_e}{\Delta m_{21}^2} \rightarrow \cos 2\theta_{12};$$

Θ_{12} mixing angle becomes maximal:

$$\sin 2\theta_{\text{eff}} \rightarrow 1$$

We have checked the resonance behaviour in the case of two non-unitary mixing neutrinos

The effective mixing angle in this case is given by:

$$\sin 2\theta_{\text{eff}} = c_{13}^2 \frac{A}{\sqrt{(B - \cos 2\theta_{12})^2 + A^2}},$$

with

$$A^2 = \left[\sin 2\theta_{12} + \frac{\sqrt{2}G_F |\vec{k}| N_n}{\Delta m_{21}^2} s_{13} \sin 2\theta_{23} \cos \delta \right]^2 + \left(\frac{\sqrt{2}G_F |\vec{k}| N_n}{\Delta m_{21}^2} \right)^2 s_{13}^2 \sin^2 2\theta_{23} \sin^2 \delta,$$

$$B = \frac{\sqrt{2}G_F |\vec{k}| (2N_e - N_n)}{\Delta m_{21}^2} c_{13}^2 + \frac{\sqrt{2}G_F |\vec{k}| N_n}{\Delta m_{21}^2} (c_{23}^2 - s_{13}^2 s_{23}^2)$$

There are two kinds of new effects

The resonance has a new position:

$$B \rightarrow \cos 2\theta_{12};$$

$$B \rightarrow \frac{2\sqrt{2}G_F|\vec{k}|}{\Delta m_{12}^2} \left(1 - \frac{\varepsilon^2}{2}\right)$$

The effective mixing is not maximal:

$$\sin 2\theta_{\text{eff}} \rightarrow c_{13}^2 = 1 - \varepsilon^2 \neq 1$$

6. CONCLUSIONS

1. Some models are able to predict visible mixing between light and heavy neutrinos, but the departure from the SM predictions, resulting from the charged lepton processes, are small,
2. Present limits postpone any observation of non – unitary mixing in the neutrino oscillation to the ν factory experiments,
3. The main signature is the observation of CP violation together with no mixing between first and third families ($\theta_{13} \rightarrow 0$),
4. Other effect, like the sum of probabilities not adding to 1 or modified resonance effects will be difficult to discriminate,
5. Ten best place to look for lack of unitarity is the μ and τ processes not involving e because there present limits are less stringent.