

# *Probing the Maximality and Octant of $\theta_{23}$ using $\nu_{\mu}$ -Disappearance*

Sandhya Choubey

University of Oxford

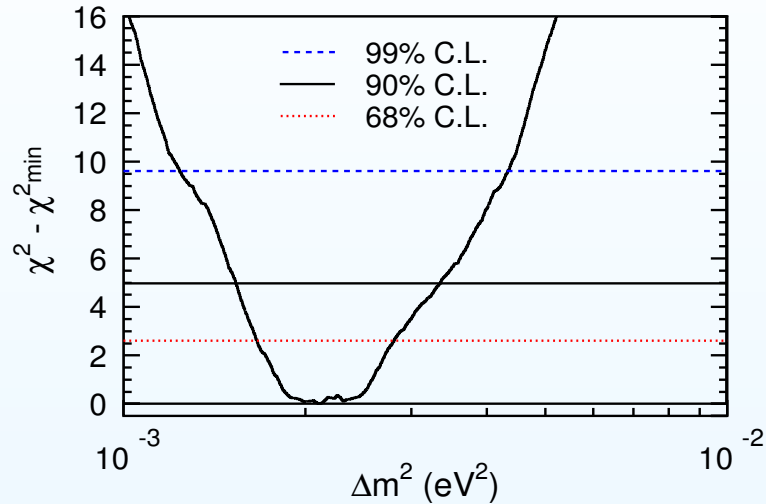


International Neutrino Factory and Superbeam Scoping Study  
Imperial College, London, 14-21 November 2005

# Atmospheric Neutrino Oscillation Parameters

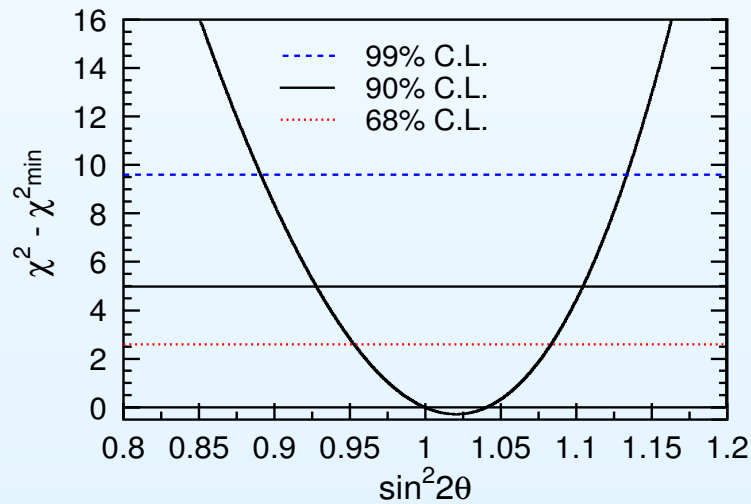
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# Atmospheric Neutrino Oscillation Parameters



● 99.73% C.L. range

$$\Delta m_{31}^2 = (1.3 - 4.2) \times 10^{-3} \text{eV}^2 ; \text{spread} = 53\%$$

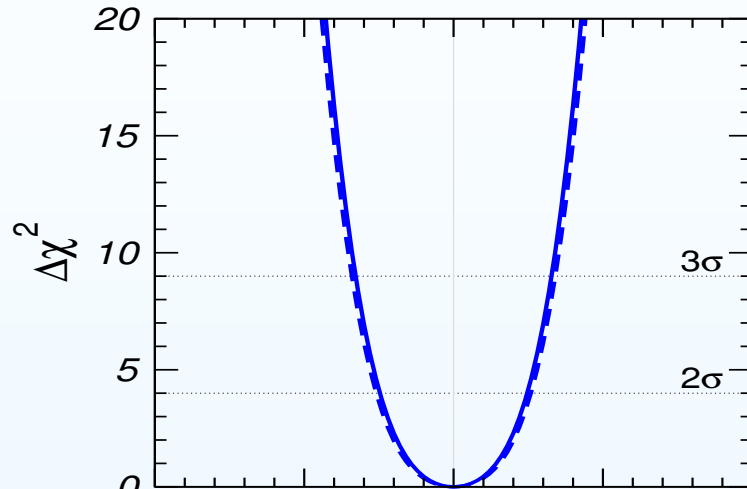


$$\sin^2 2\theta_{23} < 0.89 ; \text{spread} = 5.8\% ;$$

$$\sin^2 \theta_{23} = (0.34 - 0.66) ; \text{spread} = 34\%$$

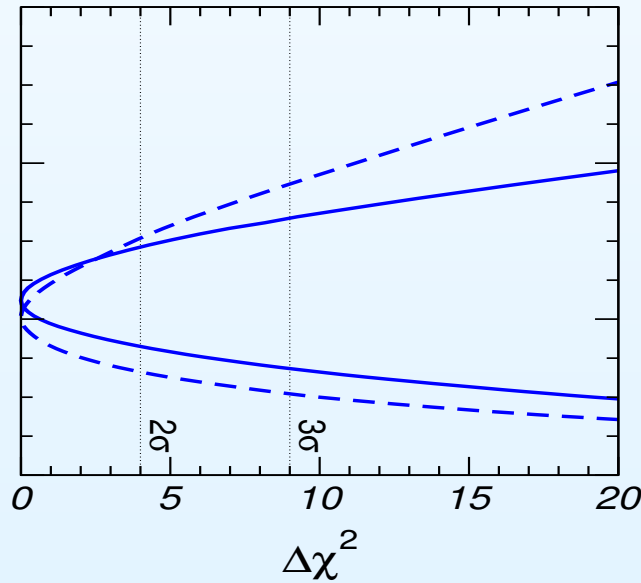
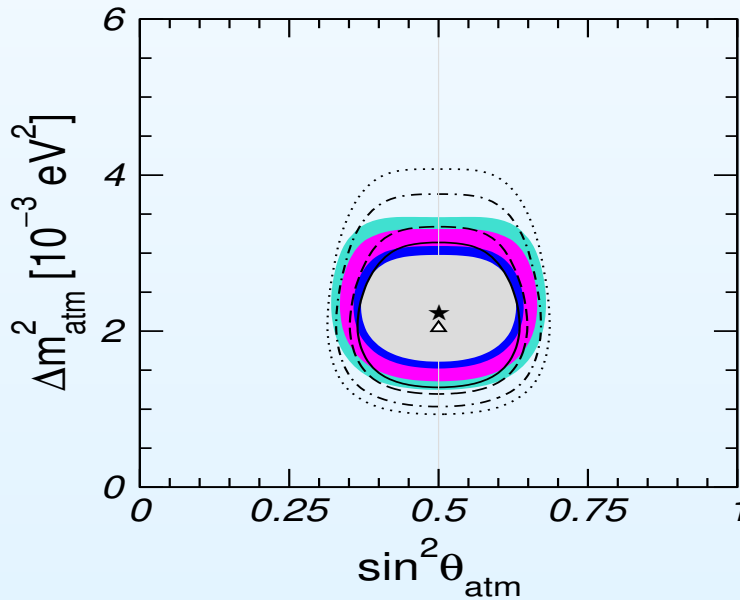
SK Collab, hep-ex/0502026

# Combined Allowed Areas from SK+K2K



$\Delta m_{31}^2 = (1.4 - 3.3) \times 10^{-3}$  ; spr=40%  
 $\sin^2\theta_{23} = (0.34 - 0.66)$  ; spread=34%

--- atmospheric only  
 — atmospheric + K2K



Maltoni et al, hep-ph/0405172

# Future Precision Measurement

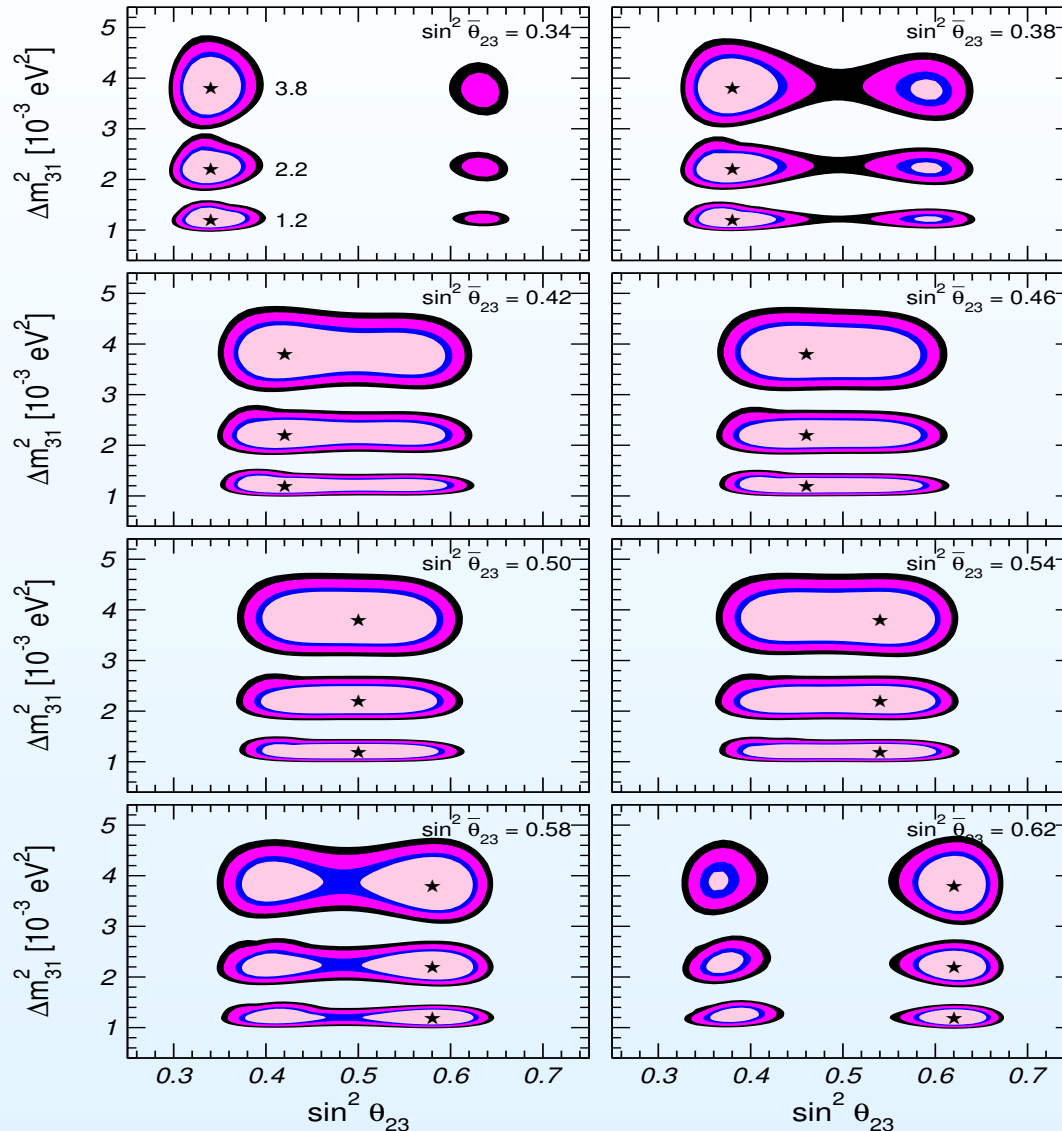
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## Potential of Long Baseline Experiments

Experiment	$ \Delta m_{31}^2 $	$\sin^2 \theta_{23}$
Current	44%	40%
MINOS+CNGS	13%	38%
T2K	6%	22%
NO $\nu$ A	13%	42%
Combination	4.5%	20%

Huber et al hep-ph/0403068

# Constraints from Future SK-like Atmospheric Data



• From 20 SKyr at  $3\sigma$ ,  
 $\Delta m_{31}^2 = 0.002 \text{ eV}^2, s_{23}^2 = 0.5$

spread in  $\Delta m_{31}^2 \sim 17\%$

spread in  $\sin^2 \theta_{23} \sim 24\%$

Gonzalez-Garcia et al, hep-ph/0408170

# Measuring the Deviation of $\sin^2 \theta_{23}$ from Maximal

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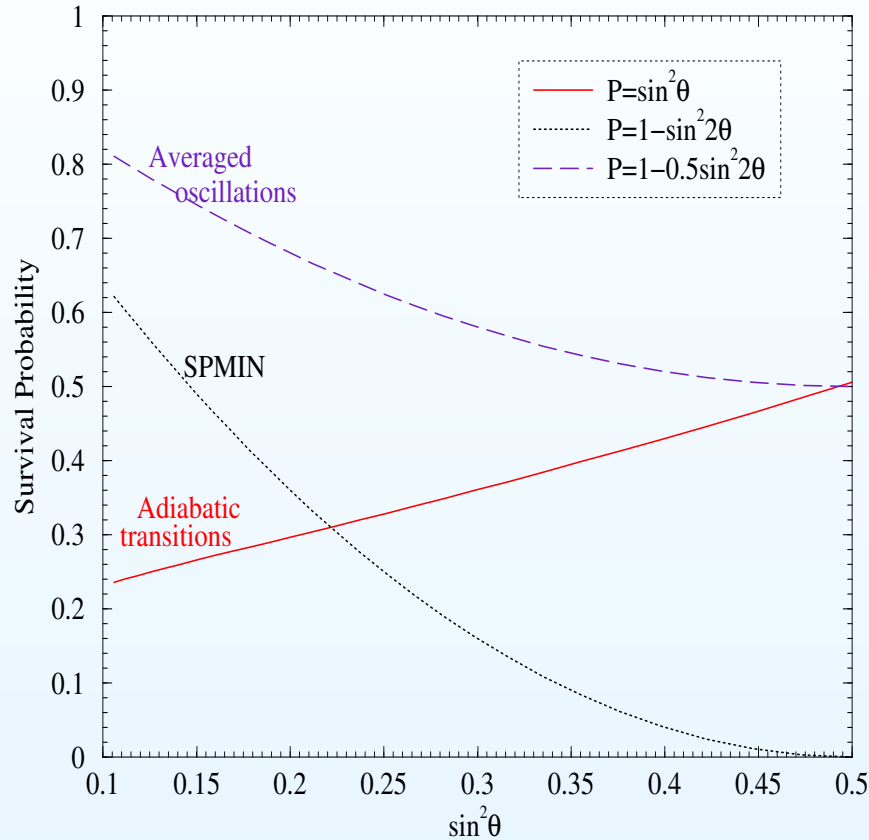
- $|D|$  gives the deviation of  $\sin^2 \theta_{23}$  from its maximal value

- $\text{sgn}(D)$  gives the octant of  $\sin^2 \theta_{23}$

- Current  $3\sigma$  limits on the magnitude and sign of  $D$ :

- ★  $|D| < 0.16$  at  $3\sigma$  from the SKI data
- ★ No robust information on  $\text{sgn}(D)$

# Problem of Measuring the Deviation $D$ from Maximal

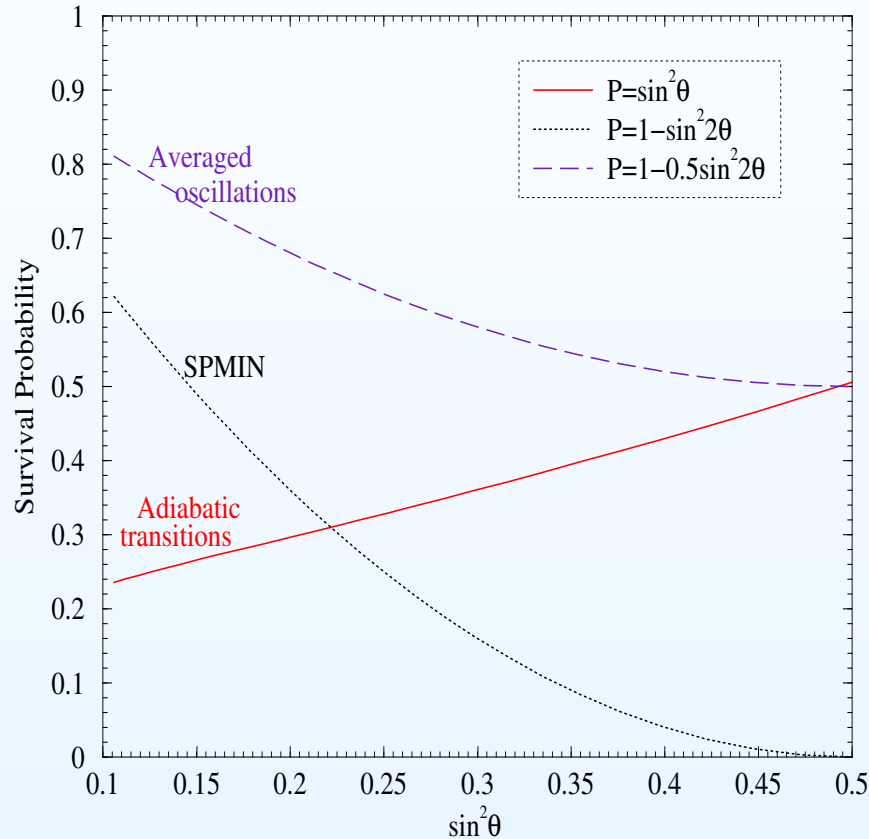


•  $(\Delta \sin^2 \theta)_{LMA} \sim \Delta P$ ; (good even if  $\sin^2 \theta \sim 0.5$ )

•  $(\Delta \sin^2 \theta)_{VAC} \sim \frac{\Delta P}{-4 \cos 2\theta \langle \sin^2 \frac{\Delta m^2 L}{4E} \rangle}$ ;

•  $(\Delta \sin^2 \theta)_{SPMIN} \sim \frac{\Delta P}{-4 \cos 2\theta}$ ; (best if  $\cos 2\theta \gtrsim 0.25$  ( $\sin^2 \theta \lesssim 0.375$ ))

# Problem of Measuring the Deviation $D$ from Maximal



- SPMIN is best for measuring  $\theta$  in vacuum oscillation experiments

- For  $\theta$  close to maximal, oscillations in vacuum is bad

- Alas! Matter effects in earth for GeV neutrinos directly affect  $\theta_{13}$

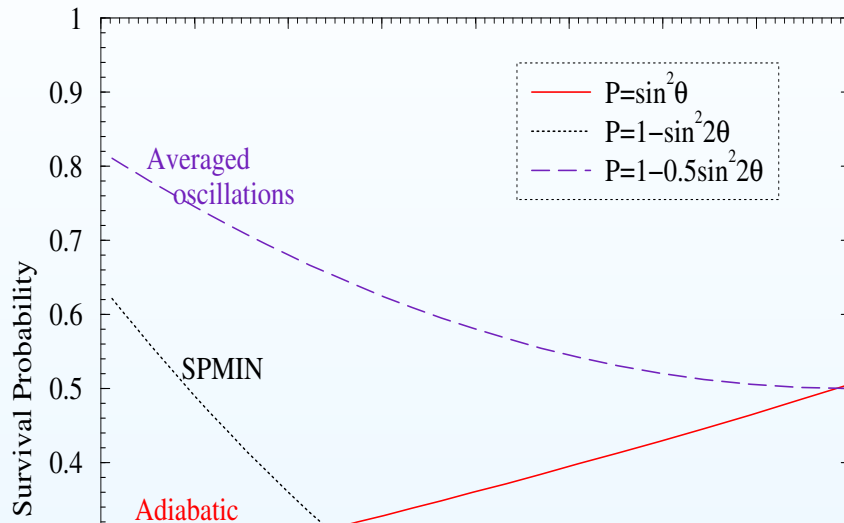
- Besides, octant cannot be determined using a disappearance experiment in vacuum

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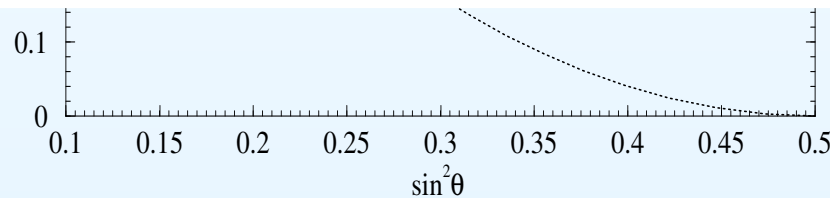


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**CAN EARTH MATTER EFFECTS HELP IN MEASURING  $\theta_{23}$ ?**



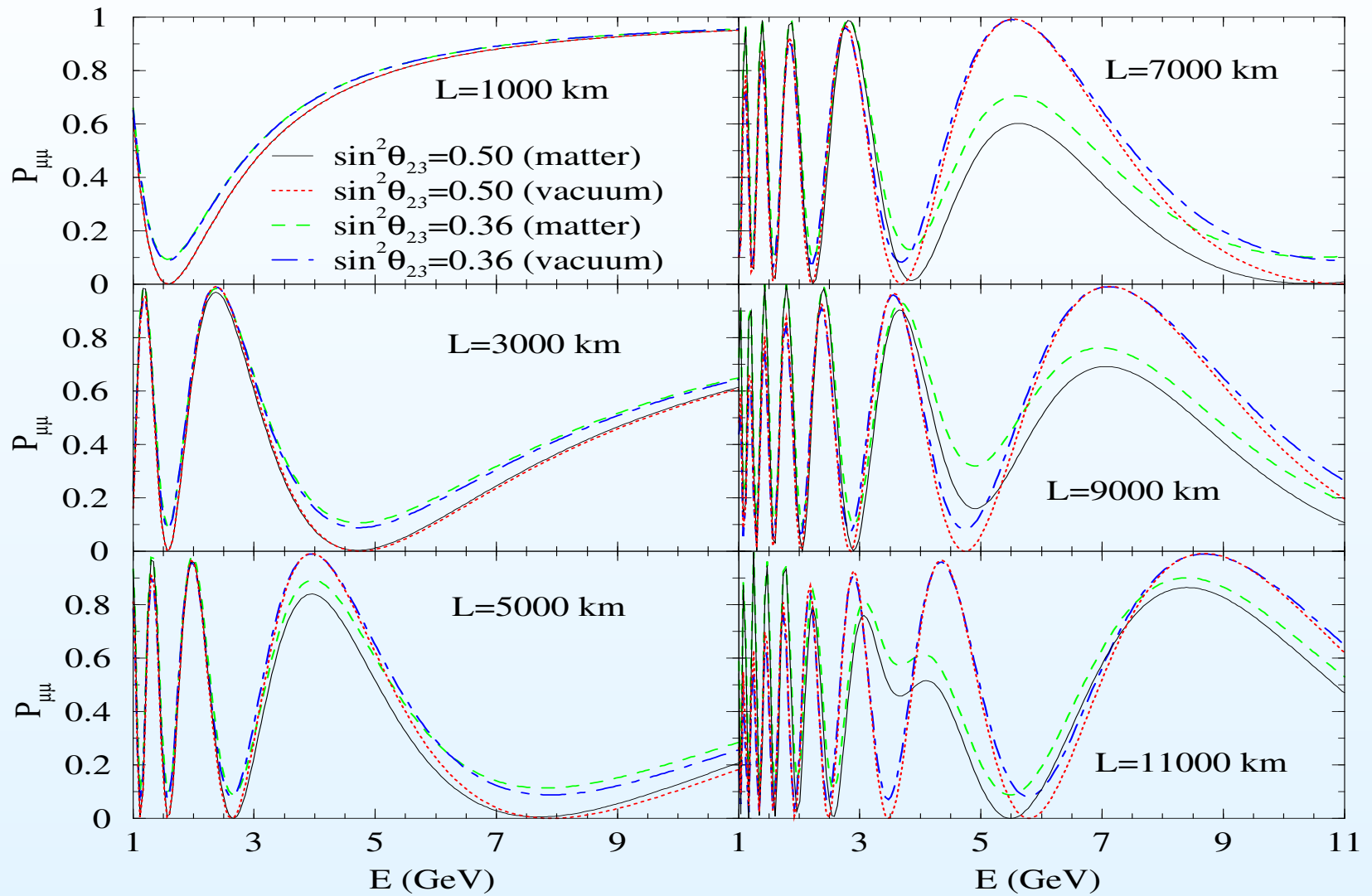
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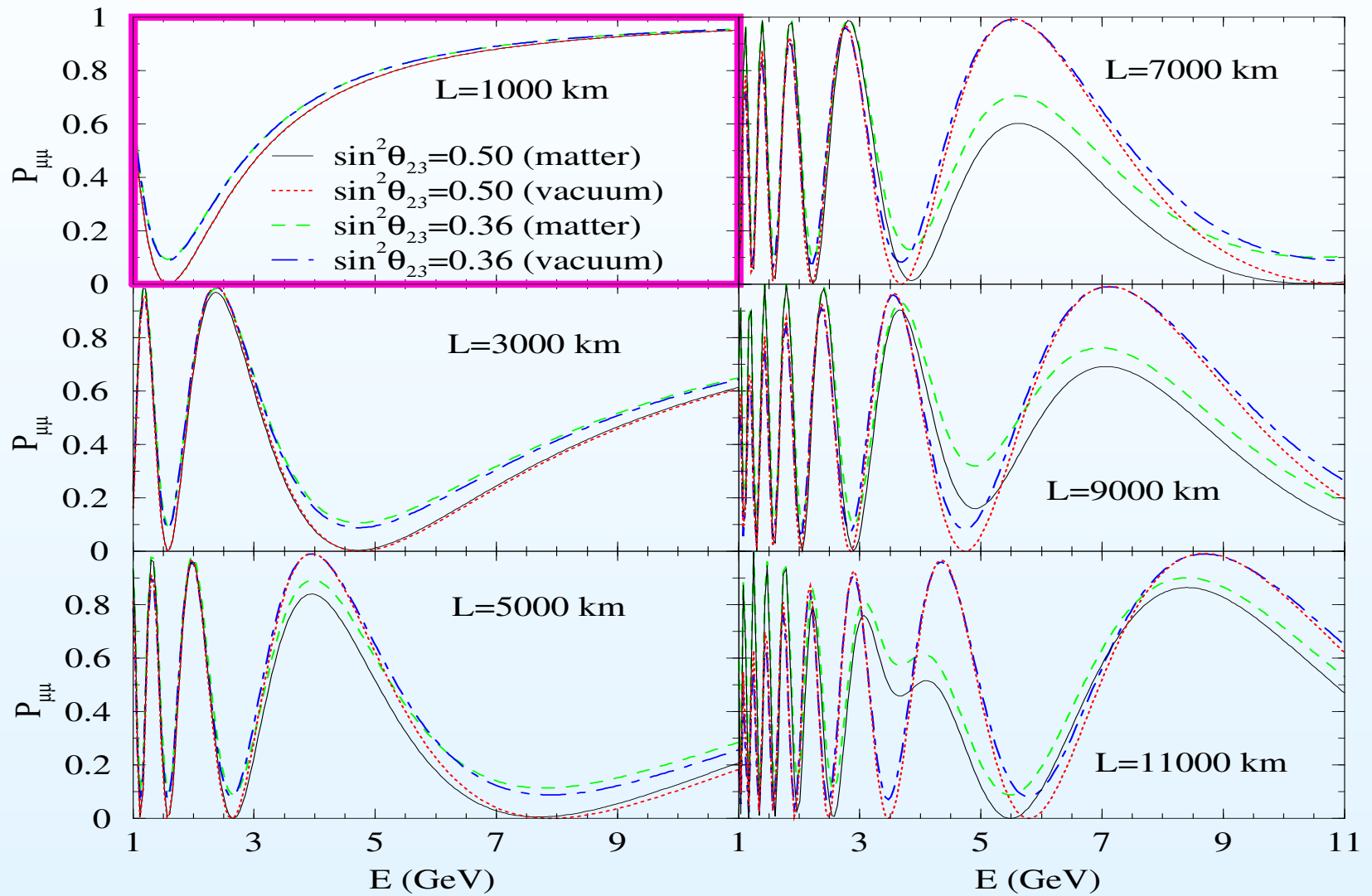
- $(\Delta \sin^2 \theta)_{SPMIN} \sim \frac{\Delta P}{-4 \cos 2\theta}$ ; (best if  $\cos 2\theta \gtrsim 0.25$  ( $\sin^2 \theta \lesssim 0.375$ ))

# Muon Neutrino Survival Probability



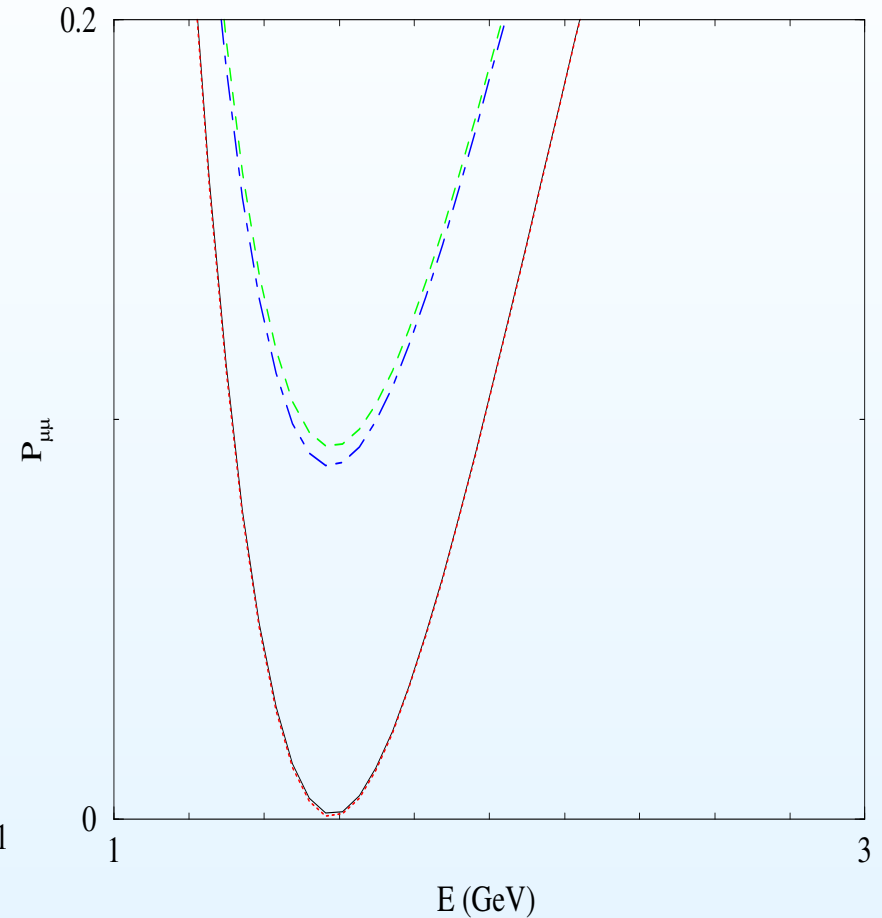
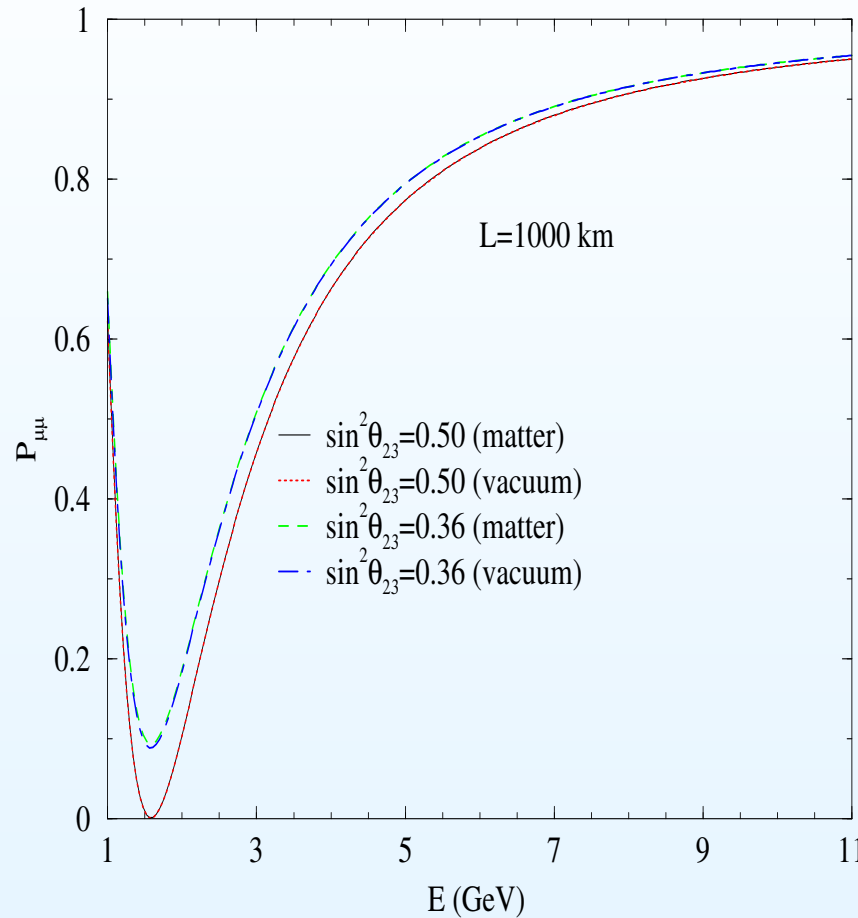
S.C. and P. Roy, hep-ph/0509197

# Muon Neutrino Survival Probability



S.C. and P. Roy, hep-ph/0509197

# Muon Disappearance in Long Baseline Experiments



- No matter effect for  $\sin^2 \theta_{23} = 0.5$
- Very small matter effects for  $\sin^2 \theta_{23} \neq 0.5$

# Muon Disappearance in Long Baseline Experiments

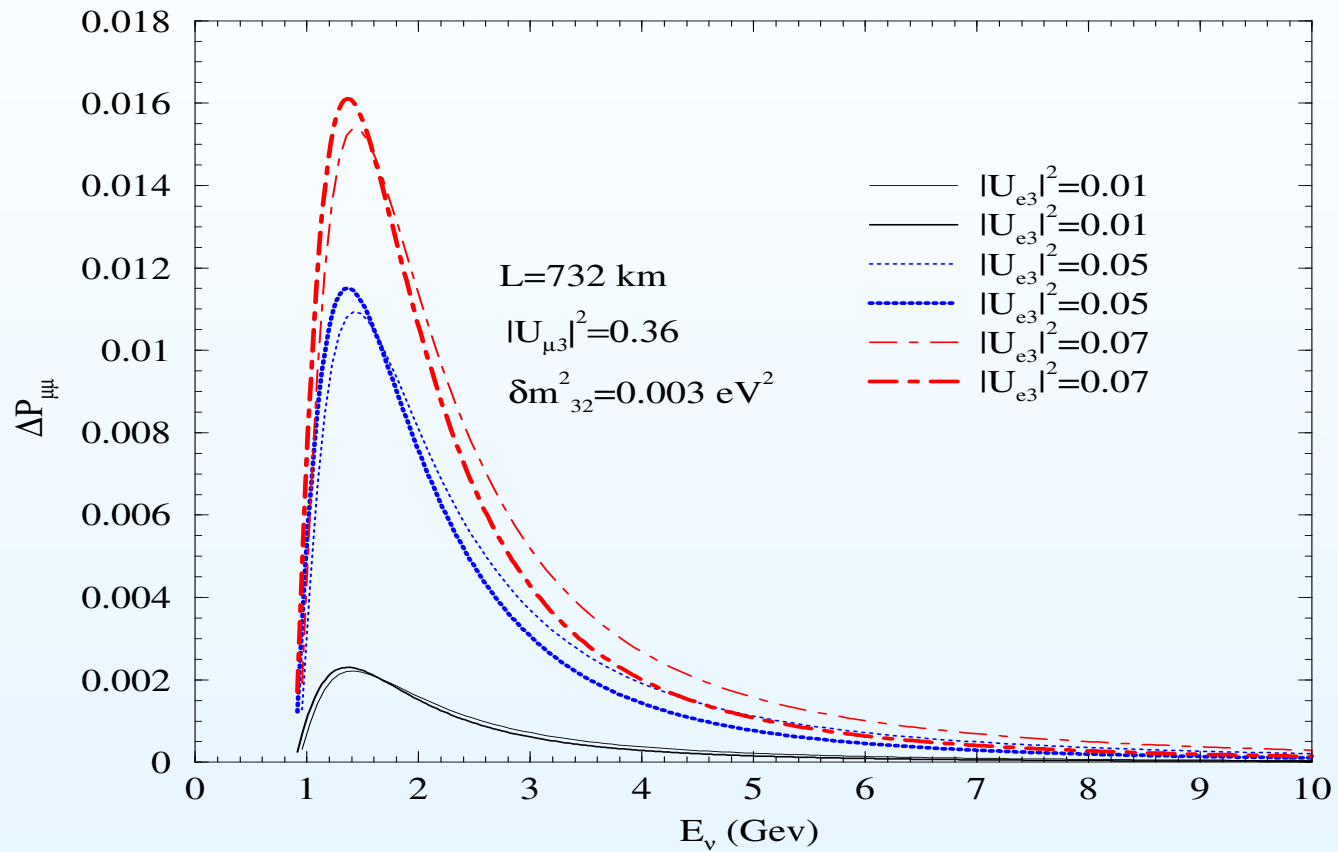
- The  $\nu_\mu$  survival probability can be written as

$$P_{\mu\mu} = P_{\mu\mu}^{\text{vac}} + 2|U_{e3}|^2|U_{\mu3}|^2(1 - 2|U_{\mu3}|^2) \\ A \left[ 4\Delta_{31}^{-1} \sin^2 \frac{\Delta_{31}L}{2} - L \sin(\Delta_{31}L) \right] + \mathcal{O}(A^2)$$

$$\Delta P_{\mu\mu} = 4|U_{e3}|^2|U_{\mu3}|^2(1 - 2|U_{\mu3}|^2) \\ A \left[ 4\Delta_{31}^{-1} \sin^2 \frac{\Delta_{31}L}{2} - L \sin(\Delta_{31}L) \right] \\ + \mathcal{O}[(U_{e3}A)^3]$$

- ( $\Delta_{31} = \Delta m_{31}^2/2E$ )

# Muon Disappearance in Long Baseline Experiments



S.C. and Roy, hep-ph/0310316

Not feasible to measure in any proposed experiment

# Muon Disappearance in Long Baseline Experiments

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- Survival probability in vacuum:

$$P_{\mu\mu} \simeq 1 - (c_{13}^2 \sin^2 2\theta_{23} + \sin^2 2\theta_{13} s_{23}^4) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

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- No octant dependence expected
- Best measurements on  $|D|$  expected near **SPMIN** where

$$\sin^2 \frac{\Delta m_{31}^2 L}{4E} \rightarrow 1$$

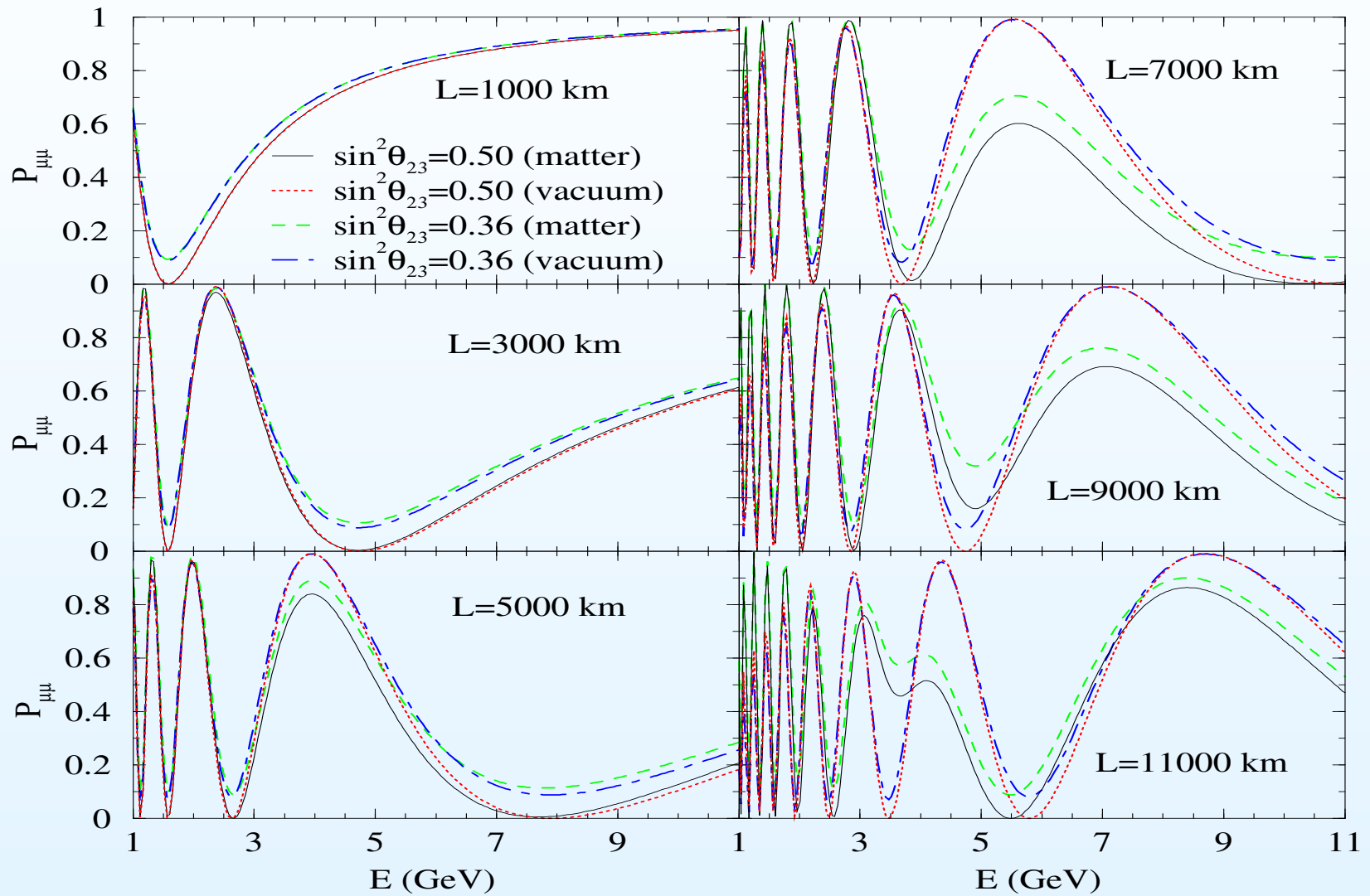
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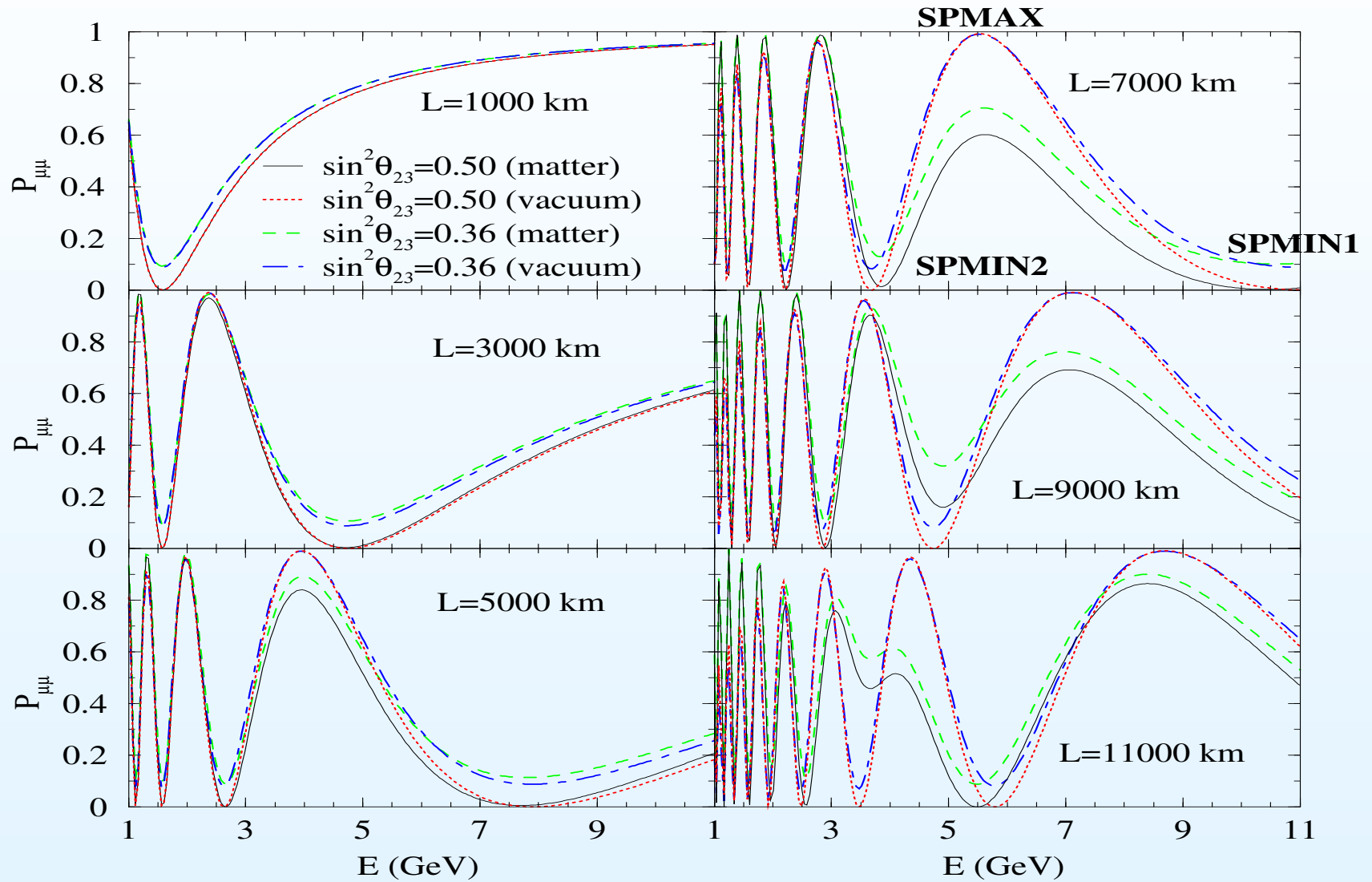
- Dependence on  $\theta_{23}$  in the form  $\sin^2 2\theta_{23}$
- Measurement expected to be harder near maximal mixing
- No octant dependence expected
- Best measurements on  $|D|$  expected near **SPMIN** where  $\sin^2 \frac{\Delta m_{31}^2 L}{4E} \rightarrow 1$
- For a given  $L$  and  $E$ , measurement sensitive to  $\Delta m_{31}^2$  (**true**)

# Large Matter Effects in $\nu_\mu$ Survival Probability



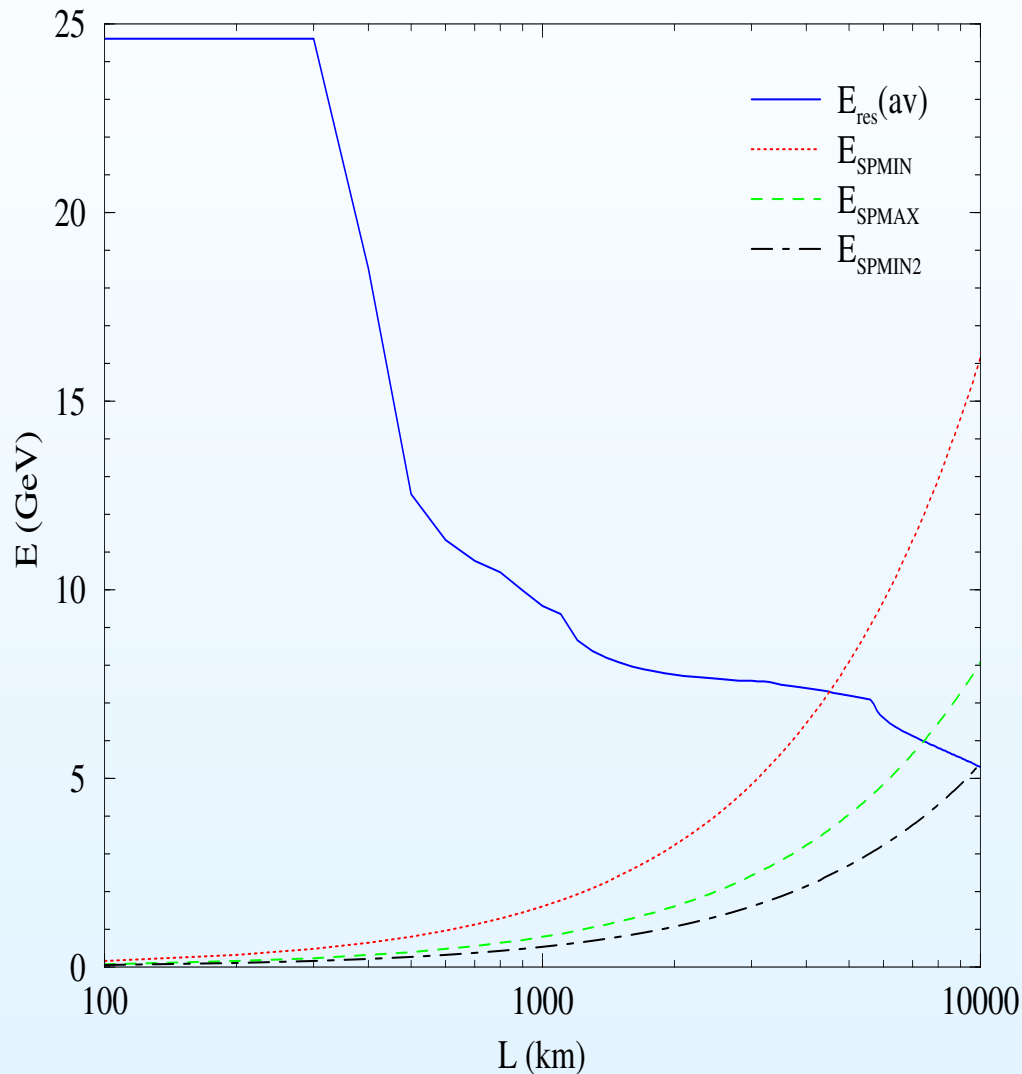
S.C. and P. Roy, hep-ph/0509197

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# Large Matter Effects in $\nu_\mu$ Survival Probability



S.C. and P. Roy, hep-ph/0509197

$$E_{\text{res}} \equiv \frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2\sqrt{2}G_F N_e}$$

$$E_{\text{SPMIN}n} = \frac{2}{2n-1} (1.27) \Delta m_{31}^2 L,$$

$$E_{\text{SPMAX}n} = \frac{2}{2n} (1.27) \Delta m_{31}^2 L$$

$$E_{\text{res}} \simeq E_{\text{SPMIN}1} \text{ for } L \simeq 4500 \text{ km}$$

$$E_{\text{res}} \simeq E_{\text{SPMIN}2} \text{ for } L \simeq 9700 \text{ km}$$

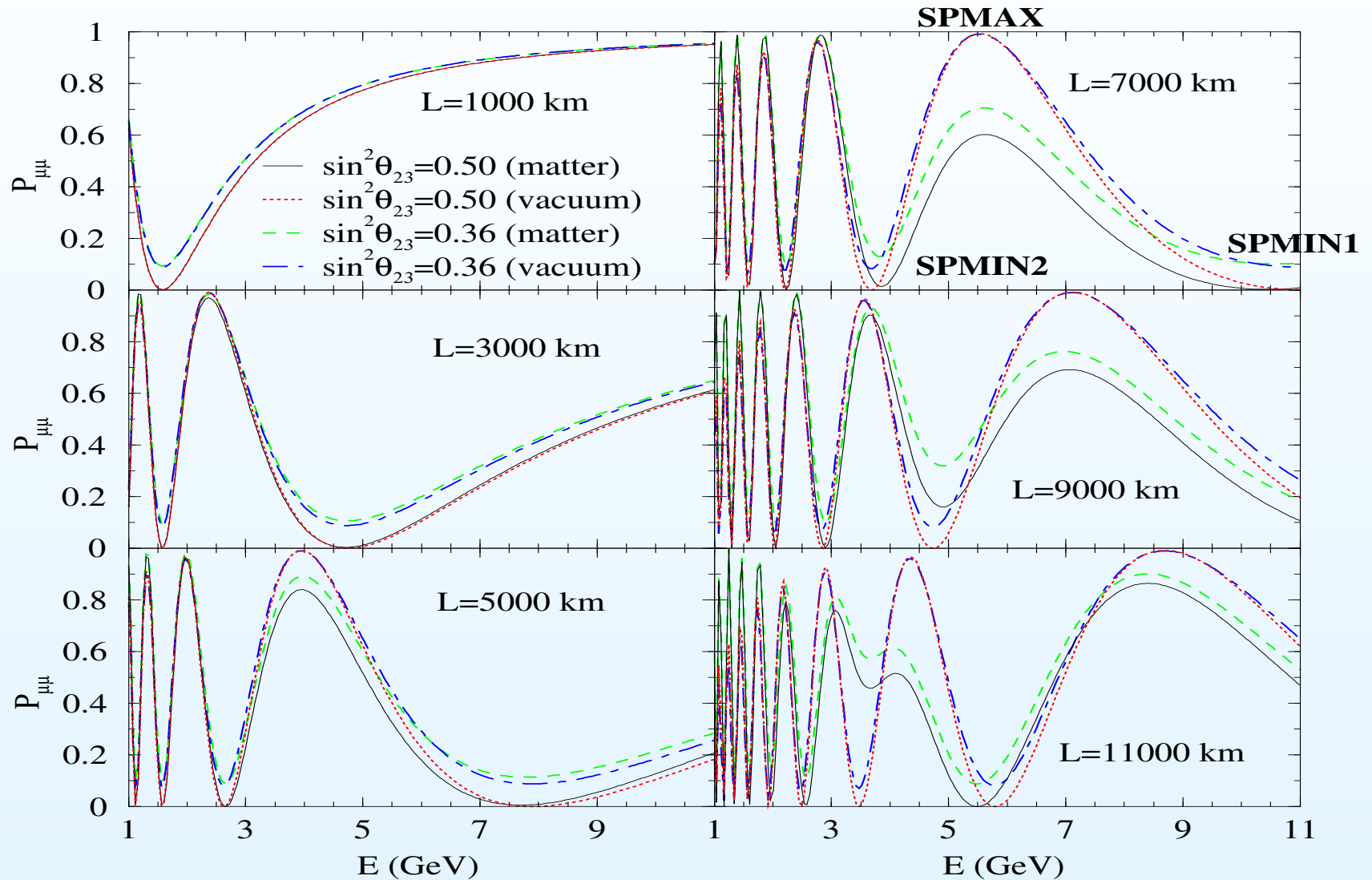
$$E_{\text{res}} \simeq E_{\text{SPMAX}} \text{ for } L \simeq 7400 \text{ km}$$

Polamares-Ruiz&Petcov

Indumathi&Murthy

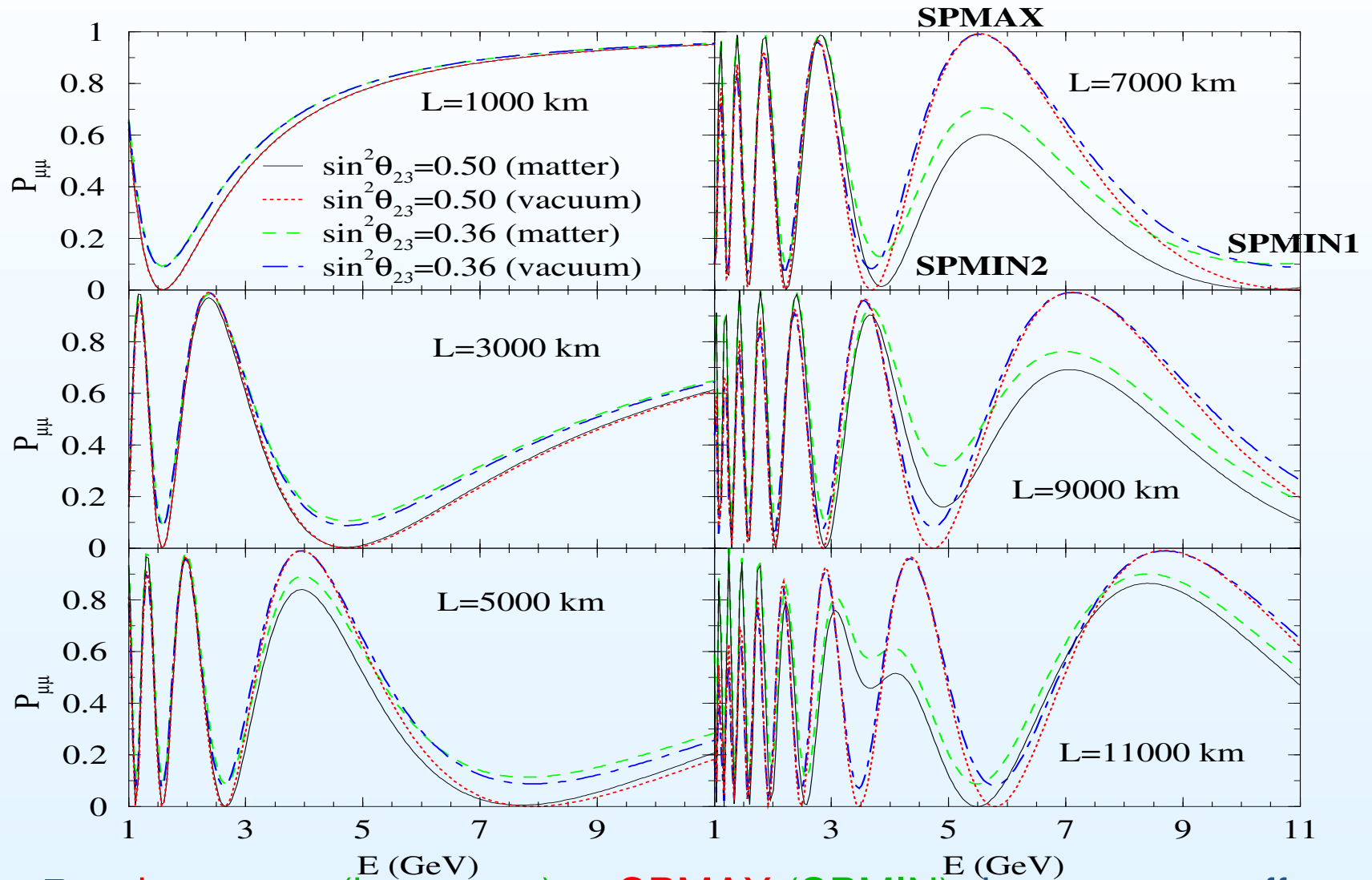
Gandhi et al

# Large Matter Effects in $\nu_\mu$ Survival Probability



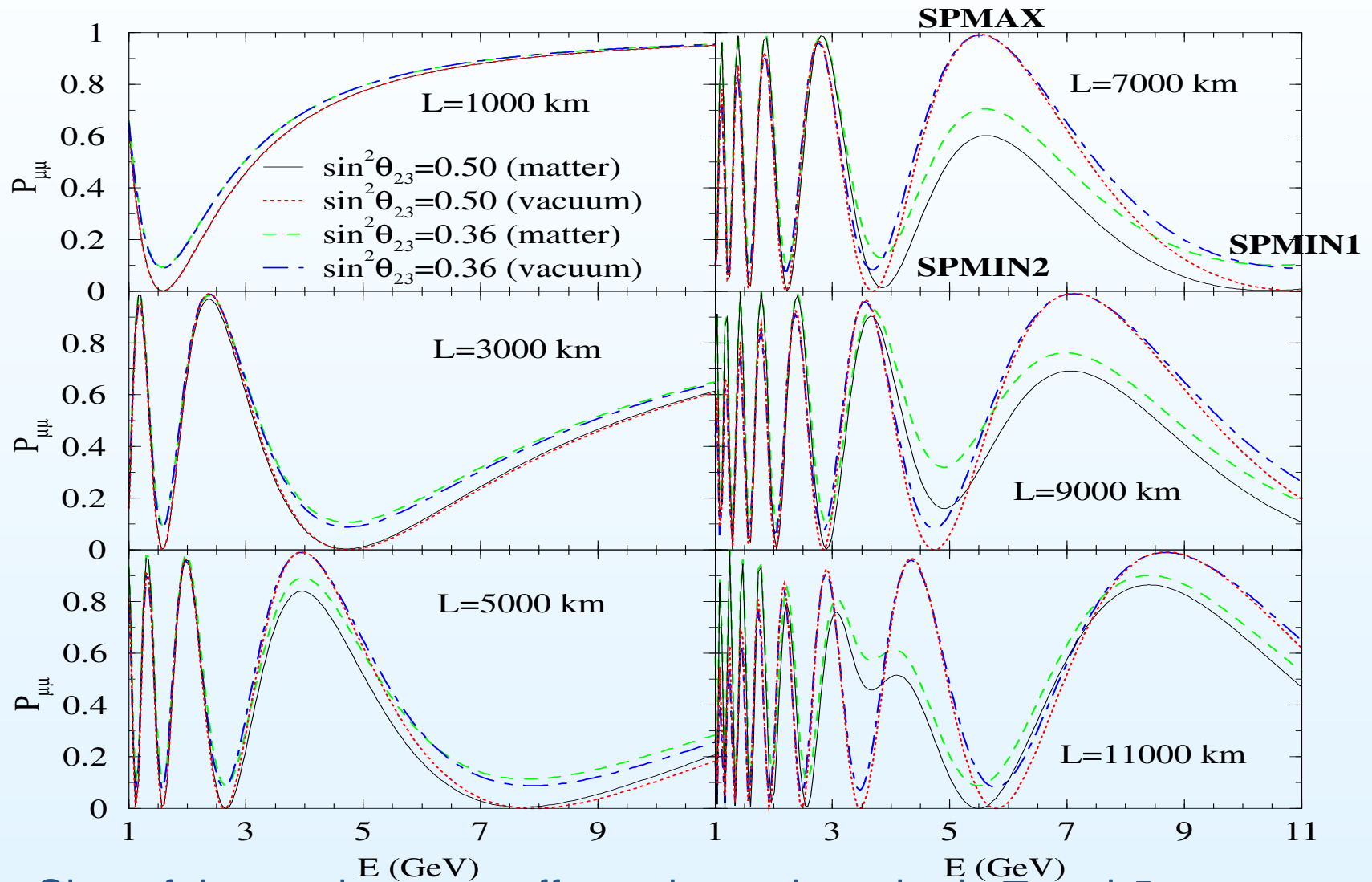
● Largest effect for  $L \simeq 7000$  km and  $E \simeq 5$  GeV  $\Rightarrow (E_{\text{SPMAX}} \simeq E_{\text{res}})$

# Large Matter Effects in $\nu_\mu$ Survival Probability



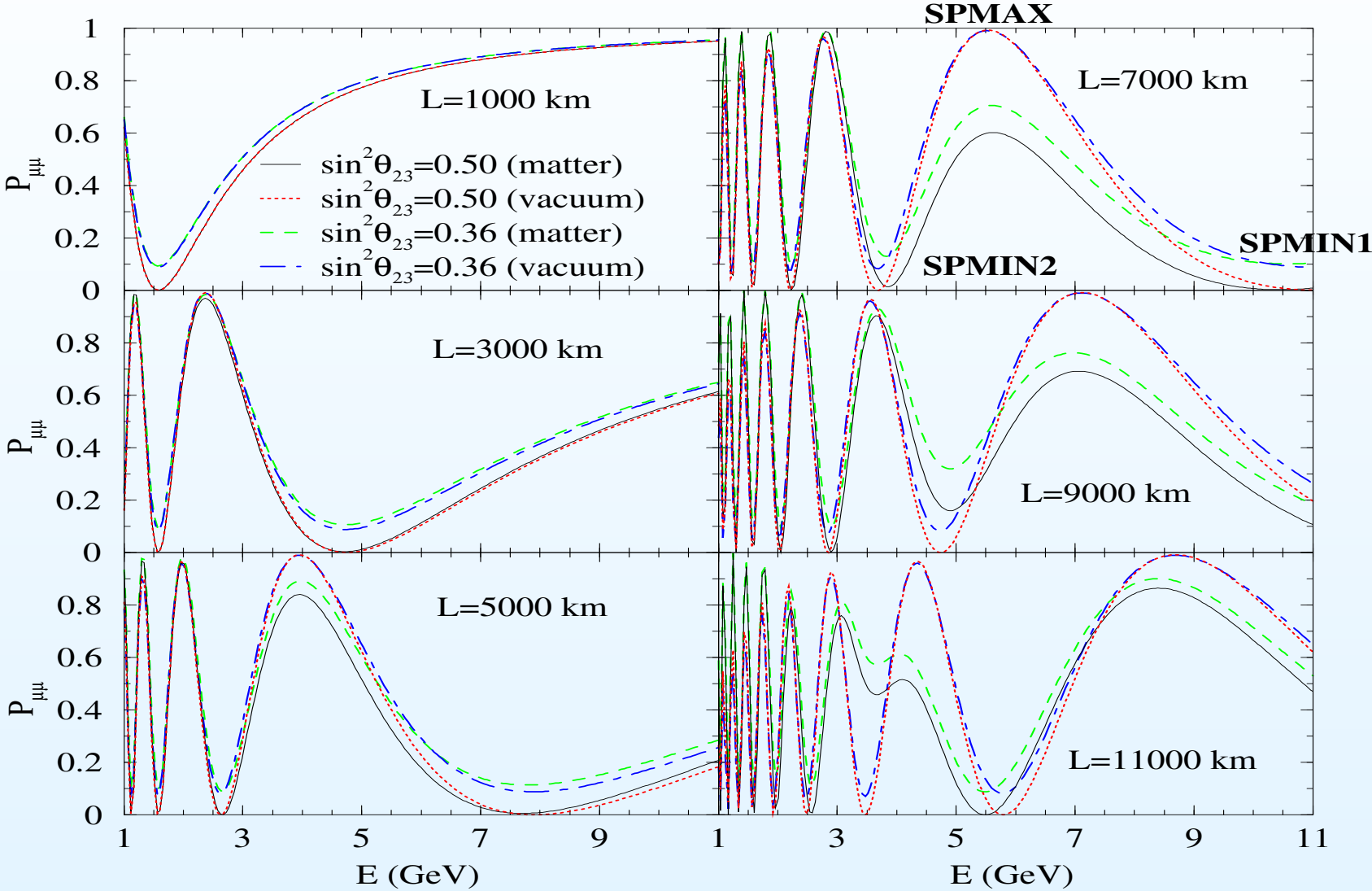
- $P_{\mu\mu}$  decreases (increases) at SPMAX (SPMIN) due to matter effects
- The spectral shape changes due to earth matter effects

# Large Matter Effects in $\nu_\mu$ Survival Probability



- Sign of the earth matter effects depends on both  $E$  and  $L$
- Most important to choose the bins properly in both  $E$  and  $L$

# Large Matter Effects in $\nu_\mu$ Survival Probability



Matter effects depend on the value of  $\sin^2 \theta_{23}$

# Muon Neutrino Survival Probability

$$\lim_{\Delta m_{21}^2 \rightarrow 0} P_{\mu\mu}(L) = 1 - P_{\mu\mu}^1(L) - P_{\mu\mu}^2(L) - P_{\mu\mu}^3(L)$$

$$P_{\mu\mu}^1(L) = \sin^2 \theta_{13}^M \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2 (\tilde{A} + 1) - (\Delta m_{31}^2)^M}{8E} L$$

$$P_{\mu\mu}^2(L) = \cos^2 \theta_{13}^M \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2 (\tilde{A} + 1) + (\Delta m_{31}^2)^M}{8E} L$$

$$P_{\mu\mu}^3(L) = \sin^2 2\theta_{13}^M \sin^4 \theta_{23} \sin^2 \frac{(\Delta m_{31}^2)^M}{4E} L$$

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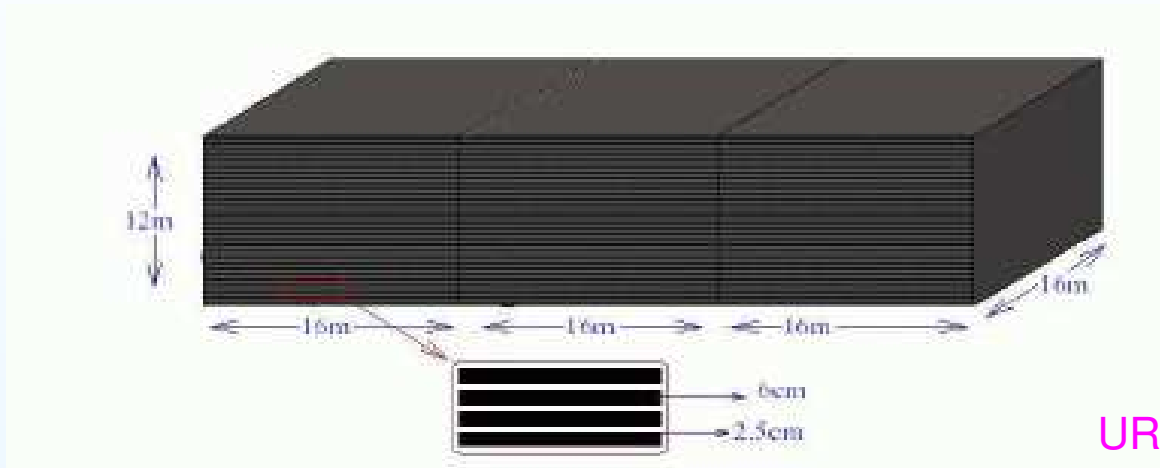
- Dependence on  $\theta_{23}$  in the form  $\sin^4 \theta_{23}$
- Octant sensitivity is expected to be good
- Need an experimental set-up that can see the matter effects

# Atmospheric $\nu_\mu$ in Large Magnetised Iron Calorimeter

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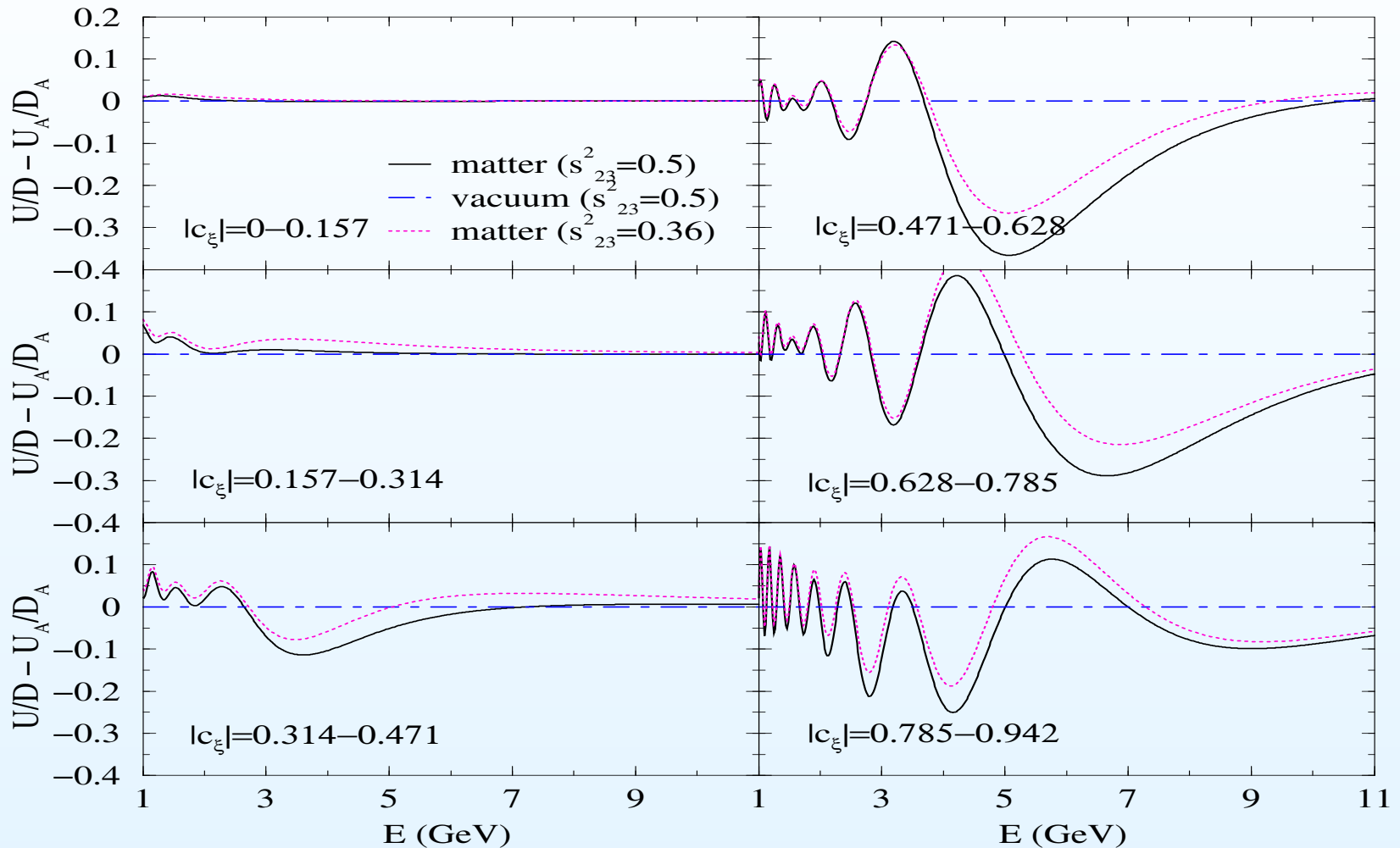
## India-based Neutrino Observatory



URL: <http://www.imsc.res.in/~ino>

- ~2200 meters underground
- 50 kton detector
- Very good energy and angle resolution  
⇒ fine binning possible in both energy and zenith angle
- It will be magnetized ⇒  $\nu_\mu$  can be distinguished from  $\bar{\nu}_\mu$
- Best place to look for matter effects in  $P_{\mu\mu}$

# Atmospheric $\nu_\mu$ in Large Magnetised Iron Calorimeter

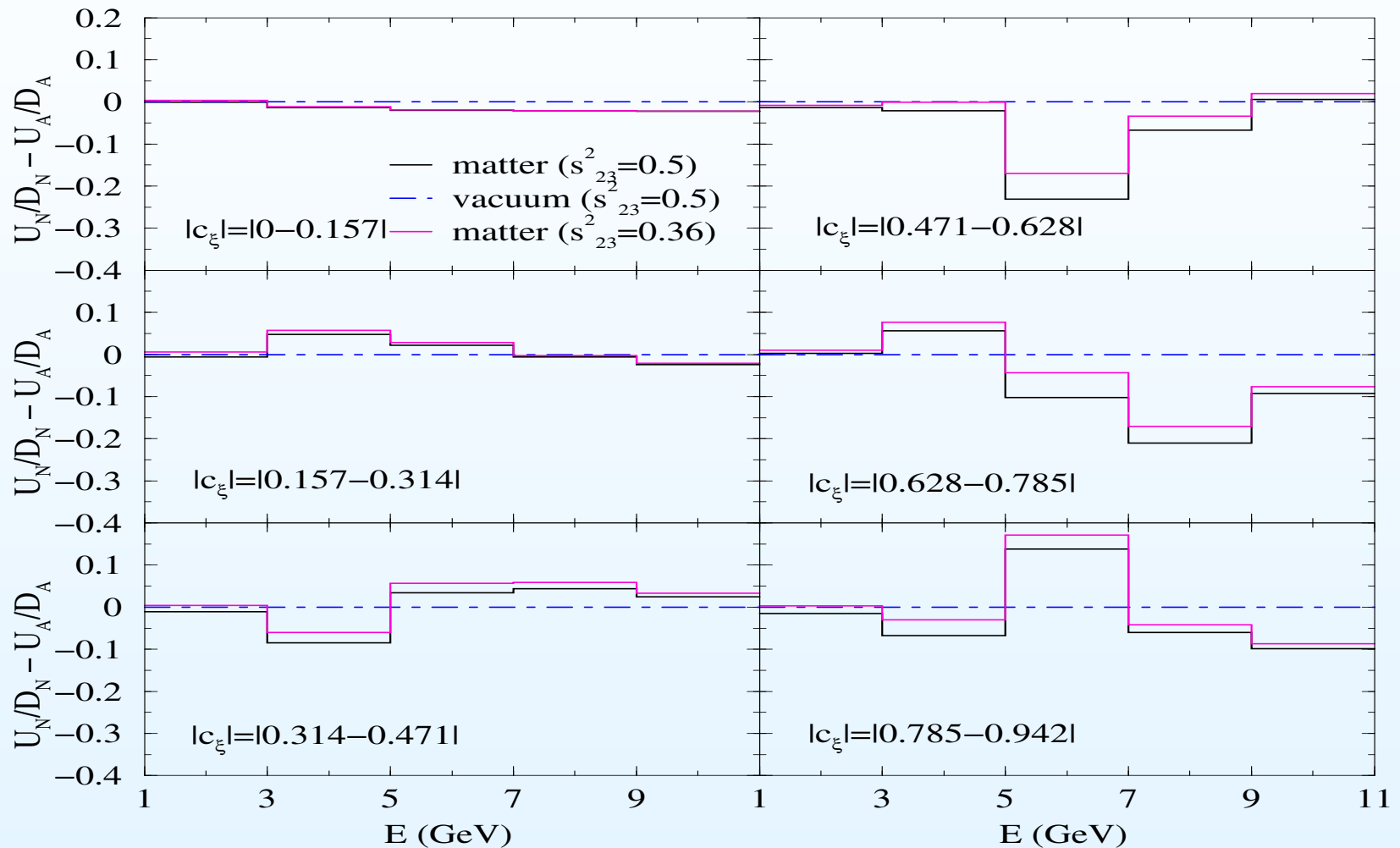


● Large matter effects in  $P_{\mu\mu}$

S.C and P. Roy hep-ph/0509197

● Matter effects are  $\sin^2 \theta_{23}$  dependent

# Atmospheric $\nu_\mu$ in Large Magnetised Iron Calorimeter



● Matter effects survive after the binning S.C and P. Roy hep-ph/0509197

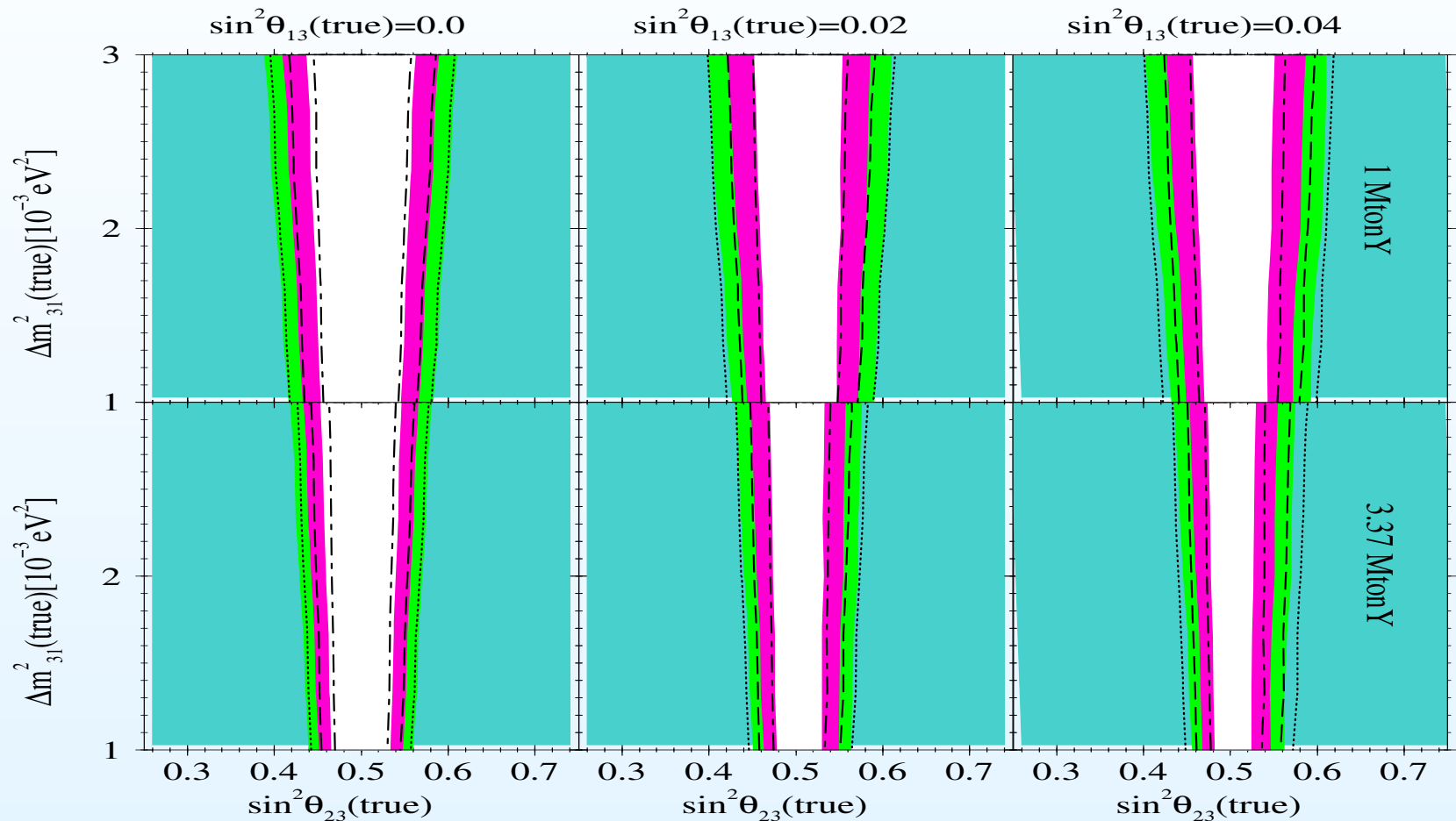
● 60+60 bins of data

● define a  $\chi^2$  taking all errors

# Is the Atmospheric Mixing Maximal?

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✓ Using atmospheric neutrino data in INO

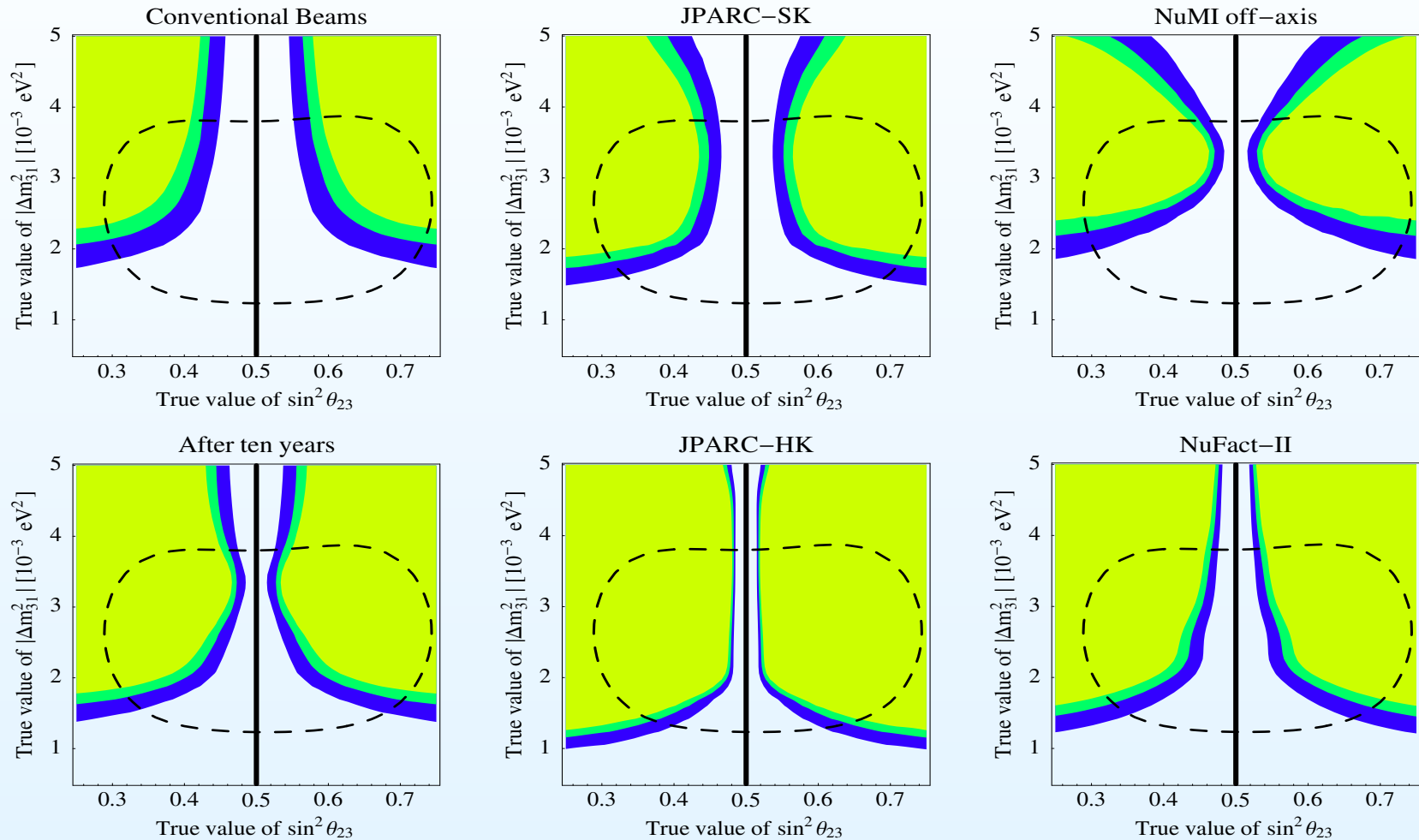


S.C. and P. Roy hep-ph/0509197

- $|D|$  can be measured to  $\sim 17\%$  ( $20\%$ ) at  $3\sigma$  for  $s_{13}^2 = 0.04$  ( $0.00$ ) with 1 MtonY exposure and 50% detector efficiency

# Is the Atmospheric Mixing Maximal?

✓ Using long baseline experiments

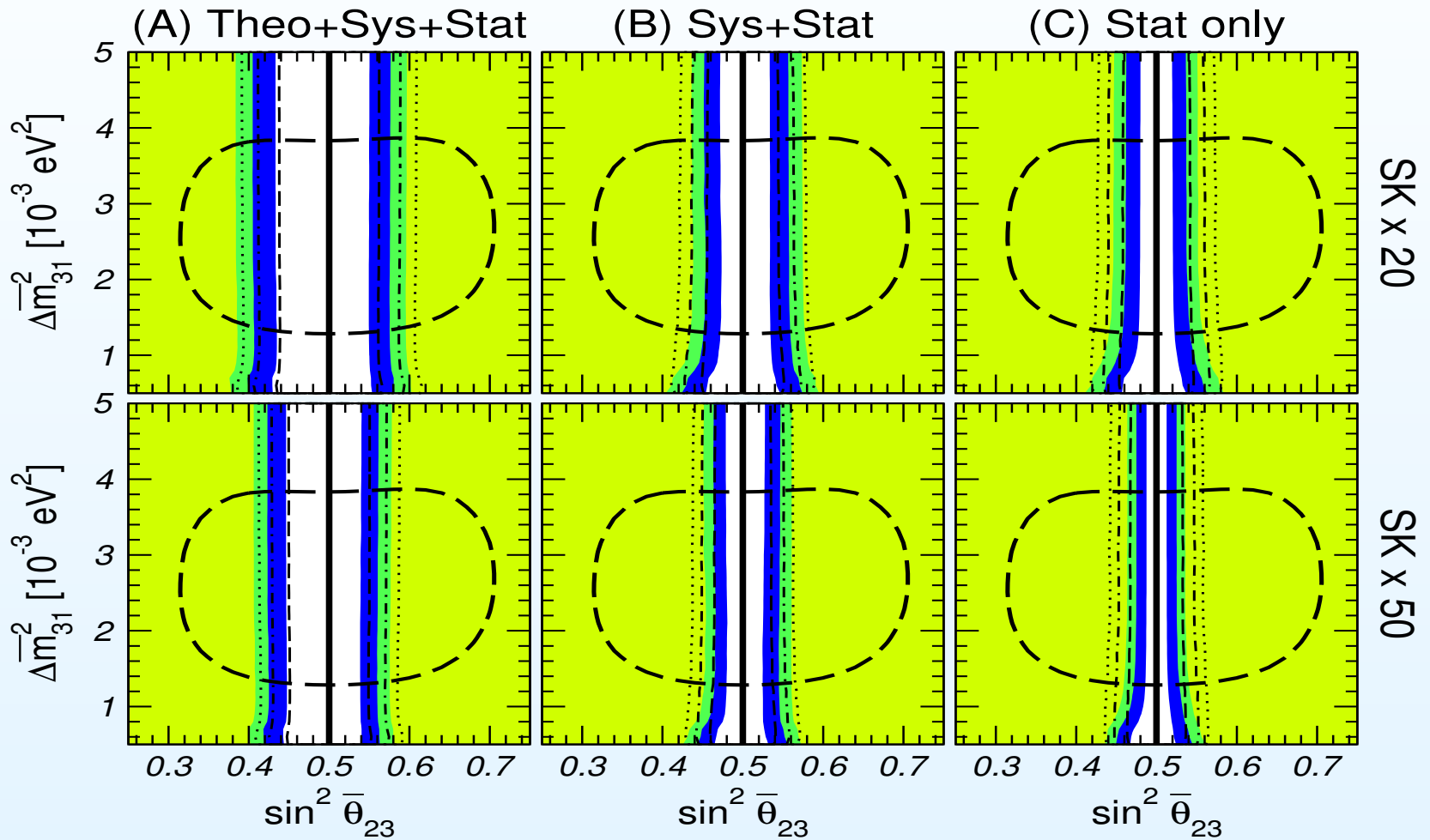


Antusch, et al, hep-ph/0404268

•  $|D|$  can be measured to  $\sim 14\%$  at  $3\sigma$  for  $\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$

# Is the Atmospheric Mixing Maximal?

✓ Using atmospheric neutrino data in SK-like detector



Gonzalez-Garcia et al, hep-ph/0408170

•  $|D|$  can be measured to  $\sim 19\%$  ( $23\%$ ) at  $3\sigma$  with SK50 (SK20)

# Resolving the Octant Ambiguity

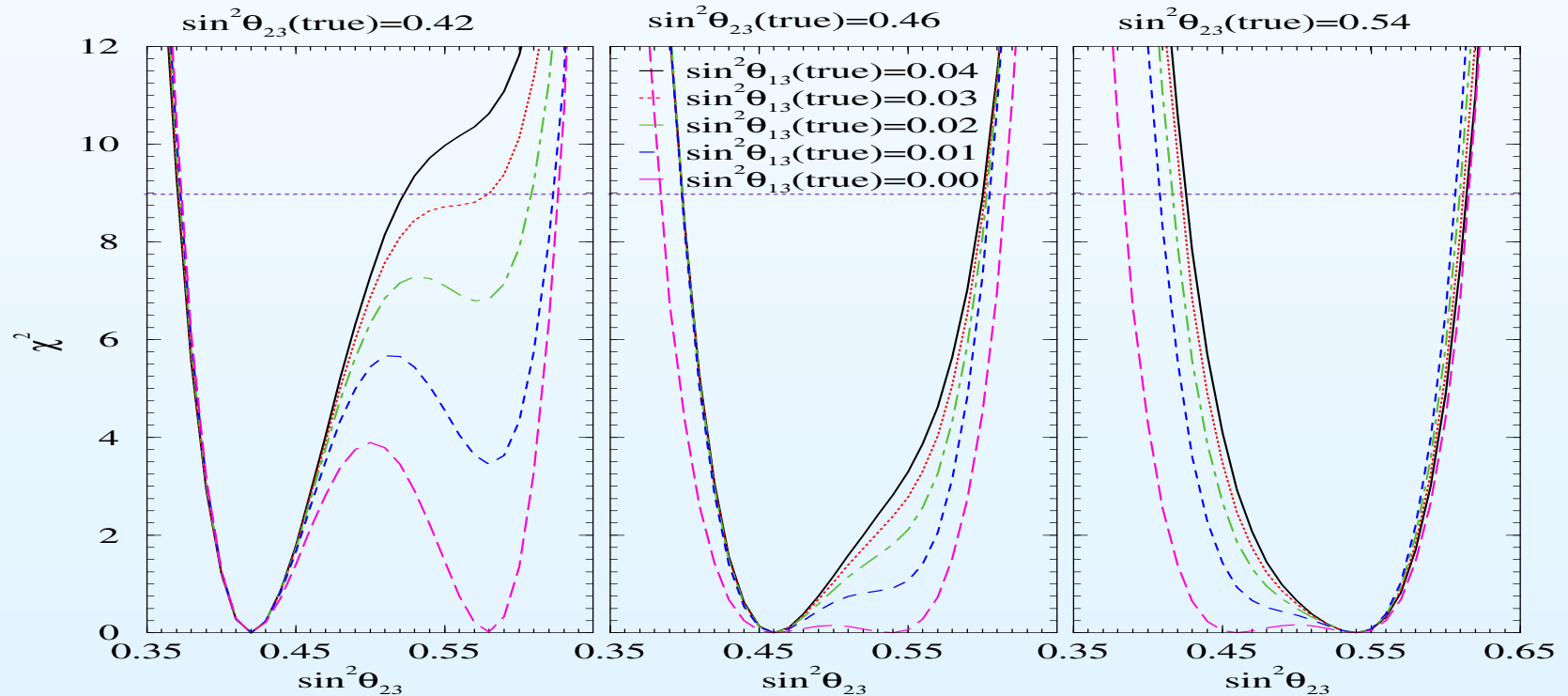
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# Resolving the Octant Ambiguity

- For every nonmaximal  $\sin^2 \theta_{23}(\text{true})$ , there exists a  $\sin^2 \theta_{23}(\text{false})$

$$\sin^2 \theta_{23}(\text{false}) = 1 - \sin^2 \theta_{23}(\text{true})$$

- Atmospheric neutrinos in magnetized iron detector



S.C and P. Roy hep-ph/0509197

## Resolving the Octant Ambiguity

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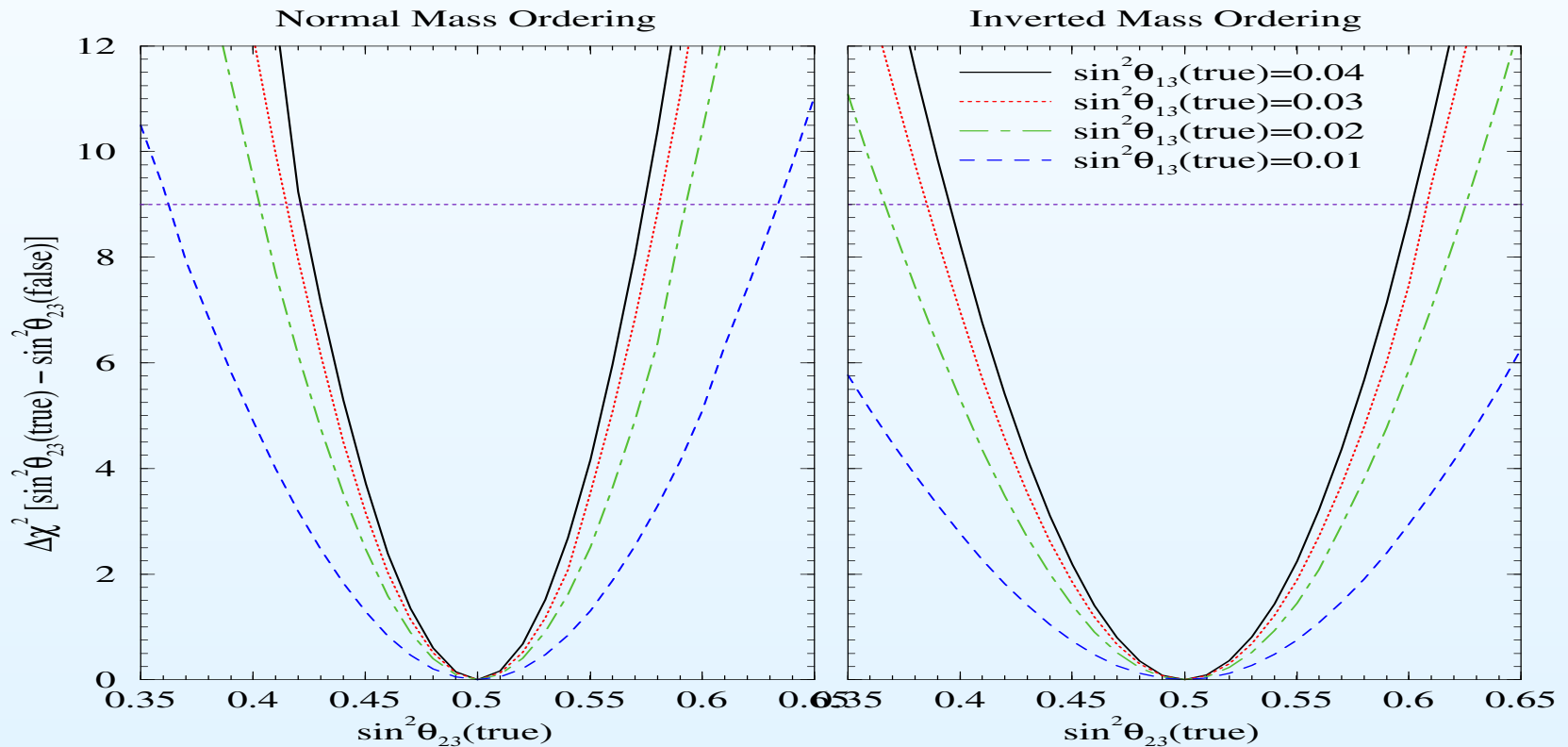
$$\Delta\chi^2 \equiv \chi^2(\sin^2 \theta_{23}(\text{false}), \sin^2 \theta_{13}, \Delta m_{31}^2, \Delta m_{21}^2, \sin^2 \theta_{12} | \omega(\text{true})) \\ - \chi^2(\sin^2 \theta_{23}(\text{true}), \sin^2 \theta_{13}, \Delta m_{31}^2, \Delta m_{21}^2, \sin^2 \theta_{12} | \omega(\text{true}))$$

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Atmospheric neutrinos in magnetized iron detector

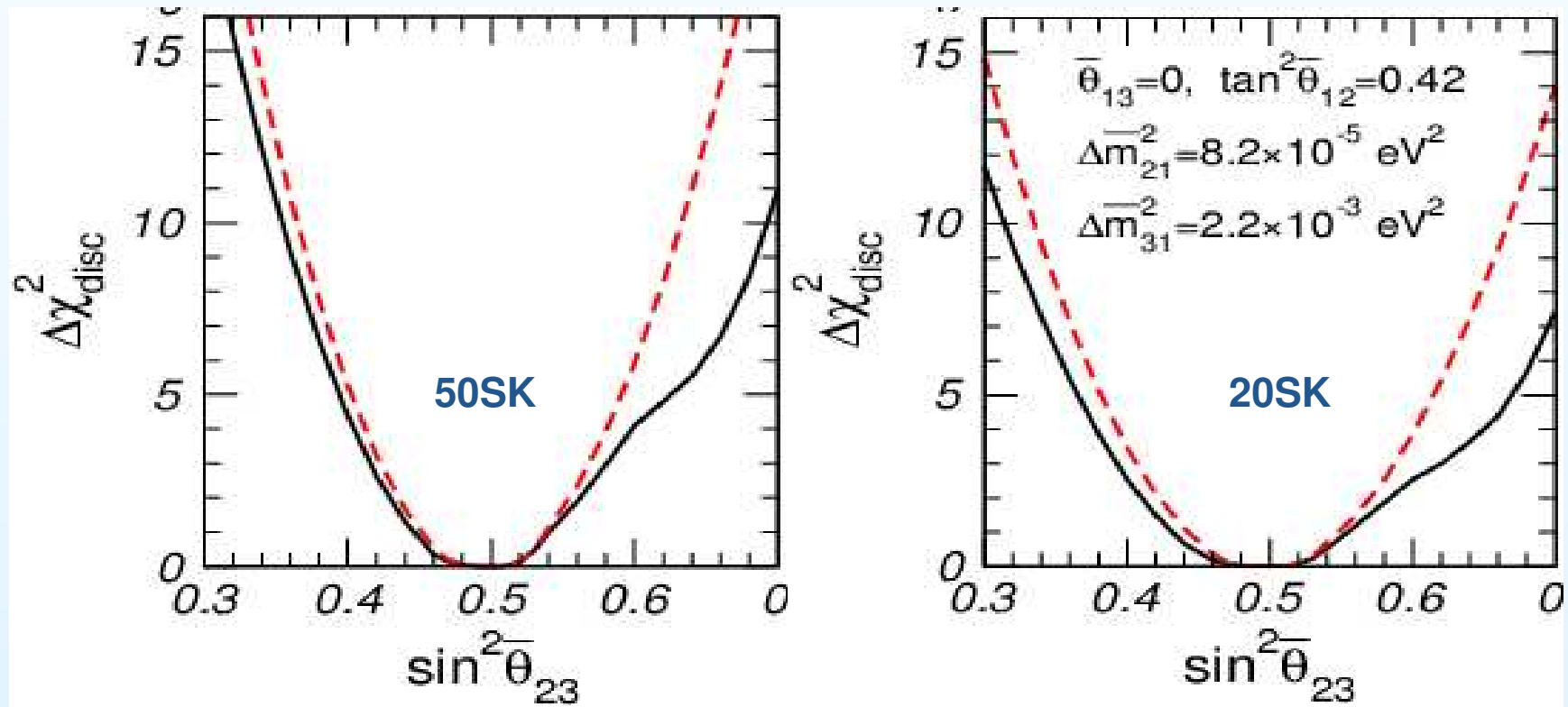


S.C and P. Roy hep-ph/0509197

# Resolving the Octant Ambiguity

$$\Delta\chi^2 \equiv \chi^2(\sin^2 \theta_{23}(\text{false}), \sin^2 \theta_{13}, \Delta m_{31}^2, \Delta m_{21}^2, \sin^2 \theta_{12} | \omega(\text{true})) - \chi^2(\sin^2 \theta_{23}(\text{true}), \sin^2 \theta_{13}, \Delta m_{31}^2, \Delta m_{21}^2, \sin^2 \theta_{12} | \omega(\text{true}))$$

- Atmospheric neutrinos in water Cerenkov detector



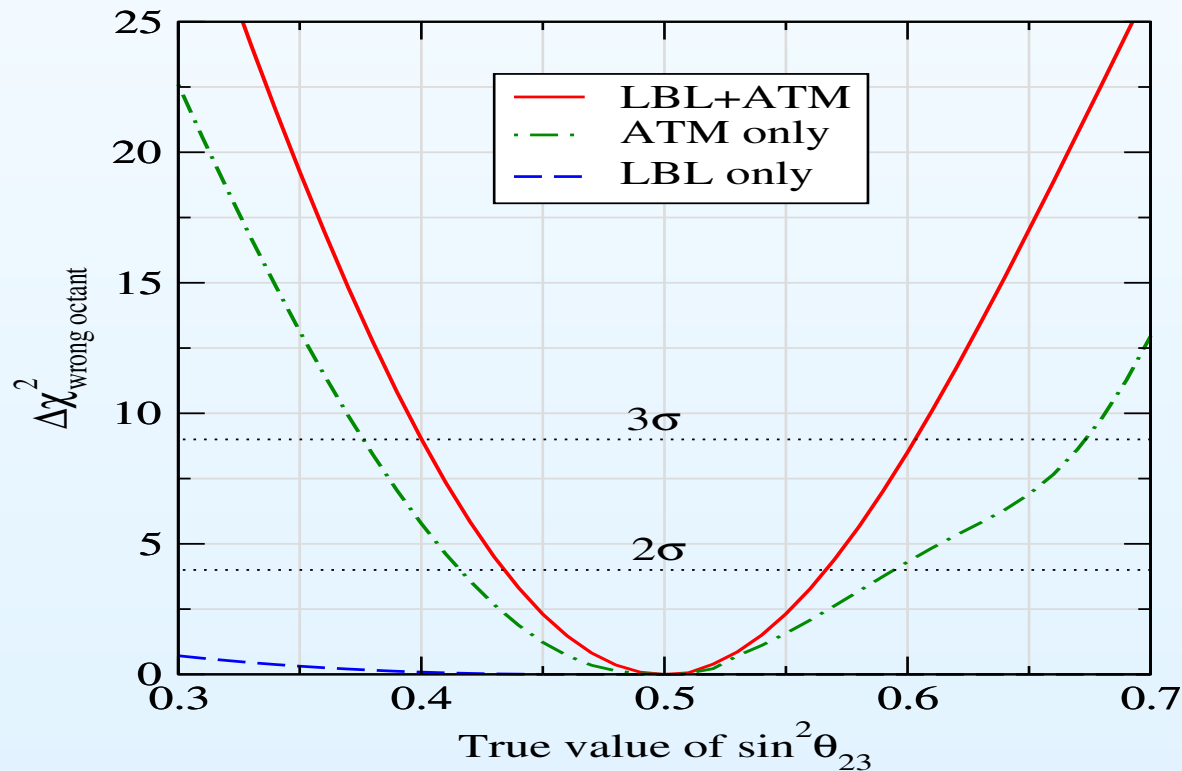
Gonzalez-Garcia et al, hep-ph/0408170

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Long baseline experiments with accelerator (super)beam



Huber et al hep-ph/0501037

# Comparing the Octant Sensitivity of the Experiments

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- Long baseline experiments

  - ★ No sensitivity at all

- Atmospheric neutrinos in water Cerenkov detectors

  - ★  $\sin^2 \theta_{23}(\text{false})$  can be excluded at  $3\sigma$  if:

$$\sin^2 \theta_{23}(\text{true}) < 0.36 \text{ or } > 0.62$$

- Atmospheric neutrinos in large magnetized iron detectors

  - ★  $\sin^2 \theta_{23}(\text{false})$  can be excluded at  $3\sigma$  if:

$$\sin^2 \theta_{23}(\text{true}) < 0.361 \text{ or } > 0.633 \text{ for } \sin^2 \theta_{13}(\text{true}) = 0.01,$$

$$\sin^2 \theta_{23}(\text{true}) < 0.402 \text{ or } > 0.592 \text{ for } \sin^2 \theta_{13}(\text{true}) = 0.02,$$

$$\sin^2 \theta_{23}(\text{true}) < 0.415 \text{ or } > 0.580 \text{ for } \sin^2 \theta_{13}(\text{true}) = 0.03,$$

$$\sin^2 \theta_{23}(\text{true}) < 0.421 \text{ or } > 0.573 \text{ for } \sin^2 \theta_{13}(\text{true}) = 0.04.$$

# Conclusions

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- Binning in both energy and zenith angle is essential
- It is also very important to distinguish neutrinos from anti-neutrinos
- Large magnetized iron detectors can achieve both
- Sensitivity to  $|D|$  in INO comparable to 20SK/50SK and LBL experiments
- Octant resolution can be done best in INO if  $\sin^2 \theta_{13} \geq 0.01$