Transverse Enhancement: Fits to e-A and application to v-A scattering



Nulnt14,

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Outline

- I. QE v-A scattering and the axial mass anomaly
- II. QE e-A scattering, response functions and superscaling
- III. Extraction of transverse enhancement from e-C data
- IV. New QE superscaling fit to $^{12}\mathrm{C}$ inclusive data
- V. Including final state nucleons *effective spectral function* VI. Summary

Number of puzzles in recent v-A cross section measurements

Nucleon modification to axial form factor $G_A(Q^2)$?

Dipole Form

 $G_A(Q^2) = -1.267 / (1 + Q^2 / M_A^2)^2$

Recent nuclear target data Indicate axial mass larger than from v-d and threshold electroproduction

Free nucleon value: $M_A \sim 1.014 \text{ GeV}$

Experiment	Target	Cut in Q^2 [GeV ²]	$M_A[GeV]$
K2K ⁴	oxygen	$Q^2 > 0.2$	1.2 ± 0.12
$K2K^5$	carbon	$Q^2 > 0.2$	1.14 ± 0.11
MINOS ⁶	iron	no cut	1.19 ± 0.17
MINOS ⁶	iron	$Q^2 > 0.2$	1.26 ± 0.17
MiniBooNE ⁷	carbon	no cut	1.35 ± 0.17
MiniBooNE ⁷	carbon	$Q^2 > 0.25$	1.27 ± 0.14
NOMAD ⁸	carbon	no cut	1.07 ± 0.07

Table from Juszczak, et.al. Arxiv: 1007.2195

Carbon data at low and high E_v appear inconsistent with dipole axial form factor.



NOMAD High energy ¹²C data consistent with bare nucleon $M_A = 1.03$ **MiniBooNE** low energy ¹²C data requires enhanced $M_A = 1.35$

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Significant theoretical efforts to understand these inconsistencies

- Neives et.al. arXiv: 1102.2777 [hep-ph]
- Martini et. al. PRC80, 065501 (2009)
- Amaro et. el., PRC82, 04601
 - ... Many more

based on NN correlations which result in 2p2h (2 particles 2 holes):

Short range correlations, Meson Exchange Currents (MECs)



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Story mirrors that from inclusive e-A QE scattering several decades ago...

- → QE cross section was found to be enhanced beyond independent nucleon Impulse Approximation (IA)
- → Enhancement is Q² dependent and predominantly in part of cross section from photoabsorption of transversely polarized (helicity +/-1) virtual photons.

QE Response and scaling functions

Photon polarization

$$\frac{d^2\sigma}{d\Omega d\omega} \frac{1}{\sigma_{Mott}} \epsilon \left(\frac{q}{Q}\right)^4 = \epsilon R_L(q,\omega) + \frac{1}{2} \left(\frac{q}{Q}\right)^2 R_T(q,\omega) \qquad \epsilon = \left(1 + \frac{2q^2}{Q^2} \tan^2 \frac{\vartheta}{2}\right)^{-1}$$

For the elementary *nucleon* electric (G_E) and magnetic (G_M) form factors

Define scaling functions:

$$f_{L,T}(\boldsymbol{\psi}') \equiv k_f \frac{R_{L,T}}{G_{L,T}}$$

$$G_T = kin.fact * (ZG_{Mp}^2 + NG_{Mn}^2)$$

With scaling variables:

$$\psi' \sim y/k_{f}$$

$$y = -q + \sqrt{\omega^2 + 2\omega m}$$

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2D distribution in nucleon momentum and removal energy

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[1, 2] introducing a dimensionless scaling variable as above,

$$\psi' \equiv \frac{1}{\sqrt{\xi_F}} \frac{\lambda' - \tau'}{\sqrt{(1 + \lambda')\tau' + \kappa}\sqrt{\tau'(\tau' + 1)}},$$

where $\lambda_{shift} \equiv E_{shift}/2m_N$, $\lambda' \equiv \lambda - \lambda_{shift}$ and $\tau' \equiv \kappa^2 - \lambda'^2$.

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→ Known for over a decade that electron QE longitudinal cross sections obey superscaling in both A, Q²



 \rightarrow Transverse cross sections exhibit enhancement, which is Q² dependent.

- → Enhancement in $f_{\tau}(\psi, Q^2)$ relative to $f_{\mu}(\psi, Q^2)$ attributed to MECs.
- → Extraction of $R_{\tau} = \prod \int d\psi f_{\tau}(\psi, Q^2) / \int d\psi f_{\tau}(\psi, Q^2) at low Q^2 by$

Carlson et al. PRC 65:024003 (2002)

"Longitudinal and transverse quasielastic response functions of light nuclei" J. Carlson, J. Jourdan, R. Schiavilla, I. Sick

Phys. Rev. C 65, 024002 (2002)



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Extraction of Transverse Enhancement

J. Carlson, J. Jourdan, R. Schiavilla, I. Sick (2002)



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The Fermi Gas superscales and

also requires a Transverse Enhancement

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2-body exchange currents



- → These lead to 2p2h final states and can interfere
- → The short range correlation (SRCs) resulting from the tensor part of the interaction also leads to 2p2h final states, but these are an intrinsic part of the single nucleon momentum distribution (e.g. $f(\psi)$).
- → The initial state correlated pair from SRC are predominantly in quasi-deuteron states.

Bosted-Mamyan Fit to inclusive e-A cross section data

Having a fit which accurately describes Cross section data is critical for input to Radiative Corrections and useful for many physics studies. Such a fit was performed by Bosted-Mamyan

→ Input to fit are the inclusive proton and deuteron (neutron) inelastic cross section fits from Christy and Bosted PRC77(08)065206, PRC81(10)055213

 \rightarrow The QE cross section is calculated in the IA using the superscaling formalism and the Bosted parametrization of the elastic from factors.

- → The inelastic smearing is performed using a Gaussian distribution with width = k_{f} .
- \rightarrow Correction factors for medium modifications Including the EMC effect are applied.
- → A broad distorted Gaussian in included for the MEC (TE) in the transverse cross section.

Preliminary E04–001, E = 4.629, Ø = 10.661 20000 18000 $Q^2 = 0.68 (GeV/c)^2$ Tota $\varepsilon = 0.98$ OE Inelastic Relative Cross s 10000 10000 $R_r = 1.7$ OE transverse QE Longitudinal 10000 8000 6000 4000 2000 1.2 0.8 1.6 0.6 1.4 1.8 $W^2(GeV^2)$



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TE model Motivated by Carlson et al

 \rightarrow Electron data strongly suggests that the enhancement is only in transverse scattering.

or v-A QE:

$$\sigma_T^{\text{vector}} \propto \tau |\mathcal{G}_M^V(Q^2)|^2; \qquad \sigma_T^{\text{axial}} \propto (1+\tau) |\mathcal{F}_A(Q^2)|^2,$$

$$\sigma_L^{\text{vector}} \propto (\mathcal{G}_E^V(Q^2))^2; \qquad \sigma_L^{\text{axial}} = 0.$$

→ Inclusive neutrino scattering experiments on typically sensitive to v (or W) dependence of enhancement, but only to total increased cross section in QE region.

=> Parameterize the TE in a nucleus as a larger *effective* magnetic form factor of the bound nucleon, with

$$G_{M}^{2} = G_{M}^{2} * R_{T} \qquad \qquad \mathcal{R}_{T} = \frac{QE_{transverse} + TE}{QE_{transverse}}$$

Where the QE and TE part of the cross section has been integrated over all ν (W)

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→ Extract enhancement in following Carlson et al. PRC 65:024003 (2002)

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We parameterize TE in a nucleus as a larger effective magnetic form factor of the bound nucleon. $G_{M_{P}}^{nuclear}(Q^{2}) = G_{M_{P}}(Q^{2}) \times \sqrt{1 + AQ^{2}e^{-Q^{2}/B}}$

$$G_{Mn}^{nuclear}(Q^2) = G_{Mn}(Q^2) \times \sqrt{1 + AQ^2 e^{-Q^2/B}}.$$

This prescription assumes that there is no enhancement in longitudinal scattering, or in the axial contribution in neutrino scattering.

Longitudinal (L) - scattering from charge. Charge is conserved, Coulomb sum rule is found to be valid in electron scattering. Since no enhancement is seen in the longitudinal scattering it implies that the charge distribution of bound nucleons is not changed in a nucleus.

Transverse (T) - Scattering from currents, **orbital angular momentum and Dirac and anomalous magnetic moments**. These are not conserved (e.g. Meson exchange currents)

Axial current is partially conserved, so we assume that axial form factor is not modified in a nucleus.

The above prescription implies that the vector amplitudes from MEC/TEC interfere with axial current in the non-TE Transverse component

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Both data sets seen to be consistent with free nucleon M_{A} !

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Antineutrino



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By including TE in IA calculation for v-A via $G_M \rightarrow G_M^* R_T$ we can resolve:

1. the 'Axial Anomoly' data consistent with free nucleon M_A

2. Apparent inconsistency between NOMAD and MiniBoone resolved



TEM describes both MINERvA ν and $\nu\,$ CH data.

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New ¹²C Inclusive QE Fit

- \rightarrow Include world data set
 - see Donal Day QE archive at

http://faculty.virginia.edu/qes-archive/index.html

 \rightarrow Include Coulomb Corrections via effective momentum Approx.

A. Aste et. al 2005

- \rightarrow Update nucleon electromagnetic Form Factor parameterizations.
- \rightarrow Allow normalization factors for each data set.
- \rightarrow Allow for optimization of scaling functions

12C Fit Results:





Normalization of data sets						
(preliminary)						
1.	Barreau	(1983)	0.980			
2.	O'Connel	(1987)	0.983			
3.	Sealock	(1989)	1.051			
4.	Baran	(1988)	0.987			
5.	Bagdasaryan	(1988)	0.982			
6.	Zelllar	(1973)	Inconsistent			
7.	Arrington	(1995)	0.983			
8.	Day	(1993)	1.006			
9.	Arrington	(1998)	0.992			
10.	Gaskell	(2008)	1 .008			
11.	Whitney	(1974)	0.988			
12.	E04-001 prelim	low Q ²	1.004			
13.	E04-001 prelim	high Q ²	1.005			
→ Only Zeller data found to have large						

→ normalization factors are generally 2% of unity
 * Exception is Sealock data.

Inconsistencies.

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Fit Results:



- (i) Shape of residuals consistent between data sets
- (ii) Residuals have no dependence on ε

=> consistent with only *transverse* enhancement

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 $Q^2 = 0.3$



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Very low Q² Results:

 $Q^2 < 0.1$ Preliminary E04-001, E = 1.207, Ø = 10.811 Baran (1988), E = 1.303, $\Theta = 11.95$ Relative Cross section Relative Cross section $Q^2 = 0.07 (GeV/c)^2$ $Q^2 = 0.05 (GeV/c)^2$ 50 50 Total Total QE QE $\varepsilon = 0.98$ $\varepsilon = 0.98$ 40 Inelastic 40 Inelastic 30 30 20 20 10 10 10 20 0.6 0.8 1.2 1.6 1.8 0.6 0.8 1.2 1.4 1.6 1.8 1 1.4 17.5 10 15 8 12.5 10 6 7.5 4 5 2 2.5 C 0 -2.5 -2 -50.6 0.8 1.2 1.4 1.6 1.8 0.6 0.8 1.2 1.4 1.6 1.8 $W^2(GeV^2)$ $W^2(GeV^2)$

- → Residuals for Q2 < 0.1 show double hump structure, but with
- → shape of residuals generally consistent between data sets.
- → Very small enhancement required (d σ is mostly longitudinal).

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12C Fit Residuals



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Updated TE factor



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How to include final state nucleons in MC generators?

- \rightarrow Rather trivial to include in terms of spectral functions (SFs)
- \rightarrow Spectral Functions are already included in v event generators

Want a SF which gives the same shape in v (W) from inclusive scattering and resonably incorporates the dependence on the removal energy.

To extract an *effective* spectral function

We vary 10 parameters until we get a prediction which closely Matches the predictions of the ψ' superscaling formalism:

(i) 8 parameters of the nucleon momentum distribution

(ii) Removal energy: In our model the off-shell energy of the spectator nucleon has only two possibilities: 1p1h process and the 2p2h processes, with the relative fractions assumed to be independent of momentum and allowed to vary in the fit (f_{1p1h})

(iii) the average effective binding energy (Δ)



Fig. 4. 1p1h process: Scattering from an off-shell bound neutron of momentum $\mathbf{P_i} = -\mathbf{k}$ in a nucleus of mass A. The onshell recoil $(A - 1)^*$ (spectator) nucleus has a momentum $\mathbf{P_{A-1}^*} = \mathbf{P_s} = \mathbf{k}$ and an average excitation energy Δ (effective binding energy). Here $M_{A-1}^* = M_A - M_n + \Delta$. The initial state off-shell neutron has energy $E'_n = M_n - \Delta - \frac{\mathbf{k}^2}{2M_{A-1}^*}$).

1p1h Mean field component.

We assume that this process has a probability of f_{1p1h} =1- f_{2p2h} (independent of momentum k) Here the recoil is an (A-1)* excited nucleus

$$\begin{split} E_n'(1p1h) &= M_A - \sqrt{k^2 + (M_{A-1}^*)^2} \\ &= M_n - \Delta - \frac{k^2}{2M_{A-1}^*} \end{split}$$

Removal energy for this process is small since a large nucleus with little energy is balancing the momentum.

$$(M_{n}^{'})^{2} = (E_{n}^{'})^{2} - \mathbf{k^{2}}$$



$$E'_{n}(2p2h) = M_{D} - 2(\Delta) - \sqrt{k^{2} + M_{p}^{2}}$$

2p2h component. Two nucleon corrections (quasideuterons)

We assume that this process has a probability of f2p2h (independent of momentum k)

Here, the recoil is a single nucleon so the removal energy is large. (momentum k is balanced by a single proton instead of a large nucleus)

$$(M_{n}^{'})^{2} = (E_{n}^{'})^{2} - \mathbf{k^{2}}$$

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$$E'_{n}(1p1h) = M_{A} - \sqrt{k^{2} + (M_{A-1}^{*})^{2}}$$
$$= M_{n} - \Delta - \frac{k^{2}}{2M_{A-1}^{*}}$$

2p2h

$$E'_{n}(2p2h) = M_{D} - 2(\Delta) - \sqrt{k^{2} + M_{p}^{2}}$$

Fig. 3. Comparison of energy for on-shell and off-shell neutrons. The on-shell energy is $E_n = \sqrt{k^2 + M_n^2}$. The off-shell energy is shown for both the 1p1h $(E'_n = M_n - BE - \frac{k^2}{2M_{A-1}^*})$ and 2p2h process $(E'_n = M_D - 2(BE) - \sqrt{k^2 + M_p^2})$ May 20, 2014 M. E. Christy - Nuint14

Benhar Fantoni Spectral Function 2D Removal energy vs momentum



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Effective Spectral Function Fit Results

Parameter	C12 Benhar-Fantoni	C12 Effective	
BE (MeV)	2Dspectral	12.5	
f_{2p2h}	2 Dspectral	0.19	
b_s	1.7	2.12	
b_p	1.77	0.7366	
α	1.5	12.94	
β	0.8	10.62	
c_1	2.823397	197.0	
c_2	7.225905	9.94	
<i>C</i> 3	0.00861524	$4.36 \text{ x} 10^{-5}$	
N	0.985	29.64	

Table 2. Parameterizations of the Momentum Distribution for Carbon 12 for the Benhar-Fantoni spectral function and for our "effective spectral function"



- We increased the fraction of high momentum components to mimic the effect of FSI.
- The fraction of 2p2h is 19%..
- The effective binding energy is 12.5 MeV. Close to the value of the ψ^{\prime} superscaling function.

Comparison for various values of Q2



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Summary

1. The Transverse Enhancement extracted from electron QE scattering data has been used to predict the enhancement due to 2p2h processes beyond the IA for neutrino scattering

2. The TEM prediction is found to resolve a number of apparent inconsistencies between v data sets and is found to be in good agreement with the first MINERvA data for both v and v.

3. A new ψ scaling fit to the available inclusive electron scattering data from ¹²C has been performed.

4. An *effective* spectral function has been developed, which produces consistent results to ψ scaling and can be easily implemented in current v generators.

Backup

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Proton $G_{\rm M}$ and $G_{\rm E}$



- → Measured to good precision in few GeV range.
- → deviations from *dipole* up to 5% or more in $G_{\rm M}$ for Q² < 2 GeV².
- → Discrepancy in $G_{\rm E}/G_{\rm M}$ polarization transfer results and Rosenbluth (ϵ) separations thought go due to 2-photon exchange terms. Under intense experimental investigation.

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Neutron G_{M}^{n} and G_{E}^{n}

[nucl-ex]



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Relations between electric and magnetic form factors and structure functions for elastic electron scattering on free

nucleons
$$\mathcal{W}_{1p}^{elastic} = \delta(
u - rac{Q^2}{2M}) au |G_{Mp}(Q^2)|^2$$

$$\mathcal{W}_{1n}^{elastic} = \delta(\nu - \frac{Q^2}{2M})\tau |G_{Mn}(Q^2)|^2$$

and

$$\begin{split} \mathcal{W}_{2p}^{elastic} &= \delta(\nu - \frac{Q^2}{2M}) \frac{[G_{Ep}(Q^2)]^2 + \tau [G_{Mp}(Q^2)]^2}{1 + \tau} \\ \mathcal{W}_{2n}^{elastic} &= \delta(\nu - \frac{Q^2}{2M}) \frac{[G_{En}(Q^2)]^2 + \tau [G_{Mn}(Q^2)]^2}{1 + \tau} \\ R_{p,n}^{elastic}(x = 1, Q^2) &= \frac{\sigma_L^{elastic}}{\sigma_T^{elastic}} = \frac{4M^2}{Q^2} \left(\frac{G_E^2}{G_M^2}\right) \end{split}$$

Here, $\tau = Q^2/4M_{p,n}^2$, where $M_{p,n}$ are the masses of proton and neutron. Therefore, G_{Mp} and G_{Mn} contribute to the transverse virtual photo-absorption cross section, and G_{Ep} and G_{En} contribute to the longitudinal cross section.

For Neutrino QE scattering: Vector form factors are known from electron scattering. But we also have axial form factors

$$\begin{split} W_{1-\text{Qelastic}}^{\nu-\text{vector}} &= \delta \left(\nu - \frac{Q^2}{2M} \right) \tau \left| \mathcal{G}_M^V(Q^2) \right|^2, \\ W_{1-\text{Qelastic}}^{\nu-\text{axial}} &= \delta \left(\nu - \frac{Q^2}{2M} \right) (1+\tau) \left| \mathcal{F}_A(Q^2) \right|^2, \\ W_{2-\text{Qelastic}}^{\nu-\text{vector}} &= \delta \left(\nu - \frac{Q^2}{2M} \right) \left| \mathcal{F}_V(Q^2) \right|^2, \\ W_{2-\text{Qelastic}}^{\nu-\text{axial}} &= \delta \left(\nu - \frac{Q^2}{2M} \right) \left| \mathcal{F}_A(Q^2) \right|^2, \\ W_{3-\text{Qelastic}}^{\nu} &= \delta \left(\nu - \frac{Q^2}{2M} \right) \left| \mathcal{G}_M^V(Q^2) \mathcal{F}_A(Q^2) \right|, \end{split}$$

where

$$\mathcal{G}_{E}^{V}(Q^{2}) = G_{E}^{p}(Q^{2}) - G_{E}^{n}(Q^{2}),$$

$$\mathcal{G}_{M}^{V}(Q^{2}) = G_{M}^{p}(Q^{2}) - G_{M}^{n}(Q^{2}).$$

and

$$\left|\mathcal{F}_{V}(Q^{2})\right|^{2} = \frac{\left[\mathcal{G}_{E}^{V}(Q^{2})\right]^{2} + \tau\left[\mathcal{G}_{M}^{V}(Q^{2})\right]^{2}}{1+\tau}.$$

$$\sigma_{T}^{\text{vector}} \propto \tau \left|\mathcal{G}_{M}^{V}(Q^{2})\right|^{2}; \quad \sigma_{T}^{\text{axial}} \propto (1+\tau)\left|\mathcal{F}_{A}(Q^{2})\right|^{2},$$
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$$\sigma_{L}^{\text{vector}} \propto \left(\mathcal{G}_{E}^{V}(Q^{2})\right)^{2}; \quad \sigma_{L}^{\text{axial}} = 0.$$

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Notes:

1. TEM parametrizes missing strength beyond impulse approximation (IA) with super-scaling Momentum distribution.

2. This could be due to a number of missing physics, including MEC, missing high-momentum components, FSI ...

 \rightarrow might expect a signature to be an additional low momentum proton, which many v experiments are blind to and can not identify as non-QE.

 \rightarrow New experiments might be able to test this. Liquid Argon TPCs (eg. Argoneut) are likely to be the most sensitive at the lowest p energies.

2p2h = 2 nucleons + 2 holes in final state



(2) Two nucleon correlations"quasideutrons)

MA-2* = MA-1+ 2BE

(on shell excited spectator nucleus)

MD* = MD- 2BE (off shell)

It is simple to model MEC as another quasideuteron processes because the process is already implemented in GENIE for the two nucleon SRC. Therefore, it should be easy to include it.

We just need

- (1)Q2 dependence of the enhanced cross section
- (2) The removal energy
- (3) Momentum distribution

We can get all three pieces of information from electron scattering data.

For simplicity we model the momentum distribution of the MEC quasideuteron as a particle in a box, i.e. a Fermi gas is. We just need to know the Fermi momentum of this MEC quasideuteron.

QE Response functions in e-A Scattering



How important are the vector form factors as input?



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FIG. 3. The Fermi momenta K_F for various nuclei of atomic weight A from Moniz *et al.* (Ref. 6).

Table I. Nuclear Fermi momentum $k_{\rm F}$ and average nucleon interaction energy $\overline{\epsilon}$ determined by least-squares fit of theory to quasielastic peak.

Nucleus	(I)	k _F /IeV/c) ^a	€ (MeV) ^b
${}^{3}{ m Li}^{6}_{6}{ m C}^{12}_{12}{ m Mg}^{24}_{20}{ m Ca}^{40}_{28}{ m Ni}^{58} \cdot ^{7}_{39}{ m Y}^{89}_{50}{ m Sn}^{118} \cdot ^{7}_{73}{ m Ta}^{181}_{82}{ m Pb}^{208}$	Moniz Fermi gas 1p1h	169 221 235 251 260 254 260 265 265	C12 25 MeV removal energy	17 25 32 28 36 39 42 42 42 44

^aThe fitting uncertainty in these numbers is approximately $\pm 5 \text{ MeV}/c$.

Table 1. Proton and neutron Fermi momenta and binding energies (in MeV) for selected nuclei

Nucle	us	$p_{ m F}^p$	$\epsilon^p_{ m b}$	$p_{ m F}^n$	$\epsilon^n_{ m b}$
${}^{12}_{6}C$		221	25.6	221	25.6
$^{14}_{7}N$		223	26.2	223	26.1
$^{16}_{8}O$	Kuzmin	225	26.6	225	26.6
${}^{19}_{9}F$	Fermi	233	28.4	233	28.3
$^{20}_{10}$ Ne	d ac	230	27.8	230	27.8
²⁷ ₁₃ Al	yas	239	29.5	239	29.4
$^{40}_{18}{ m Ar}$		242	30.7	259	35.0
$^{56}_{26}$ Fe		251	33.0	263	36.1
$^{80}_{35}\mathrm{Br}$	lplh	245	31.5	270	38.1

. Bodek

Resonance Proton fit

M.E.C. and P.E. Bosted, PRC 81,055213



L/T Separations on d, C, Al, Cu, Fe

