The CTEQ Nuclear PDFs

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• PDF = parton distribuion (density) function



- describe how nucleons compose of partons
- PDF: f(x,Q), x momentum fraction, Q scale
- x-dep. not calculable in perturbative QCD
- evolution in Q accodring to DGLAP
- generalization to nuclei: nuclear PDFs (nPDFs)

Neutrino physics



Nuclear physics





Neutrino physics



σ_{νA} small
heavy targets
nPDFs



- nuclear effects
 - Pauli principle
 - o Fermi motion
 - Multiple scattering
- nuclear corrections
 PDFs => nPDFs

Nuclear physics





- hardon collider predictions require PDFs
 - ${\rm o}~pp$ collisions ... free-proton PDFs
 - o AA collisions ... nPDFs
 - data from collisions of nuclei used also in free-proton analysis





Neutrino-nucleon interactions

• Quasi-elastic scattering



• Deep inelastic scattering (DIS)



CC: $\nu(\bar{\nu}) + N \rightarrow l^-(l^+) + X$ NC: $\nu(\bar{\nu}) + N \rightarrow \nu(\bar{\nu}) + X$

• Resonance production



Charged Current (CC): $\nu(\bar{\nu}) + N \rightarrow l^{-}(l^{+}) + N + \pi^{\pm,0}$

Neutral Current (NC): $\nu(\bar{\nu}) + N \rightarrow \nu(\bar{\nu}) + N + \pi^{\pm,0}$

Neutrino-nucleon interactions



DIS coss section dominates above a few GeV

Deep Inelastic Scattering (DIS) $W^{\mu\nu} = -g^{\mu\nu}W_1 + \frac{p^{\mu}p^{\nu}}{M_p^2}W_2$ $-i\frac{\epsilon^{\mu\nu\alpha\beta}p_{\alpha}q_{\beta}}{2M_p^2}W_3 + \cdots$ $F_1 = W_1, \ F_2 = \frac{\nu}{M_p} W_2, \ F_3 = \frac{\nu}{M_p} W_3$ $\frac{d^2\sigma(\nu,\bar{\nu})}{dxdy} \sim L \cdot W \sim g_{+l} \left[xF_1y^2 + F_2 \left[(1-y) - \left(\frac{M_pxy}{2E_\nu} \right) \right] \right]$ $\pm g_{-l} \left[x F_3 y \left(1 - \frac{y}{2} \right) \right]$ $x \equiv \frac{Q^2}{2M_{\rm m}\nu} \quad \nu \equiv E_L - E_{L'} \quad y \equiv \frac{E_L - E_{L'}}{E_L}$

Goal for this talk

- explain:
 - extraction of nPDFs from experiment
 - estimation of uncertainities
 - present:
 - most recent fit including uncertainities
 - previous fit including neutrino data
 - the compatibility of neutrino DIS data with other data sets

Available nuclear PDFs

► Multiplicative nuclear correction factors

$$f_i^{\mathbf{p/A}}(x_N,\mu_0) = R_i(x_N,\mu_0,\mathbf{A}) f_i^{free\ proton}(x_N,\mu_0)$$

- ▶ Hirai, Kumano, Nagai [PRC 76, 065207 (2007), arXiv:0709.3038]
- Eskola, Paukkunen, Salgado [JHEP 04 (2009) 065, arXiv:0902.4154]
- de Florian, Sassot, Stratmann, Zurita
 [PRD 85, 074028 (2012), arXiv:1112.6324]

▶ Native nuclear PDFs

▶ nCTEQ [PRD 80, 094004 (2009), arXiv:0907.2357]

$$f_i^{p/A}(x_N, \mu_0) = f_i(x_N, A, \mu_0)$$
$$f_i(x_N, A = 1, \mu_0) \equiv f_i^{free\ proton}(x_N, \mu_0)$$

nCTEQ framework

[PRD 80, 094004 (2009), arXiv: 0907.2357]

 Functional form of the bound proton PDF same as for the free proton (~CTEQ61 [hep-ph/0702159], x restricted to 0 < x < 1)

$$xf_i^{p/A}(x,Q_0) = c_0 x^{c_1} (1-x)^{c_2} e^{c_3 x} (1+e^{c_4} x)^{c_5}, \qquad i = u_v, d_v, g, \dots$$

$$\bar{d}(x,Q_0)/\bar{u}(x,Q_0) = c_0 x^{c_1} (1-x)^{c_2} + (1+c_3 x)(1-x)^{c_4}$$

• A-dependent fit parameters (reduces to free proton for A = 1)

$$c_k \to c_k(A) \equiv c_{k,0} + c_{k,1} \left(1 - A^{-c_{k,2}} \right), \quad k = \{1, \dots, 5\}$$

• PDFs for nucleus (A, Z)

$$f_i^{(A,Z)}(x,Q) = \frac{Z}{A} f_i^{p/A}(x,Q) + \frac{A-Z}{A} f_i^{n/A}(x,Q)$$

(bound neutron PDF $f_i^{n/A}$ by isospin symmetry)

Data sets

- NC DIS & DY
 CERN BCDMS & EMC & NMC
 N = (D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W)
 FNAL E-665
 N = (D, C, Ca, Pb, Xe)
 DESY Hermes
 N = (D, He, N, Kr)
 SLAC E-139 & E-049
 N = (D, Ag, Al, Au, Be,C, Ca, Fe, He)
 FNAL E-772 & E-886
 N = (D, C, Ca, Fe,W)
- ► Single pion production (new)



RHIC - PHENIX & STAR N = Au



► Neutrino (to be included later)



Schematics of Global Analysis

1. Parametrize PDFs at low initial scale $\mu = Q_0 = 1.3$ GeV:

$$f(x,Q_0) = f(x;a_0,a_1,\dots) = a_0 x^{a_1} (1-x)^{a_2} P(x;a_3,\dots)$$

- 2. Use DGLAP equation to evolve $f(x, \mu)$ from $\mu = Q_0$ to $\mu = Q_{\text{max}}$.
- 3. Define and minimize appropriate χ^2 function (with respect to parameters a_0, a_1, \ldots)

$$\chi^{2}(\{a_{i}\}) = \sum_{\text{experiments}} w_{n}\chi^{2}_{n}(\{a_{i}\})$$
$$\chi^{2}_{n}(\{a_{i}\}) = \sum_{\text{data points}} \left(\frac{\text{data} - \text{theory}(\{a_{i}\})}{\text{uncertainty}}\right)^{2}$$
(by default $w_{n} = 1$)

Fit details

Fit properties:

- ► fit @NLO
- $Q_0 = 1.3 \text{GeV}$
- ▶ using ACOT heavy quark scheme
- kinematical cuts: Q > 2 GeV, W > 3.5 GeV
- ► 708 (DIS & DY) + 32 (single π^0) = 740 data points after cuts
- ▶ 16 free parameters
- 7 gluon 7 valence 2 sea $\chi^2 = 618$, giving $\chi^2/dof = 0.85$

Error analysis:

▶ use Hessian method

$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij} (a_i - a_i^0) (a_j - a_j^0)$$
$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- tolerance $\Delta \chi^2 = 35$ (every nuclear target within 90% C.L.)
- ▶ eigenvalues span 10 orders of magnitude → require numerical precision
- use noise reducing derivatives

• **Expand** χ^2 function around minimum, $\{a_i^0\}$,

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2} (a_i - a_i^0) (a_j - a_j^0) \left(\frac{\partial^2 \chi^2}{\partial a_i \partial a_j}\right)_0$$





• Expand χ^2 function around minimum, $\{a_i^0\}$, and diagonalize

$$\chi^{2} = \chi_{0}^{2} + \sum_{ij} \frac{1}{2} (a_{i} - a_{i}^{0})(a_{j} - a_{j}^{0}) \left(\frac{\partial^{2} \chi^{2}}{\partial a_{i} \partial a_{j}}\right)_{0} = \chi_{0}^{2} + \sum_{i} z_{i}^{2}$$





• Expand χ^2 function around minimum, $\{a_i^0\}$, and diagonalize

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- Choose tolerance criteria $\Delta \chi^2 = \chi^2 \chi_0^2$ value (defining 1- σ uncertainty),
 - ideal case $\Delta \chi^2 = 1$
 - ► realistic global analysis $\Delta \chi^2 \sim 1 100$



• Expand χ^2 function around minimum, $\{a_i^0\}$, and diagonalize

$$\chi^{2} = \chi_{0}^{2} + \sum_{ij} \frac{1}{2} (a_{i} - a_{i}^{0})(a_{j} - a_{j}^{0}) \left(\frac{\partial^{2} \chi^{2}}{\partial a_{i} \partial a_{j}}\right)_{0} = \chi_{0}^{2} + \sum_{i} z_{i}^{2}$$

- Choose tolerance criteria $\Delta \chi^2 = \chi^2 \chi_0^2$ value (defining 1- σ uncertainty),
 - ideal case $\Delta \chi^2 = 1$
 - \blacktriangleright realistic global analysis $\Delta\chi^2 \sim 1-100$
- Construct error PDFs corresponding to each eigenvector direction:

$$f_i^{\pm} = f(\{z_i\}) = f(0, \dots, z_i) = \pm \sqrt{\Delta \chi^2}, \dots, 0)$$
$$z_i = \pm \sqrt{\Delta \chi^2}$$

Calculate errors of observable X:

$$\Delta X = \frac{1}{2} \sqrt{\sum_{i} \left[X(f_i^+) - X(f_i^-) \right]^2}$$

nCTEQ results impact of single π^0 production data



Compare nCTEQ fits: with π^0 data (violet) without π^0 data (gray)



comparison with other nPDF fits

Nuclear correction factors (Q = 10 GeV) $R_i(Pb) = \frac{f_i^{Pb}(x,Q)}{f_i^p(x,Q)}$

- different solution for *d*-valence &
 u-valence compared to EPS09 & DSSZ
- sea quark nuclear correction factors similar to EPS09
- nuclear correction factors depend largely on underlying proton baseline



 F_2 ratios

Structure function ratio

$$R = \frac{F_2^{Fe}(x,Q)}{F_2^D(x,Q)}$$

- ▶ good data description
- despite different u-valence & d-valence ratios are similar to EPS09



π^0 production

Pion production, ratio

 $R_{\rm dAu}^{\pi} = \frac{\frac{1}{2A}d^2\sigma_{\pi}^{\rm dAu}/dp_T dy}{d^2\sigma_{\pi}^{\rm pp}/dp_T dy}$

- good data description, however big experimental uncertainities do not allow for strong constraints on PDFs
- despite different u-valence & d-valence ratios are similar to EPS09



[Phys.Rev.Lett. 106(2011) 122301, arXiv: 1012.1178]

Neutrino DIS cross-section data



Challenges in combining the neutrino & charged lepton data deal with the disparity of number of data points - assigning weights to neutrino data neutrino DIS data only with 2 heavy nuclei - insufficient to get a reliable A-dependance

[Phys.Rev.Lett. 106(2011) 122301, arXiv: 1012.1178]

Comparison of charged lepton and neutrino fits



Fit to charged lepton data DIS & DY

 $\chi^2/{\rm d.o.f} = 0.89$





can we explain the difference and fit all data together in a global fit ?

[Phys.Rev.Lett. 106(2011) 122301, arXiv: 1012.1178]

- It was argued in the literature that vA DIS data is compatible with l[±]A data: Paukkunen, Salgado, JHEP 07, 32 2010, arXiv:1004.3140 de Florian, Sassot, Stratmann, Zurita, PRD85, 074028 (2012), arXiv:1112.6324
- However these results were obtained using uncorrelated errors for NuTeV experiment, which were crucial for the nCTEQ conclusions PRL106, 122301 (2011).

w	$l^{\pm}A$	$ \chi^2(/{ m pt}) $	νA	$\chi^2(/\mathrm{pt})$	total $\chi^2(/\text{pt})$
1-corr	708	736(1.04)	3134	4246(1.36)	4983(1.30)
1-uncorr	708	809(1.14)	3110	3115(1.00)	3924 (1.02)

[Phys.Rev.Lett. 106(2011) 122301, arXiv: 1012.1178]

▶ Recent paper

Paukkunen, Salgado, PRL110, 212301 (2013), arXiv:1302.2001 suggests a method to renormalize the NuTeV data so that the tension between νA and $l^{\pm}A$ data sets are removed

Instead of looking at the ratio of cross-sections $R^{\nu}(x, y, E) \equiv \frac{\sigma_{\exp}^{\nu}(x, y, E)}{\sigma_{CTEQ6.6}^{\nu}(x, y, E)}$

they suggest considering

$$\overline{R}^{\nu}(x,y,E) \equiv \frac{\sigma_{\exp}^{\nu}(x,y,E)/I_{\exp}^{\nu}(E)}{\sigma_{\text{CTEQ6.6}}^{\nu}(x,y,E)/I_{\text{CTEQ6.6}}^{\nu}(E)}$$

where

$$I_{\exp}^{\nu}(E) \equiv \sum_{i \in \text{fixed } E} \sigma_{\exp,i}(x, y, E) \times B_i(x, y)$$

 $B_i(x, y)$ is size of the experimental (x, y)-bin

 However this analysis still do not take into account correlated errors provided by NuTeV collaboration.



• when correlated errors taken into account νA and $l^{\pm} A$ prefer different nuclear corrections

Summary and outlook

- ▶ We have updated the nCTEQ error PDFs (still preliminary).
- nCTEQ PDFs features larger uncertainties but they are still underestimated.
- To have reliable estimate of nuclear corrections we need more data (LHC *lead* run can help).
- Nuclear component important not only for heavy ion collisions, but also for the free-proton analysis.
- Plans for future:
 - ▶ official release of current analysis
 - revisit question of neutrino nuclear corrections
 - include new data (LHC, JLAB, Miner ν a)
 - high x region, deuterium nuclear corrections, higher twist effects – CJ collaboration in JLAB

nCTEQ nuclear parton distribution functions

nCTEQ project is an extension of the CTEQ collaborative effort to determine parton distribution functions inside of a free proton. It generalizes the free-proton PDF framework to determine densities of partons in bound protons (hence nCTEO which stands for nuclear CTEO). More details on the framework and the first results can be found in arXiv:09072357 [hep-ph]. The effects of the nuclear environement on the parton densities can be shown as modified parton densities



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PDF grids & code

where all black curves stand for free proton PDF and red, green, blue, cyan, pink, yellow, magenta and brown curves show PDF in protons bound in nuclei - from deuterium (red) to lead (brown).

An alternative way how effects of nuclear environement can be displayed is in ratios of Deep Inelastic Scattering (DIS) structure functions e.g. ratios of of the structure function F 2 for a neutral current DIS as in the figure below on the left or ratios of of the same structure function F 2 but for a charged current DIS.

Assumptions entering the nuclear PDF analysis

1. Factorization & DGLAP evolution

- allow for definition of universal PDFs
- make the formalism predictive
- needed even if it is broken

2. Isospin symmetry
$$\begin{cases} u^{n/A}(x) = d^{p/A}(x) \\ d^{n/A}(x) = u^{p/A}(x) \end{cases}$$

3. $x \in (0,1)$ like in free-proton PDFs [instead of (0, A)]

Then observables \mathcal{O}^A can be calculated as:

$$\mathcal{O}^A = Z \, \mathcal{O}^{p/A} + (A - Z) \, \mathcal{O}^{n/A}$$

With the above assumptions we can use the free proton framework to analyze nuclear data

Nuclear PDFs
$$(Q = 10 \text{GeV})$$

 $xf_i^{Pb}(x,Q)$

Compare nCTEQ fits:

- with π^0 data (violet)
- without π^0 data (gray)



Nuclear correction factors (Q = 10 GeV)

$$R_i(Pb) = \frac{f_i^{Pb}(x,Q)}{f_i^p(x,Q)}$$

Compare nCTEQ fits:

- with π^0 data (violet)
- without π^0 data (gray)



nCTEQ vs. EPS09

nCTEQ

$$\begin{split} &xu_v^{p/A}(Q_0) = x^{c_1^u}(1-x)^{c_2^u}e^{c_3^ux}(1+e^{c_4^u}x)^{c_5^u} \\ &xd_v^{p/A}(Q_0) = x^{c_1^d}(1-x)^{c_2^d}e^{c_3^dx}(1+e^{c_4^d}x)^{c_5^d} \end{split}$$

EPS09

$$\begin{split} u_v^{p/A}(Q_0) &= R_v(x,A,Z) \, u(x,Q_0) \\ d_v^{p/A}(Q_0) &= R_v(x,A,Z) \, d(x,Q_0) \end{split}$$

$$c_{k}^{u_{v}} = c_{k,0}^{u_{v}} + c_{k,1}^{u_{v}} \left(1 - A^{-c_{k,2}^{u_{v}}} \right)$$

$$c_{k}^{d_{v}} = c_{k,0}^{d_{v}} + c_{k,1}^{d_{v}} \left(1 - A^{-c_{k,2}^{d_{v}}} \right)$$

$$R_{v} = \begin{cases} a_{0} + (a_{1} + a_{2}x)(e^{-x} - e^{-x_{a}}) & x \le x_{a} \\ b_{0} + b_{1}x + b_{2}x^{2} + b_{3}x^{3} & x_{a} \le x \le x_{e} \\ c_{0} + (c_{1} - c_{2}x)(1 - x)^{-\beta} & x_{e} \le x \le 1 \end{cases}$$



we set:

 $c_k^{u_v}$:

$$\begin{cases} c_1^{d_v} = c_1^{u_v} \\ c_2^{d_v} = c_2^{u_v} \end{cases}$$

Variables: DIS of nuclear target $eA \to e'X$

► DIS variables in case on nucleons in nucleus $\begin{cases} Q^2 \equiv -q^2 \\ x_A \equiv \frac{Q^2}{2 p_A \cdot q} \end{cases}$



- p^A nucleus momentum
- ▶ $x_A \in (0, 1)$ analog of Bjorken variable (fraction of the nucleus momentum carried by a nucleon)
- ▶ Analogue variables for partons:
 - $p_N = \frac{p_A}{A} average$ nucleon momentum
 - $x_N \equiv \frac{Q^2}{2p_{N'q}} = A x_A$ parton momentum fraction with respect to the avarage nucleon momentum p_N
 - ▶ $x_N \in (0, A)$ parton can carry more than the average nucleon momentum p_N .

Pion production, ratio

$$R_{\rm dAu}^{\pi} = \frac{\frac{1}{2A} d^2 \sigma_{\pi}^{\rm dAu} / dp_T dy}{d^2 \sigma_{\pi}^{\rm pp} / dp_T dy}$$

- good data description, however big experimental uncertainities do not allow for strong constraints on PDFs
- despite different u-valence & d-valence ratios are similar to EPS09



Neutrino DIS cross-section data



NuTeV & di-muon → 2310 data points N = FeCHORUS → 824 data points N = Pb

- All charged lepton DIS & Drell-Yan data → 708 data points
- Challenges in combining the neutrino & charged lepton data
 - deal with the disparity of number of data points assigning weights to neutrino data
 - neutrino DIS data only with 2 heavy nuclei insufficient to get a reliable A-dependance
 - o all neutrino data show the different behavior or only NuTeV ?
- Different neutrino observables

$$\frac{d\sigma^{\nu A}}{dxdQ^2}~\&~\frac{d\sigma^{\bar{\nu}A}}{dxdQ^2}$$

$$\sim {d\sigma^{\bar{
u}A}\over dx dQ^2}$$
 vs.

$$F_2^{\nu+\bar{\nu}}(x,Q^2)$$
 & $xF_3^{\nu+\bar{\nu}}(x,Q^2)$

needs theory assumptions to extract

- Nuclear correction factors
 - $R[F_2^{\nu}] = F_2^{\nu A} / F_2^{\nu A, \text{free}}$ we show correction factors defined e.g. as





- NLO QCD calculation of $\frac{F_2^{\nu A} + F_2^{\bar{\nu} A}}{2}$ in the ACOT-VFN scheme
 - comparison of nCTEQ only neutrino fit against extracted NuTeV data at different Q²
 - charge lepton fit undershoots low-x data & overshoots mid-x data
 - Iow-Q² and small-x data cause tension with the shadowing observed in charged lepton data



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[Phys.Rev.Lett. 106(2011) 122301, arXiv: 1012.1178]

Analysis of fits with different weights of neutrino DIS (correlated errors)

w	$l^{\pm}A$	$\chi^2 \ (/\mathrm{pt})$	νA	$\chi^2 \ (/\mathrm{pt})$	total $\chi^2(/\text{pt})$
0	708	630(0.89)	-	-	630 ± 58
1/7	708	645 (0.91)	3134	4681 (1.50)	5326 ± 203
1/2	708	$680 \ (0.96)$	3134	4375(1.40)	5055 ± 192
1	708	736(1.04)	3134	4246(1.36)	4983 ± 190
∞	-	_	3134	4167(1.33)	4167 ± 176



0.01

0.1 x

[Phys.Rev.Lett. 106(2011) 122301, arXiv: 1012.1178]

Analysis of fits with neutrino DIS (uncorrelated errors)



0.01

0.1

[Phys.Rev.Lett. 106(2011) 122301, arXiv: 1012.1178]

Properties of neutrino fits

CHORUS data are in good agreement with the charged lepton data

combined: χ^2 /pt=1.03

NuTeV data (with correlated errors) difficult to fit alone or with the charged lepton data alone: χ^2 /pt=1.35 combined: χ^2 /pt=1.33



