RG Corrections: From Unified Theories to Precision Neutrino Experiments

Talk by Stefan Antusch
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International scoping study of a future
Neutrino Factory and super-beam facility
Physics working group

Workshop #1

Imperial College London, November 2005

Challenges for Future Neutrino Experiments ...

Open fundamental questions ...

- What is the nature of neutrino masses? Majorana or (Pseudo-)Dirac?
- What is the neutrino mass scale?
- How many light neutrino species?

See-saw: theory gives these values at high energy (~ M_{GUT})

Flavour puzzle ...

- What is the neutrino mass scheme (sign of Δm^2_{31}) – normal or inverted?
- What is the value of θ_{13}? How small?
- What is the precise value of θ_{23}? How 'maximal'?
- What is the precise value of θ_{12}? Signatures of quark-lepton unification? ... ?
- What is the value of δ_{MNS}? Connected to baryogenesis via leptogenesis?

New physics, surprises ...

- Precise oscillation experiments can test: 3ν-oscillation hypothesis, MSW effect, CPT and Lorentz violation, Non-Standard ν-Interactions, ...

RG Corrections: Compare Predictions with Experiments

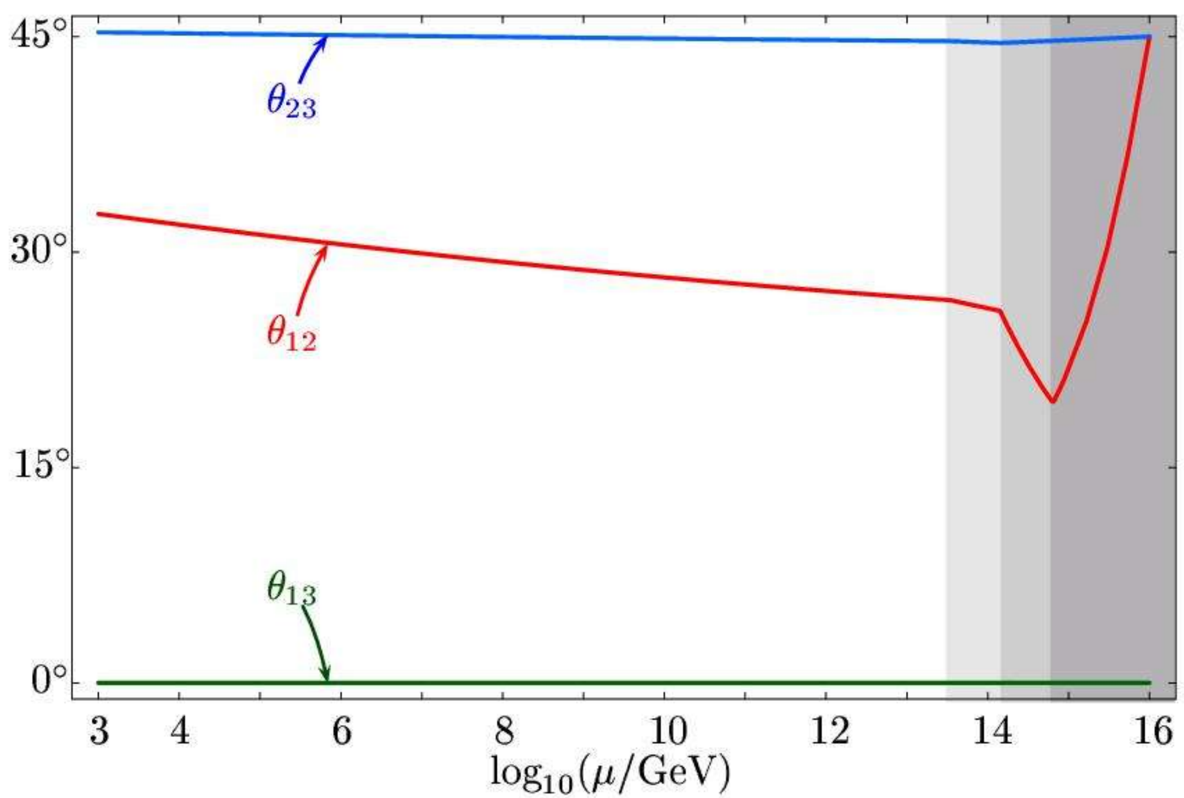
Low energy:

- Future: precision ν exp.!
  - θ_{13}? θ_{23} maximal?
    $\theta_{12} + \theta_C \sim 45°$? δ?
  - ν mass scale/ scheme?

RG Running

High energy:

- Unified Theories?
- Origin of ν masses?
- Flavour symmetries?
- ...



Content

Renormalization Group (RG) running of neutrino parameters in see-saw models

- RG evolution above and between the see-saw scales
- Analytic approximations: understanding and estimating RG effecs

Consequences for model building (very brief ...)

- New possibilities from large RG effects

Consequences with respect to future precision neutrino experiments

- RG corrections to $\theta_{23} = 45°$,
- to $\theta_{13} = 0°$ and
- to predictions for θ_{12} at M_{GUT}

Summary and a few discussion points ...

Mixing Angles and CP Phases in the Lepton Sector

Definition:

Charged lepton masses

ν-masses (assumed: 3 Majorana ν's)

$$U_{e_L} M_e U_{e_R}^\dagger = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad U_{\nu_L} m_{LL}^\nu U_{\nu_L}^T = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Charged EW current

$$\overline{e'_L}^f \gamma^\mu \nu'^f_L W_\mu^- \stackrel{!}{=} \overline{e_L}^f \gamma^\mu U_{MNS} \nu_L^f W_\mu^- \quad \Rightarrow \quad \boxed{U_{\text{MNS}} = U_{e_L} U_{\nu_L}^\dagger}$$

Mixing matrix in the lepton sector

Parametrization:

$$U_{MNS} = \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}) \cdot U \cdot \text{diag}(e^{i\frac{\varphi_1}{2}}, e^{i\frac{\varphi_2}{2}}, 1)$$

Majorana CP phases

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Dirac CP phase for the leptons

Known:

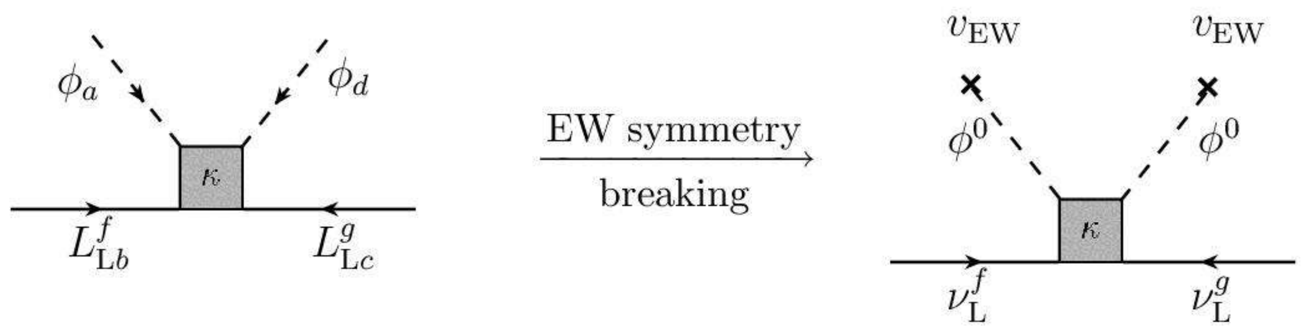
- $\theta_{12} \approx 33°$, Δm^2_{21}
- $\theta_{23} \approx 45°$, Δm^2_{31}
- $\theta_{13} < 13°$

Unknown:

- ν mass scale and mass scheme
- CP phases, ...

The Lowest Dimensional Neutrino Mass Operator

No renormalizable mass term for neutrinos in the SM (MSSM)



$$\mathscr{L}_\kappa^{\mathrm{SM}} = \frac{1}{4}\kappa_{gf}\,\overline{L^{\mathrm{C}g}_{\mathrm{L}c}}\varepsilon^{cd}\phi_d\, L^f_{\mathrm{L}b}\varepsilon^{ba}\phi_a + \mathrm{h.c.}\,,$$

$$\mathscr{L}_\kappa^{\mathrm{MSSM}} = -\frac{1}{4}\kappa_{gf}\,\hat{L}^g_c\varepsilon^{cd}(\hat{H}_u)_d\,\hat{L}^f_b\varepsilon^{ba}(\hat{H}_u)_a\big|_{\theta\theta} + \mathrm{h.c.}$$

$$\mathscr{L}_{\nu_{\mathrm{L}}\nu_{\mathrm{L}}} = \frac{1}{2}(m^\nu_{\mathrm{LL}})_{gf}\,\overline{\nu}^g_{\mathrm{L}}\nu^{\mathrm{C}f}_{\mathrm{L}}$$

Neutrino mass matrix of Majorana type

Smallness of neutrino masses:

$$\kappa \sim \frac{1}{\Lambda}$$

High realisation scale Λ (scale of new physics)

Alternative: radiative mechanism

Neutrinos provide a window to physics at very high energies!

Beyond the SM: LR Symmetric Unified Theories

Pati-Salam Unification: $\mathrm{G}_{422} = \mathrm{SU}(4)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$

RH neutrinos

$$f_\mathrm{L}^f = \begin{pmatrix} u_{\mathrm{L}r}^f & u_{\mathrm{L}y}^f & u_{\mathrm{L}b}^f & \nu_\mathrm{L}^f \\ d_{\mathrm{L}r}^f & d_{\mathrm{L}y}^f & d_{\mathrm{L}b}^f & e_\mathrm{L}^f \end{pmatrix} , \quad f_\mathrm{R}^f = \begin{pmatrix} u_{\mathrm{R}r}^f & u_{\mathrm{R}y}^f & u_{\mathrm{R}b}^f & \nu_\mathrm{R}^f \\ d_{\mathrm{R}r}^f & d_{\mathrm{R}y}^f & d_{\mathrm{R}b}^f & e_\mathrm{R}^f \end{pmatrix}$$

$\mathrm{SU}(2)_\mathrm{L}$ triplet Higgs (neutrino masses from coupling $(y_\mathrm{L})_{fg}\, L^f\, \Delta_\mathrm{L}\, L^g$)

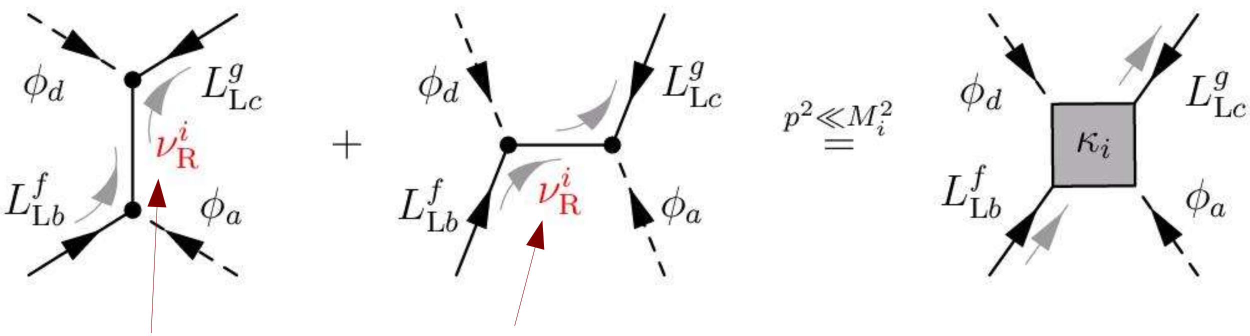
Field	f_L^f	$f_\mathrm{R}^{\mathrm{C}f}$	Φ	Φ'	χ_L	χ_R^*	Δ_L	Δ_R^*
$\mathrm{SU}(4)_\mathrm{C}$	$\mathbf{4}$	$\overline{\mathbf{4}}$	$\mathbf{1}$	$\mathbf{15}$	$\mathbf{4}$	$\overline{\mathbf{4}}$	$\mathbf{10}$	$\overline{\mathbf{10}}$
$\mathrm{SU}(2)_\mathrm{L}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$
$\mathrm{SU}(2)_\mathrm{R}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$

$$\mathrm{G}_{422} \xrightarrow{\langle\Delta_\mathrm{R}\rangle \text{ or } \langle\chi_\mathrm{R}\rangle} G_{321} \xrightarrow{\langle\Phi\rangle,\ \langle\Phi'\rangle} \mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{e}$$

Large masses for the RH neutrinos (from ren. coupling $(y_\mathrm{R})_{fg}\, R^f\, \Delta_\mathrm{R}\, R^g$)

SO(10) GUTs: $\mathbf{16}_{\mathrm{SO}(10)}^f = (\mathbf{4},\mathbf{2},\mathbf{1})_{G_{422}}^f + (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{G_{422}}^f = f_\mathrm{L}^f + f_\mathrm{R}^{\mathrm{C}f}$

Generating the Neutrino Mass Operator



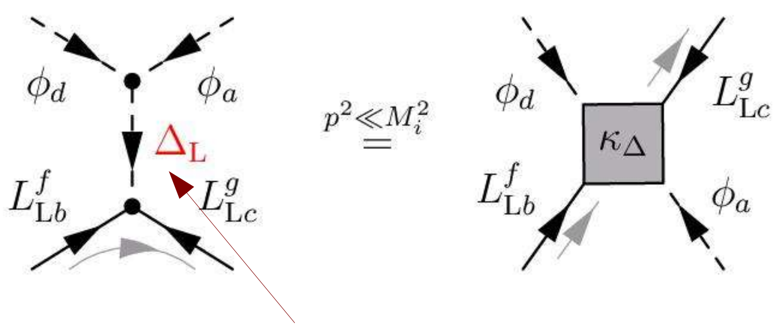
heavy RH neutrinos

P. Minkowski (1977), Gell-Mann, Glashow, Mohapatra, Ramond, Senjanovic, Slanski, Yanagida (1979/1980)

Type I see-saw

$$\overset{(1)}{\kappa} = \sum_i \kappa_i = 2(Y_\nu^T)_{gi}(M^{-1})_{ij}(Y_\nu)_{jf}$$

Type I +



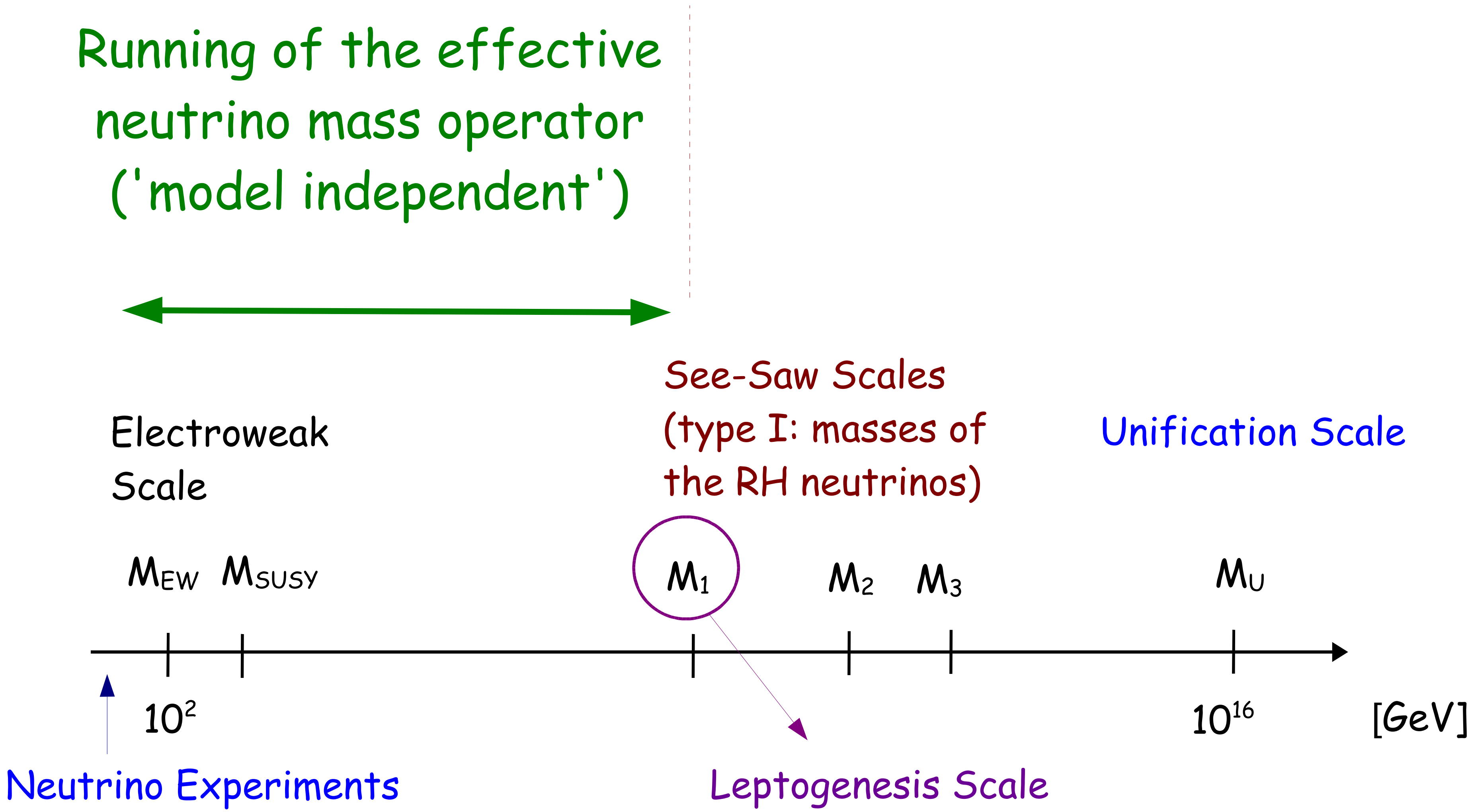
heavy $SU(2)_L$-triplet

Lazarides, Magg, Mohapatra, Schechter, Senjanovic, Shafi, Valle, Wetterich (1981)

Type II see-saw

$$\begin{aligned}\overset{(1)}{\kappa} &= \kappa_\Delta + \sum_i \kappa_i \\ &= -M_{\Delta_{\mathrm{L}}} + 2(Y_\nu^T)_{gi}(M^{-1})_{ij}(Y_\nu)_{jf}\end{aligned}$$

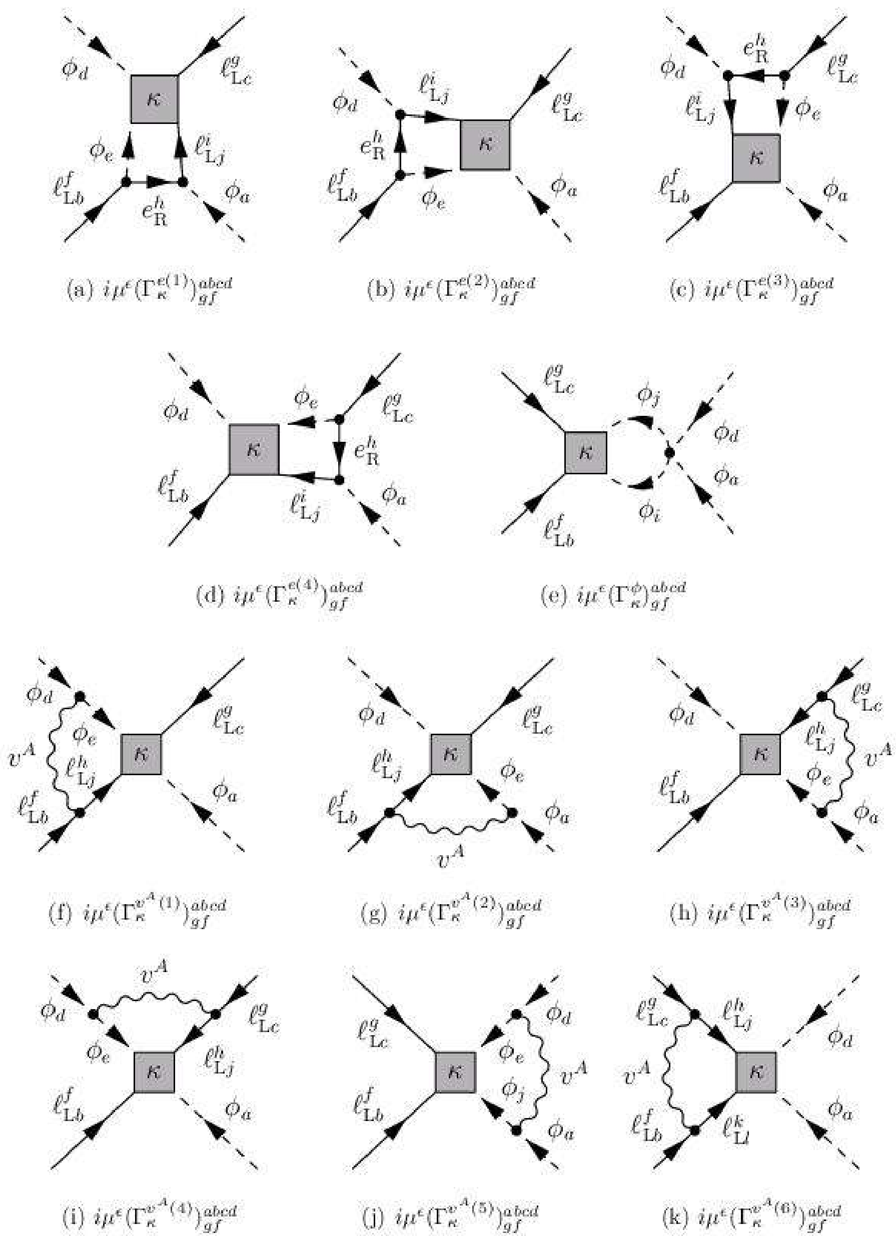
'Minimal' SeeSaw: Running in the Effective Theory



'minimal': there could of course be additional new particles ...

Stefan Antusch

β-Functions for the dim. 5 Neutrino Mass Operator



(a) $i\mu^{\epsilon}(\Gamma_{\kappa}^{e(1)})_{gf}^{abcd}$ (b) $i\mu^{\epsilon}(\Gamma_{\kappa}^{e(2)})_{gf}^{abcd}$ (c) $i\mu^{\epsilon}(\Gamma_{\kappa}^{e(3)})_{gf}^{abcd}$

(d) $i\mu^{\epsilon}(\Gamma_{\kappa}^{e(4)})_{gf}^{abcd}$ (e) $i\mu^{\epsilon}(\Gamma_{\kappa}^{\phi})_{gf}^{abcd}$

(f) $i\mu^{\epsilon}(\Gamma_{\kappa}^{v^A(1)})_{gf}^{abcd}$ (g) $i\mu^{\epsilon}(\Gamma_{\kappa}^{v^A(2)})_{gf}^{abcd}$ (h) $i\mu^{\epsilon}(\Gamma_{\kappa}^{v^A(3)})_{gf}^{abcd}$

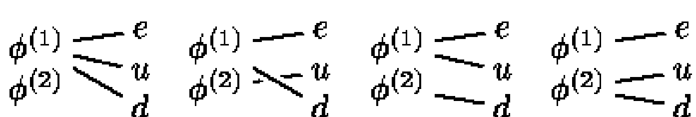
(i) $i\mu^{\epsilon}(\Gamma_{\kappa}^{v^A(4)})_{gf}^{abcd}$ (j) $i\mu^{\epsilon}(\Gamma_{\kappa}^{v^A(5)})_{gf}^{abcd}$ (k) $i\mu^{\epsilon}(\Gamma_{\kappa}^{v^A(6)})_{gf}^{abcd}$

β-Function in the Standard Model:

$$16\pi^2\beta_\kappa = -\frac{3}{2}\kappa\left(Y_e^\dagger Y_e\right) - \frac{3}{2}\left(Y_e^\dagger Y_e\right)^T\kappa + \lambda\kappa - 3g_2^2\kappa + 2\mathrm{Tr}\left(Y_e^\dagger Y_e\right)\kappa + 6\mathrm{Tr}\left(Y_u^\dagger Y_u\right)\kappa + 6\mathrm{Tr}\left(Y_d^\dagger Y_d\right)\kappa$$

S.A., M. Drees, J. Kersten, M. Lindner, M. Ratz
Phys. Lett. B 519, (2001), (hep-ph/0108005)

β-Functions in 2-Higgs-Doublet Models:



(i) (ii) (iii) (iv)

S.A., M. Drees, J. Kersten, M. Lindner, M. Ratz
Phys. Lett. B 525, (2001), (hep-ph/0110366);
For multi-Higgs models:
W. Grimus, L. Lavoura, Eur.Phys.J.C39:219-227,2005, (hep-ph/0409231)

β-Functions for the dim. 5 Neutrino Mass Operator in the MSSM

1-loop part:

$$16\pi^2 \beta_\kappa^{(1)} = \kappa\,(Y_e^\dagger Y_e) + (Y_e^\dagger Y_e)^T \kappa - 2g_1^2\kappa - 6g_2^2\kappa + 6\,\mathrm{Tr}\,(Y_u^\dagger Y_u)\,\kappa$$

Chankowski, Pluciennik, Phys. Lett. B316 (1993);
Babu, Leung, Pantaleone, Phys. Lett. B319, (1993).

2-loop part:

$$\begin{aligned}(4\pi)^4 \beta_\kappa^{(2)} = &\Big[-6\,\mathrm{Tr}\,(Y_u^\dagger Y_d Y_d^\dagger Y_u) - 18\,\mathrm{Tr}\,(Y_u^\dagger Y_u Y_u^\dagger Y_u) + \frac{8}{5} g_1^2\,\mathrm{Tr}\,(Y_u^\dagger Y_u) \\ &+ 32\, g_3^2\,\mathrm{Tr}\,(Y_u^\dagger Y_u) + \frac{207}{25}\, g_1^4 + \frac{18}{5}\, g_1^2 g_2^2 + 15\, g_2^4\Big]\,\kappa \\ &- \left[2\,(Y_e^\dagger Y_e Y_e^\dagger Y_e)^T - \left(\frac{6}{5} g_1^2 - \mathrm{Tr}\,(Y_e Y_e^\dagger) - 3\,\mathrm{Tr}\,(Y_d Y_d^\dagger)\right)(Y_e^\dagger Y_e)^T\right]\kappa \\ &- \kappa\left[2\,Y_e^\dagger Y_e Y_e^\dagger Y_e - \left(\frac{6}{5} g_1^2 - \mathrm{Tr}\,(Y_e Y_e^\dagger) - 3\,\mathrm{Tr}\,(Y_d Y_d^\dagger)\right) Y_e^\dagger Y_e\right]\end{aligned}$$

S.A., M. Ratz, JHEP 0207 (2002) 059

Analytical Approximations: Below M_{R1}

Example: Formulae for the mixing angles

J.A. Casas, J.R. Espinosa, A. Ibarra I. Navarro ('99)
P.H. Chankowski, W. Krolikowski, S. Pokorski ('99)
S.A., J. Kersten, M. Lindner, M. Ratz ('03)

Strong enhancement possible due to small mass squared difference

Dependence on CP phases

$t := \ln(\mu/\mu_0)$

$$\dot{\theta}_{12} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12}\, s_{23}^2 \frac{|m_1\, e^{i\varphi_1} + m_2\, e^{i\varphi_2}|^2}{\Delta m^2_{\mathrm{sol}}} + \mathscr{O}(\theta_{13})$$

$$\dot{\theta}_{13} \approx \frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_3}{\Delta m^2_{\mathrm{atm}}} \left[m_1 \cos(\varphi_1 - \delta) - m_2 \cos(\varphi_2 - \delta)\right] + \mathscr{O}(\theta_{13})$$

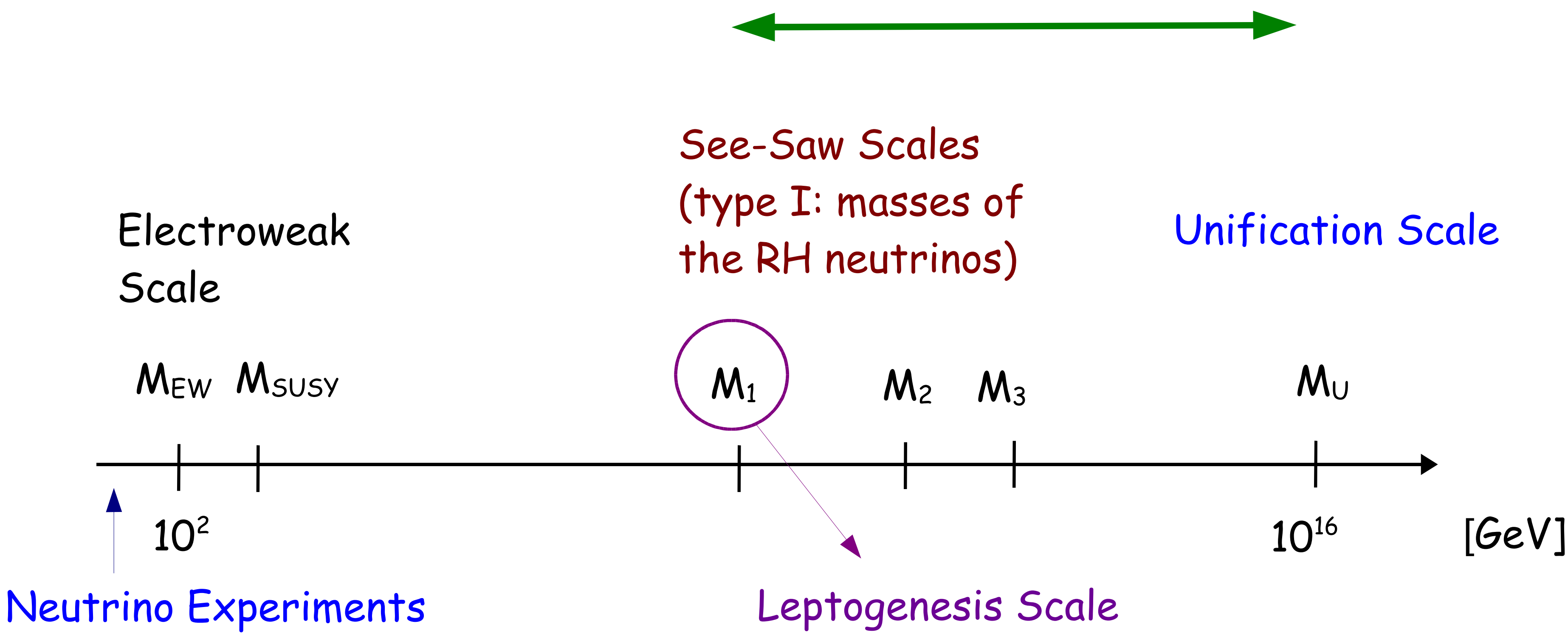
$$\dot{\theta}_{23} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{23} \frac{1}{\Delta m^2_{\mathrm{atm}}} \left[c_{12}^2\, |m_2\, e^{i\varphi_2} + m_3|^2 + s_{12}^2\, |m_1\, e^{i\varphi_1} + m_3|^2\right] + \mathscr{O}(\theta_{13})$$

(shown: MSSM, leading order in θ_{13} and in $\Delta m^2_{\mathrm{sol}} / \Delta m^2_{\mathrm{atm}}$)

Generically:
- Running enhanced for larger neutrino masses
- Running unsuppressed because mixings are large
- Running in the MSSM enhanced for large tan β

Running in 'Minimal' SeeSaw Scenarios

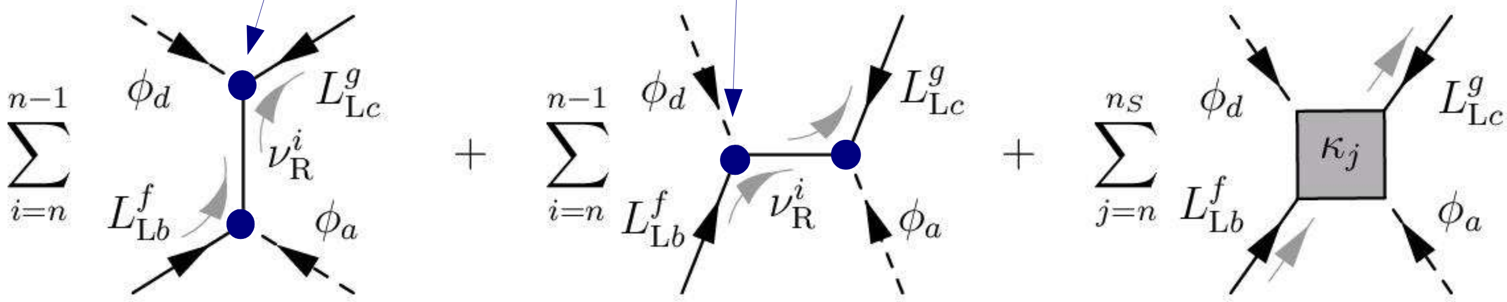
Running above M_{R1} (depends on neutrino Yukawa matrix Y_ν)



'minimal': there could of course be additional new particles ...

Between the See-Saw Scales: ν_{Ri} partly integrated out

Neutrino Yukawa couplings



$M_i < \mu$
(ν_{Ri} propagate in the theory)

$M_i > \mu$
(ν_{Ri} effectively integrated out)

$$Y_\nu \to \left(\begin{array}{ccc} (Y_\nu)_{1,1} & \cdots & (Y_\nu)_{1,n_F} \\ \vdots & & \vdots \\ (Y_\nu)_{n-1,1} & \cdots & (Y_\nu)_{n-1,n_F} \\ \hline 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{array}\right)$$

$$\left.\begin{array}{ccc} (Y_\nu)_{1,1} & \cdots & (Y_\nu)_{1,n_F} \\ \vdots & & \vdots \\ (Y_\nu)_{n-1,1} & \cdots & (Y_\nu)_{n-1,n_F} \end{array}\right\} =: \overset{(n)}{Y}_\nu$$

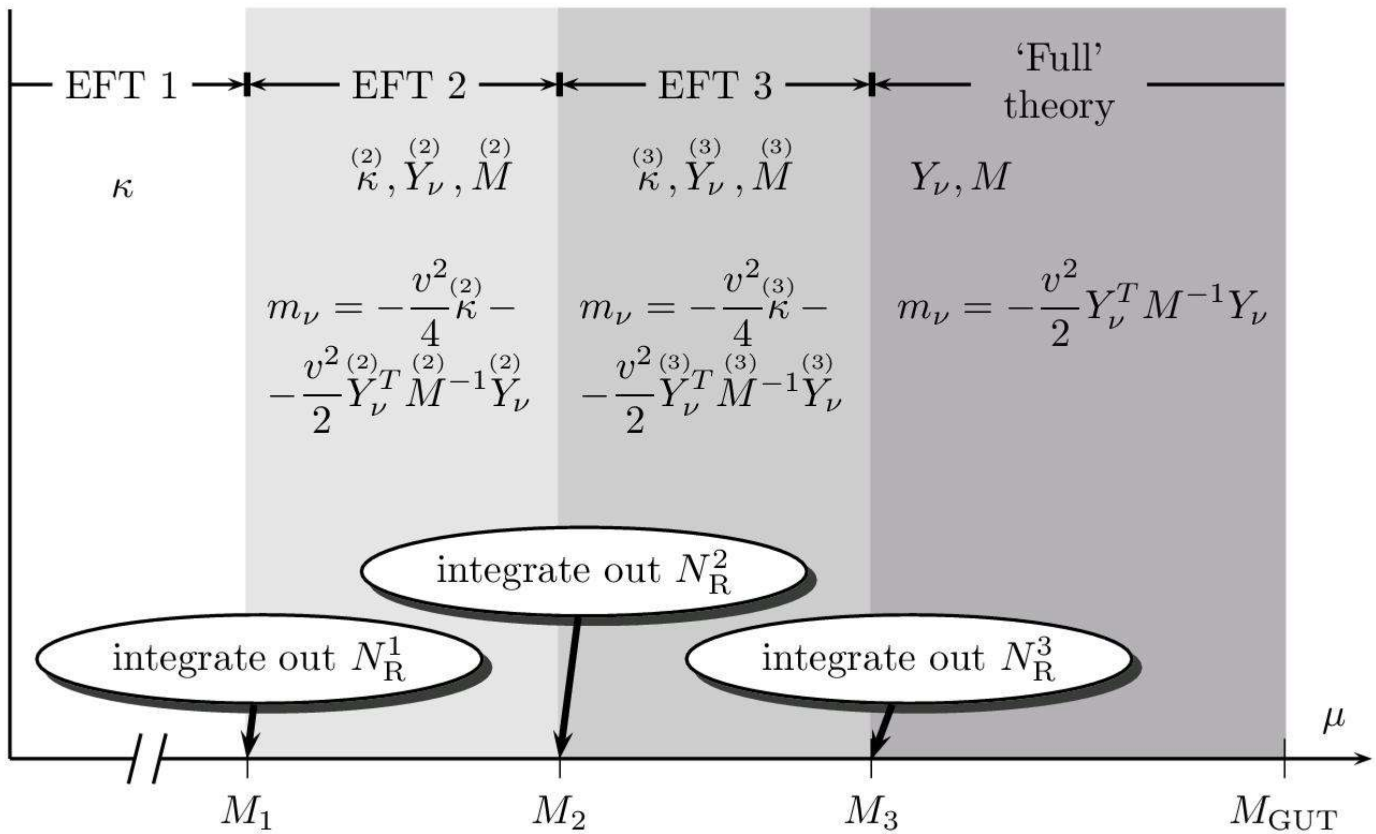
Neutrino Yukawa matrix between the see-saw scales

(n): below n-th see-saw scale

S.A., J. Kersten, M. Lindner, M. Ratz, Phys. Lett. B538 (2002)

Numerical RG Evolution: REAP/MPT Software Packages

introduced in: S.A., J. Kersten, M. Lindner, M. Ratz, M. A. Schmidt JHEP 0503 (2005) 024, (hep-ph/0501272)



- REAP (Renormalization group Evolution of Angles and Phases): Mathematica package for numerical RG evolution (heavy RH neutrinos successively integrated out)
- MPT (Mixing Parameter Tools): package for extracting masses, mixing angles and CP phases from the quark and lepton mass matrices

public, download: www.ph.tum.de/~rge

Implications/Possiblities for Model Building from RG Running

Some possibilities for model building from RG running:

- Radiative magnification of mixing angles

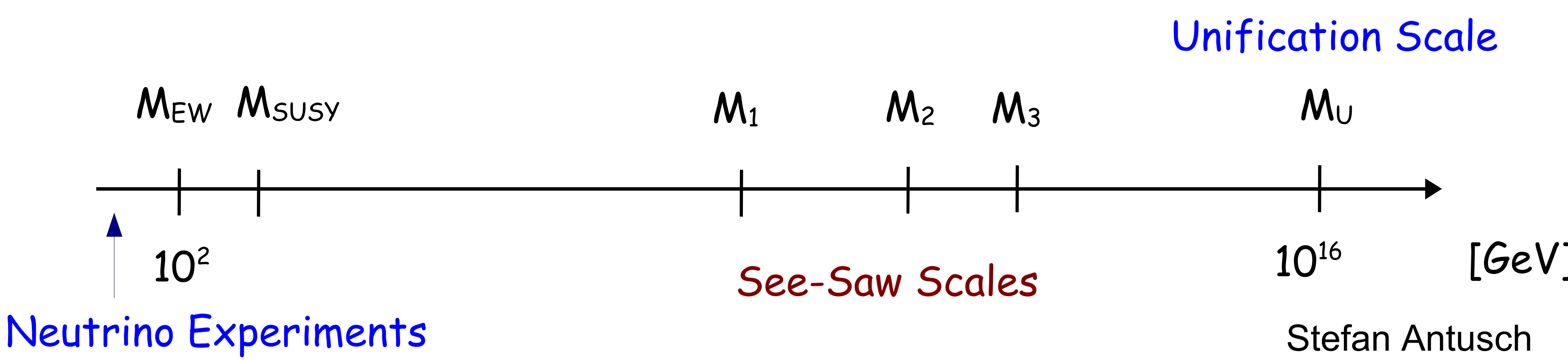
  see e.g.: M. Tanimoto ('95); K. R. S. Balaji, A.S. Dighe, R.N. Mohapatra, M.K. Parida (''00); T. Miura, E. Takasugi, M. Yoshimura ('00); S.A., M. Ratz ('02); H. S. Goh, R. N. Mohapatra ('03); ...

- Bi-maximal mixing scenarios (at M_{GUT}) can be compatible with experiments

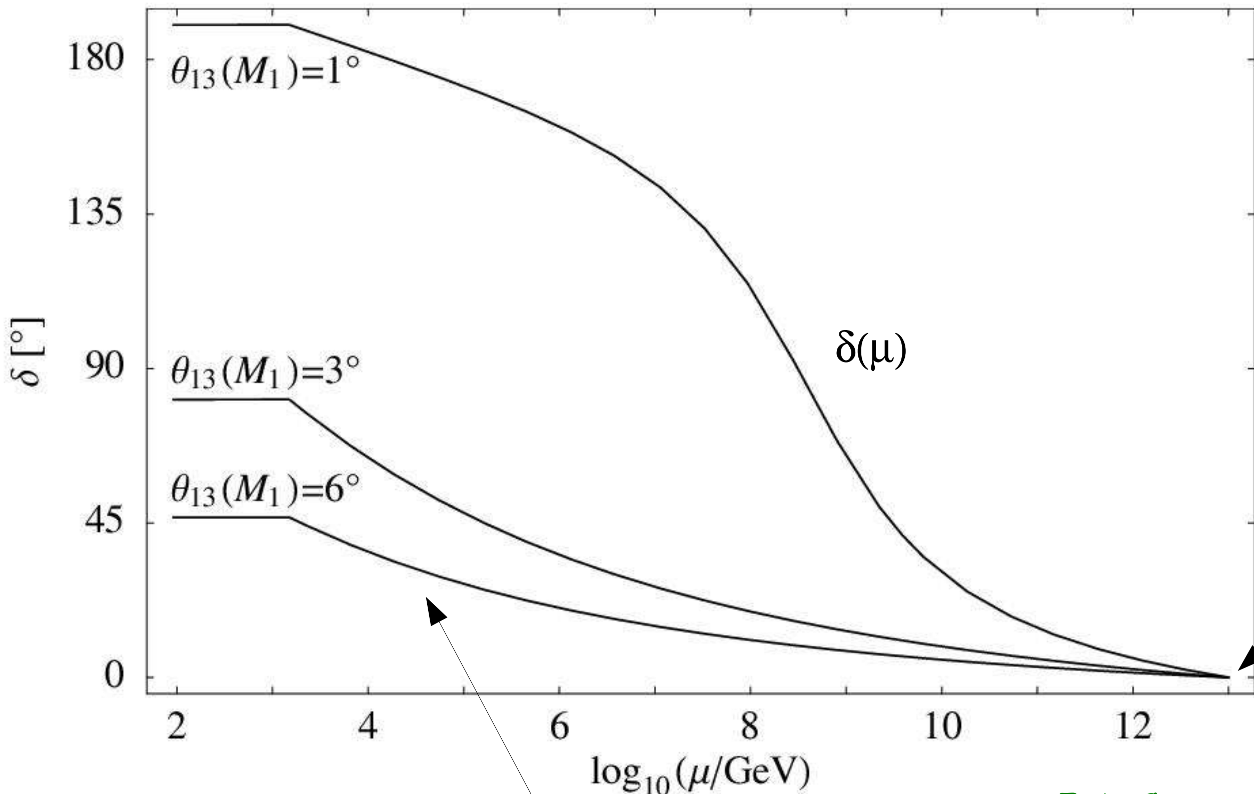
  S.A., J. Kersten, M. Lindner, M. Ratz ('02),T. Miura, T. Shindou, E. Takasugi ('03), T. Shindou, E. Takasugi ('04), ...

- Radiative generation of small mass splittings

  P.H. Chankowski, A. Ioannisian, S. Pokorski, J. W. F. Valle ('00); M.-C. Chen, K.T. Mahanthappa ('01); A.S. Joshipura, S.D. Rindani, N.N. Singh ('02), A. S. Joshipura, S.D. Rindani ('03), N.N. Singh, M.K. Das ('04); ...



Radiative Generation of the Dirac CP Phase δ



(MSSM, tan β= 50,
m_1= 0.17eV)

Initial condition at Λ:

δ = 0°, $\varphi_1 - \varphi_2$ = 120°

J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro ('99)
S.A., J. Kersten, M. Lindner, M. Ratz ('03)

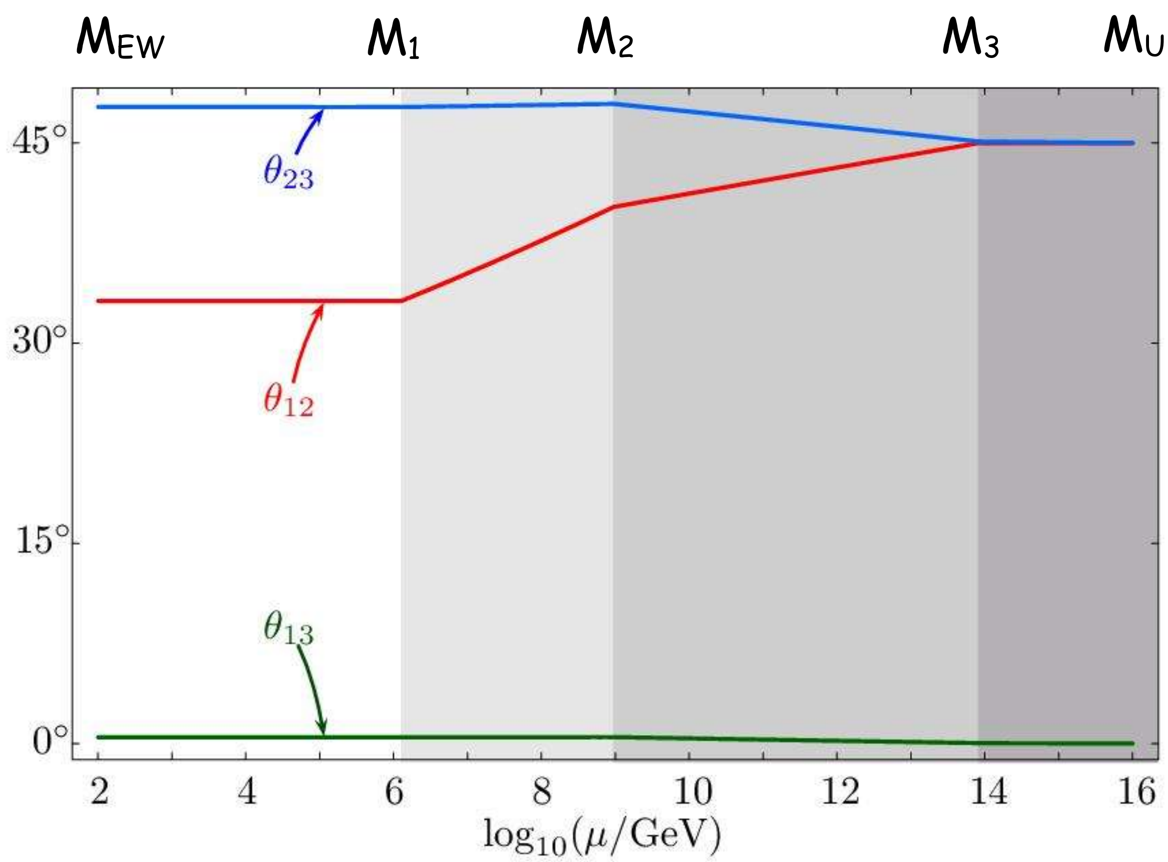
Dirac CP phase δ = 0 at M_{GUT},
non-zero δ at low energy

$$\dot{\delta} = \frac{C y_\tau^2}{32\pi^2} \frac{\delta^{(-1)}}{\theta_{13}} + \frac{C y_\tau^2}{8\pi^2} \delta^{(0)} + \mathscr{O}(\theta_{13})$$

From Bi-maximal Mixing to LMA solution by Running

S.A., J. Kersten, M. Lindner, M. Ratz, Phys. Lett. B544 (2002)

LMA solution ($\theta_{12} \approx 33°$)



Bi-maximal Mixing

normal neutrino mass ordering

$m_1 = 4 \cdot 10^{-3}$ eV

Example: SM,

$Y_\nu = \mathrm{diag}(\varepsilon^2, \varepsilon, 1)$

RG Corrections and Future Precision Neutrino Experiments

- RG corrections to θ_{13} vs. experimental sensitivities: Special case: $\theta_{13} = 0$.

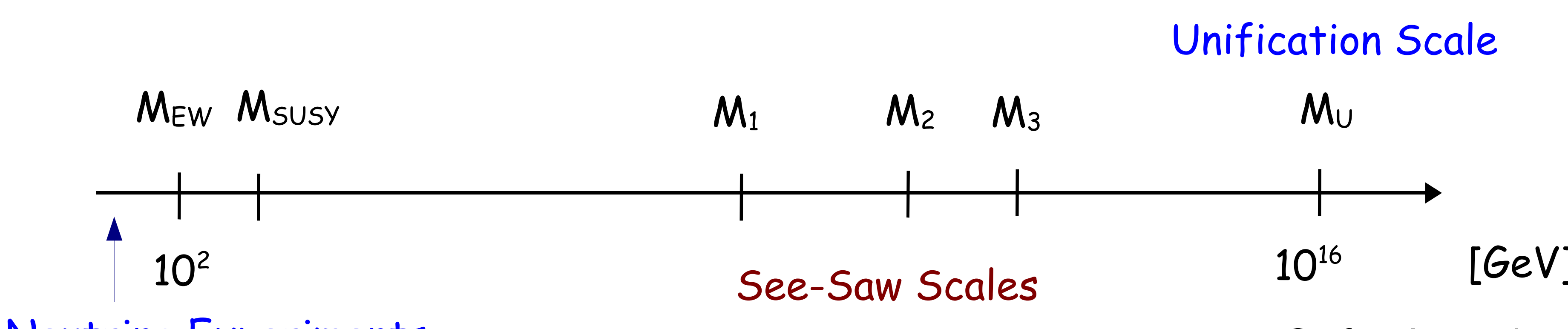
  S.A., J. Kersten, M. Lindner, M. Ratz ('03); Jian-wei Mei, Zhi-zhong Xing ('04);
  S.A., J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt ('05);

- How 'maximal' is θ_{23}? Deviation $\theta_{23} - \pi/4$ induced by RG running.

  S.A., J. Kersten, M. Lindner, M. Ratz ('03); S.A., M. Huber, J. Kersten, T. Schwetz, W. Winter ('04)

- RG corrections to θ_{12}. Are there signatures of unified theories/flavour symmetries?

  Papers on QLC: M. Raidal ('04), H. Minakata, A. Y. Smirnov ('04), ...
  RG effects (+ above M_{Ri}), e.g.: S.A., J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt ('05); ...



RG Corrections to θ_{13}

Analytical Approximation (below see-saw scales): $\Delta\theta_{13} = \dot{\theta}_{13} \ln(\Lambda/M_{EW})$

quasi-degenerate ν's: $\varphi_1 = \varphi_2$ can damp large RG effects

$$\dot{\theta}_{13} \approx \frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_3}{\Delta m^2_{\mathrm{atm}}} [m_1 \cos(\varphi_1 - \delta) - m_2 \cos(\varphi_2 - \delta)] + \mathscr{O}(\theta_{13})$$

$\dot{\theta}_{13} = 0$ possible for $m_3 = 0$ (or for 'conspiracy' of CP phases)

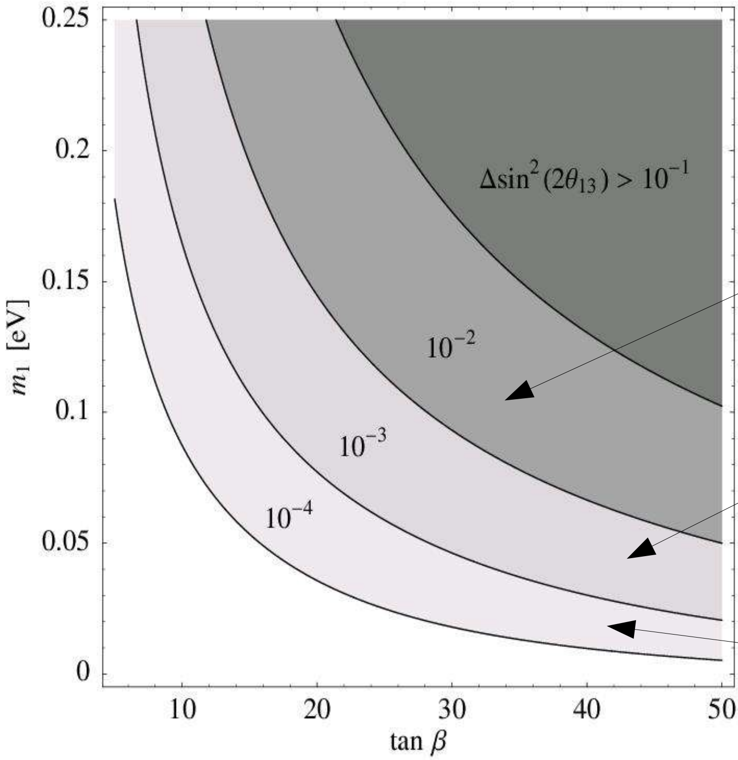
- In general: RG effects larger for $\tan\beta \uparrow$ and for $m_1 \uparrow$

S.A., J. Kersten, M. Lindner, M. Ratz (hep-ph/0305273)

RG Corrections to $\theta_{13} = 0$ @ High Energy

Graphical illustration of RG corrections ($\Delta\sin^2(2\theta_{13})$)

(running of dim. 5 operator between $\Lambda = 10^{12}$ GeV and M_{EW} in the MSSM)



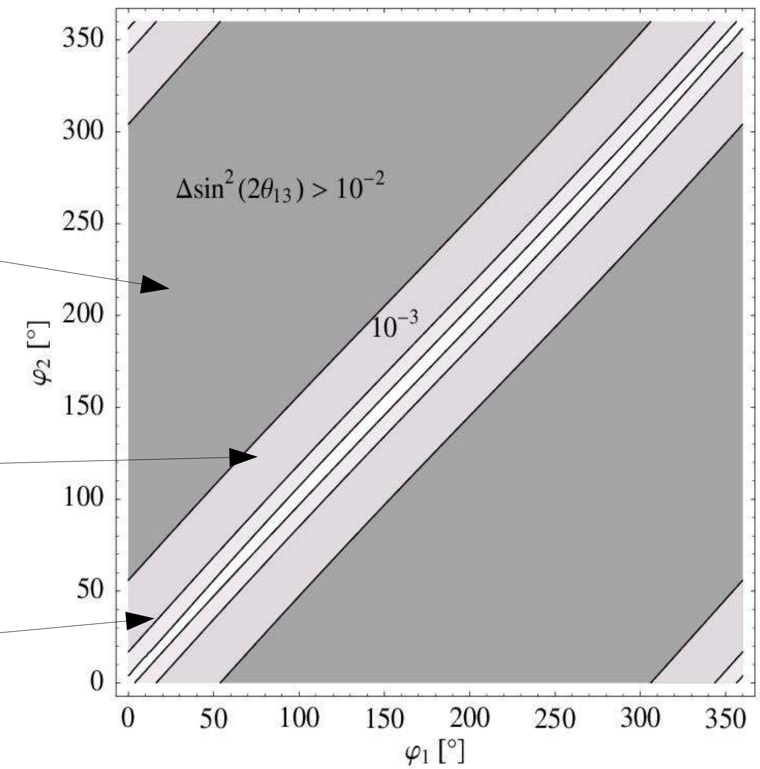
($\varphi_1 - \varphi_2 = 180°$, $\delta = 0$, normal scheme)

Expected sensitivities:

Reactor and superbeam exp.

Superbeam exp. (upgrade)

Neutrino factory



($\tan\beta = 50$, $m_1 = 0.08$eV, $\delta = 0$)

Even for $\theta_{13} = 0$ @ high energy, RG running $\Rightarrow$ in general non-zero θ_{13} @ low energy

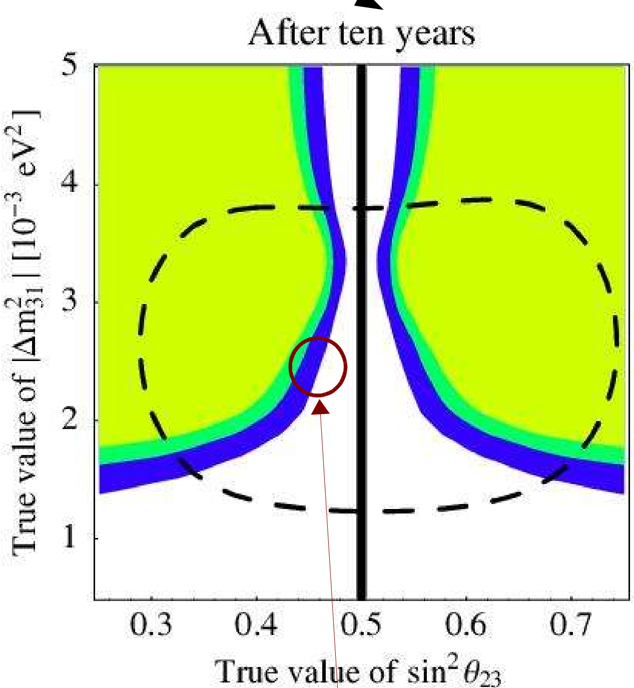
S.A., J. Kersten, M. Lindner, M. Ratz (hep-ph/0305273)

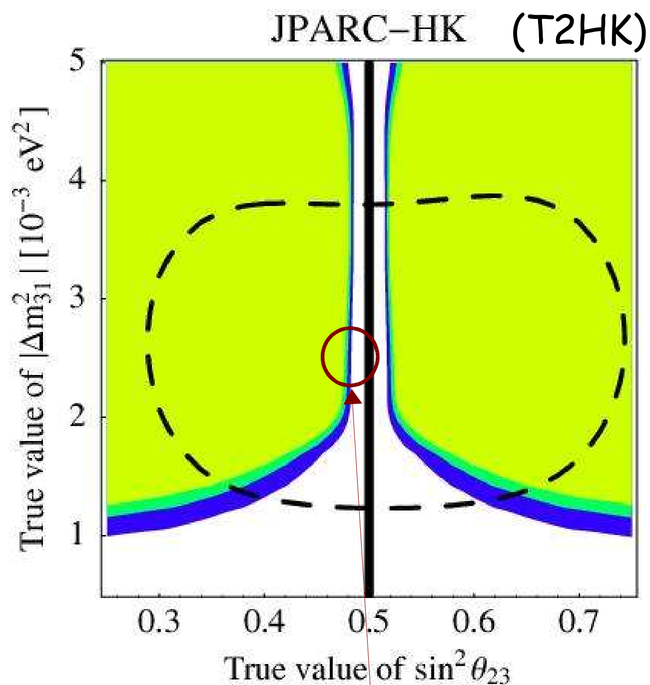
Estimated Exp. Sensitivties for Excluding θ_{23} = 45°

Combined:
MINOS,
ICARUS, OPERA,
T2K, NuMI

Long Baseline Experiments

4 MW, 1 Mt





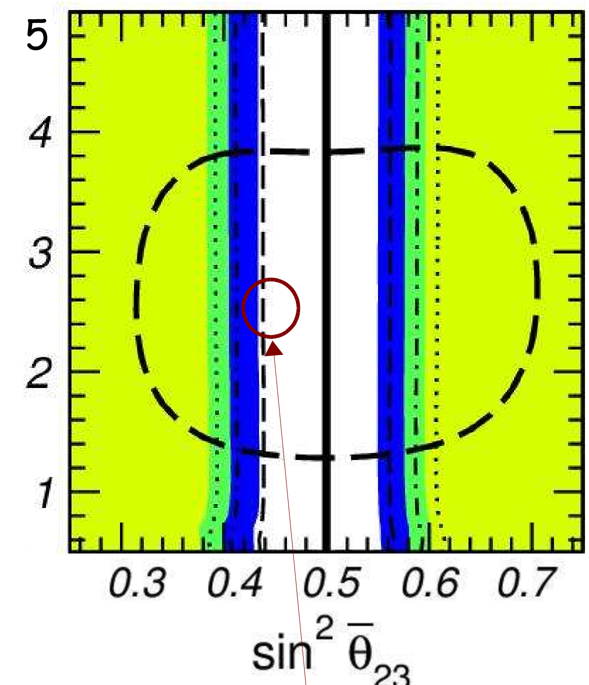
(T2HK)

$\Delta\theta_{23} \approx 3°$ @ 90% CL

$\Delta\theta_{23} \approx 1°$ @ 90% CL

S.A., P. Huber, J. Kersten, T. Schwetz,
W. Winter (hep-ph/0404268)

Atm. neutrino experiment
with statistics SK x 20



$\Delta\theta_{23} \approx 3°$ @ 90% CL

and θ_{23} > or < $\pi/4$

M.C. Gonzalez-Garcia, M. Maltoni,
A.Yu. Smirnov (hep-ph/0408170)

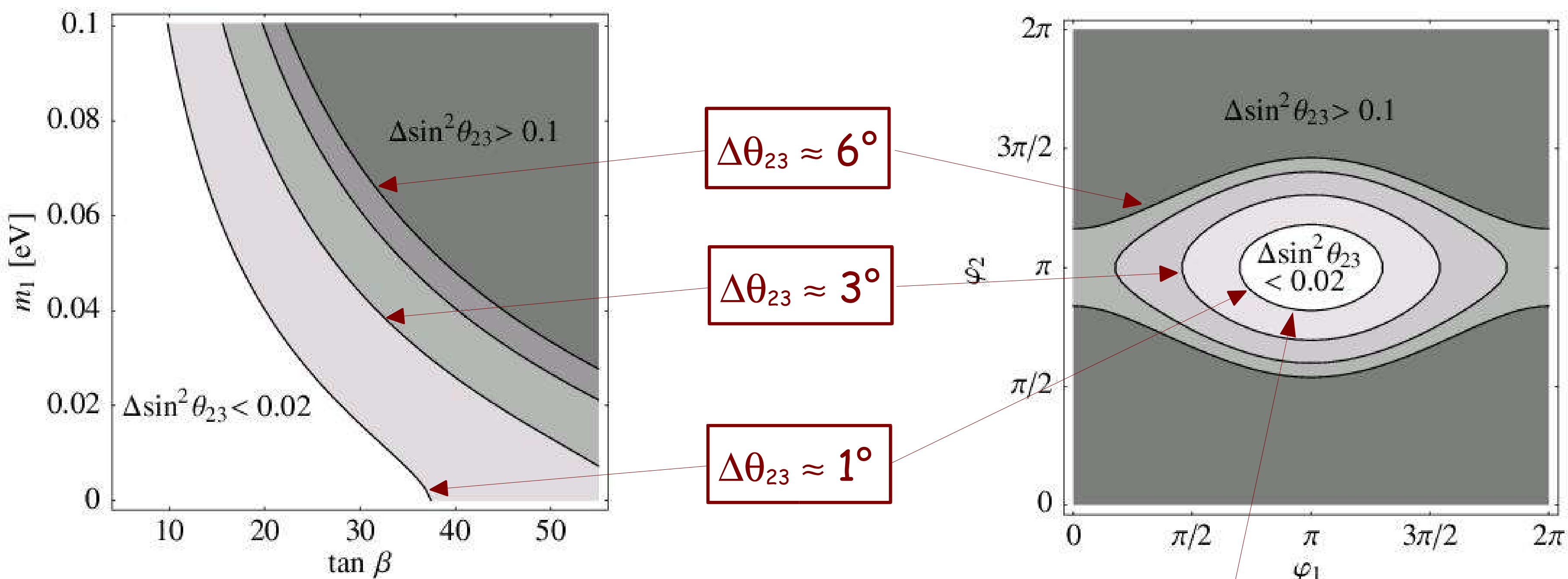
RG Corrections to Maximal Mixing θ_{23} = 45°

Analytical Approximation (below see-saw scales): $\Delta\theta_{23} = \dot{\theta}_{23} \ln(\Lambda/M_{EW})$

$$\dot{\theta}_{23} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{23} \frac{1}{\Delta m^2_{\rm atm}} \left[c_{12}^2 \left|m_2 e^{i\varphi_2} + m_3\right|^2 + s_{12}^2 \left|m_1 e^{i\varphi_1} + m_3\right|^2\right] + \mathscr{O}(\theta_{13})$$

Note: $\Delta\theta_{23}$ > (<) 0 for Δm^2_{atm} < (>) 0

Example: Conservative estimate (ignore Y_ν contributions) of RG corrections (MSSM)



(MSSM, $\varphi_1 = \varphi_1 = 0$, $\theta_{13} = 0$)

Majorana CP phases (m1 = 0.075 eV)
(can damp RG effects, but $\Delta\theta_{23}$ always ≠ 0!)

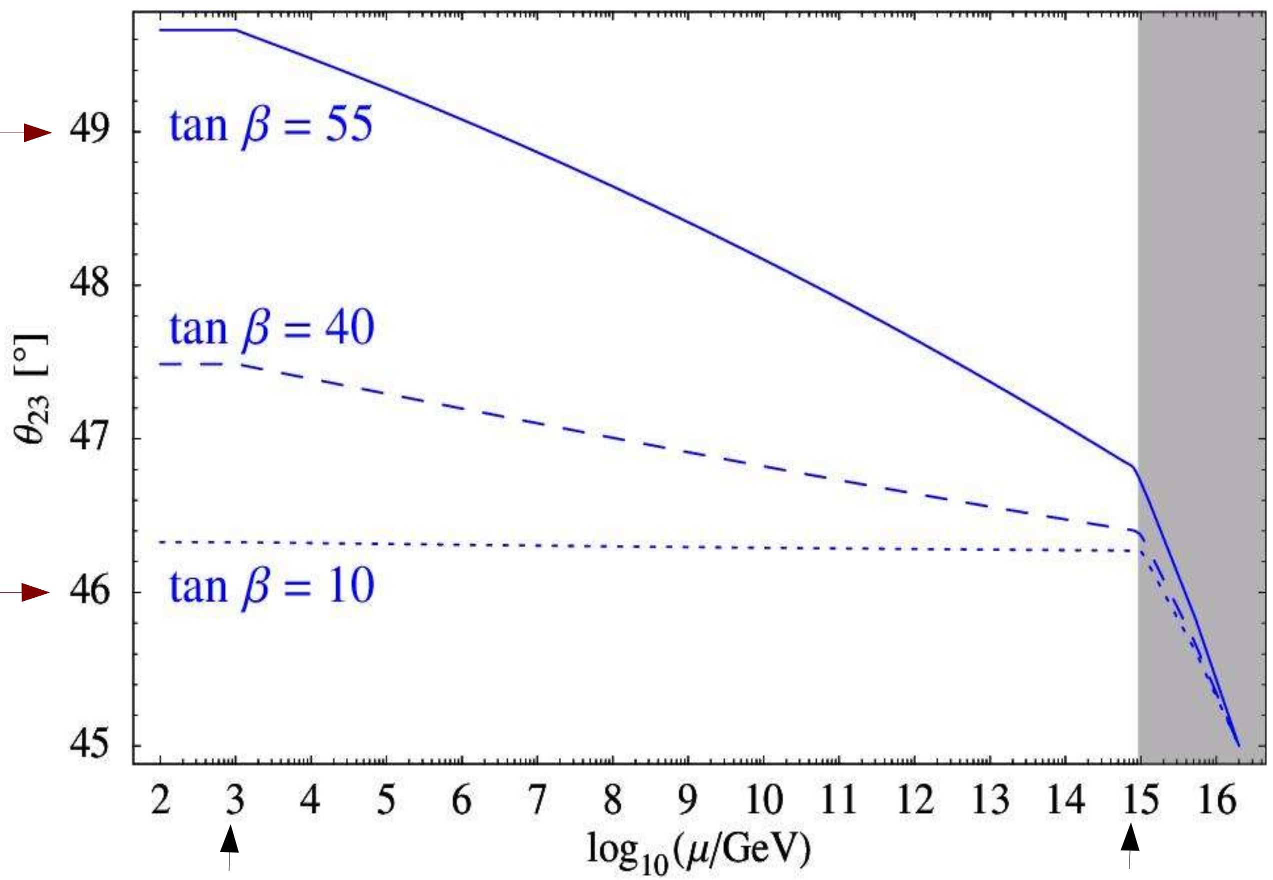
S.A., J. Kersten, M. Lindner, M. Ratz (hep-ph/0305273)
S.A., M. Huber, J. Kersten, T. Schwetz, W. Winter (hep-ph/0404268)

RG Corrections to Maximal Mixing $\theta_{23} = 45° \ @ \ M_{GUT}$

Even if maximal mixing is predicted @ high energy, RG running will lead to deviation:

$\Delta\theta_{23} \approx 4°$

$\Delta\theta_{23} \approx 1°$



M_{SUSY} (MSSM)

M_3

$$Y_\nu \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

leading order
(RL convention)

$\theta_{23} = 45°$
at M_U

n. hierarchy
$m_1 \approx 0$ eV

S.A., J. Kersten, M. Lindner, M. Ratz, M. A. Schmidt ('05)

Signatures of Unified Theories & θ_{12}, θ_{13}

Quark-Lepton Unification via Pati-Salam Gauge Group:

'Similar' structure of Y_u, Y_d, Y_e, Y_ν (hierarchical, small entries originate from effective operators – we expect them to be related, but not equal)

$Y_u \xleftrightarrow{V_{CKM}} Y_d$

Often: $\theta_{12}^d \gg \theta_{12}^u$

$\theta_{12}^d \approx \theta_C \sim 13°$

'Similar' structure

See-saw $\Rightarrow$

$m_\nu = v_{EW}^2\, Y_\nu M_R^{-1} Y_\nu^T$

$m_\nu \xleftrightarrow{V_{MNS}} Y_e$

Then:

$\theta_{12}^e \approx X_{CG}\, \theta_{12}^d \approx X_{CG}\, \theta_C$

(group theory factor X_{CG}, e.g. $X_{CG} = 1/3$ for Georgi-Jarlskog relation)

Correction to θ_{12}^ν (e.g. ~ 45° or 35.26°) and θ_{13}^ν!

$$\boxed{\begin{aligned}\theta_{12} &\approx \theta_{12}^\nu - 1/\sqrt{2}\, X_{CG}\, \theta_C \cos(\delta_{MNS} - \pi)\\ \theta_{13} &\approx 1/\sqrt{2}\, X_{CG}\, \theta_C\end{aligned}}$$

for small θ_{13}^ν and small charged lepton mixings

S.A., S.F. King ('05)

RG Correction to θ_{12}

$\Delta\theta_{12} \approx \dot{\theta}_{12} \ln(\Lambda/M_{EW})$

$|\varphi_1 - \varphi_2| = \pi$ can damp large RG effects for $m_1 \sim m_2$

$$\dot{\theta}_{12} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12}\, s_{23}^2 \frac{|m_1\, e^{i\varphi_1} + m_2\, e^{i\varphi_2}|^2}{\Delta m^2_{\mathrm{sol}}} + \mathscr{O}(\theta_{13})$$

Can be dominant for inverted hierarchy with $|\varphi_1 - \varphi_2| = \pi$

- Example: predictions of a unified model for QLC with inverted hierarchy and $|\varphi_1 - \varphi_2| = \pi$

quantity	θ_{12}	θ_{13}
prediction at M_U	$\frac{\pi}{4} - 1.06\,\theta_C$	$1.06\,\theta_C$
prediction at M_{EW}	$\frac{\pi}{4} - 1.06\,\theta_C - 0.8° \approx 30.5°$	$1.06\,\theta_C - 0.5° \approx 13.2°$

RG corrections

(calculated with REAP)

S.A., S.F. King, R. N. Mohapatra ('05)

Neutrinos can provide a window to physics at very high energies ...

One interesting aspect:

Precision tests of flavour models by future neutrino experiments

Requires:

- High precision experiments
- High accuracy of model predictions
- Inclusion of RG corrections