

THE RELATIVISTIC GREEN'S FUNCTION MODEL FOR CCQE and NCE SCATTERING

Carlotta Giusti and Andrea Meucci
Università and INFN, Pavia

NuInt14 19-24 May 2014, Selsdon Park Hotel Surrey, UK

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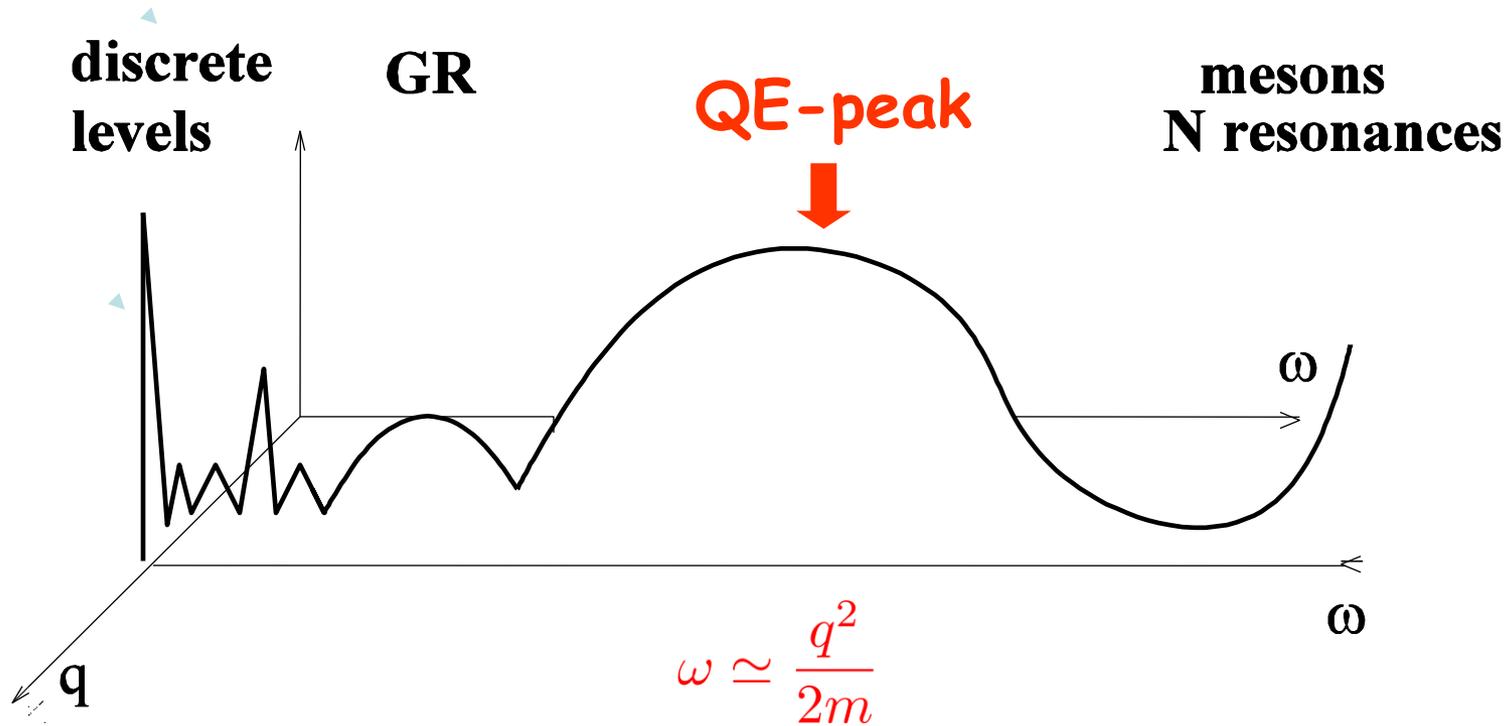
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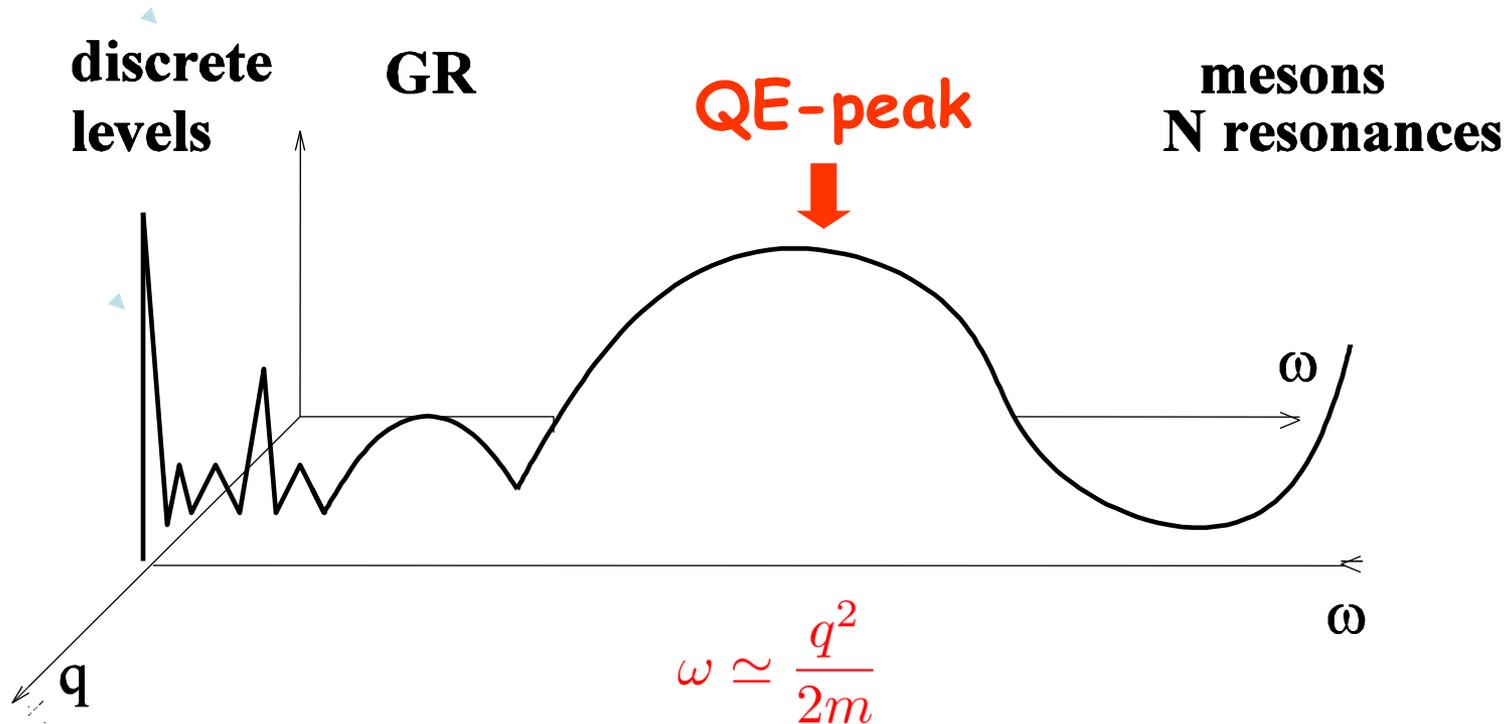
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nuclear response to the electroweak probe



nuclear response to the electroweak probe



QE-peak dominated by one-nucleon knockout

QE e-nucleus scattering

$$e + A \Rightarrow e' + N + (A - 1)$$

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- both e' and N detected $(A-1)$ discrete eigenstate n exclusive $(e,e'p)$

QE e-nucleus scattering

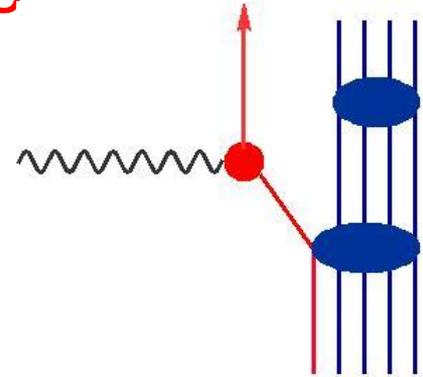
$$e + A \Rightarrow e' + N + (A - 1)$$

- both e' and N detected ($A-1$) discrete eigenstate n **exclusive** ($e,e'p$)
- only e' detected, all final nuclear states included **inclusive** (e,e')

IMPULSE APPROXIMATION

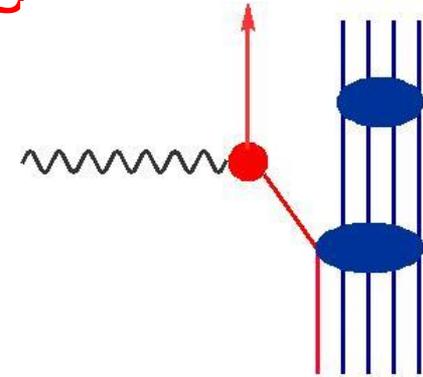
IMPULSE APPROXIMATION

- ✱ EXCLUSIVE SCATTERING: interaction through a 1-body current on a quasi-free nucleon, direct 1NKO

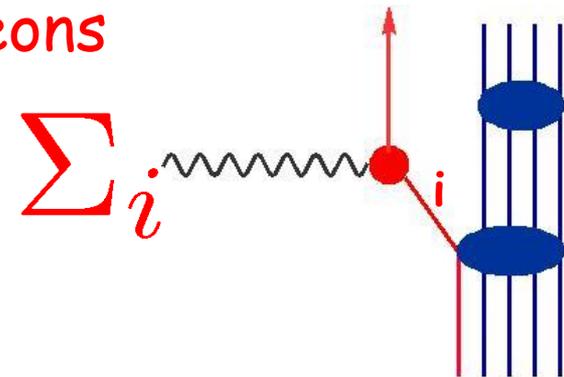


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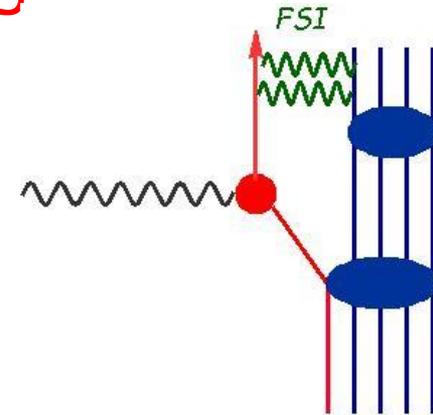


- ✱ **INCLUSIVE SCATTERING:** c.s given by the sum of integrated direct 1NKO over all the nucleons

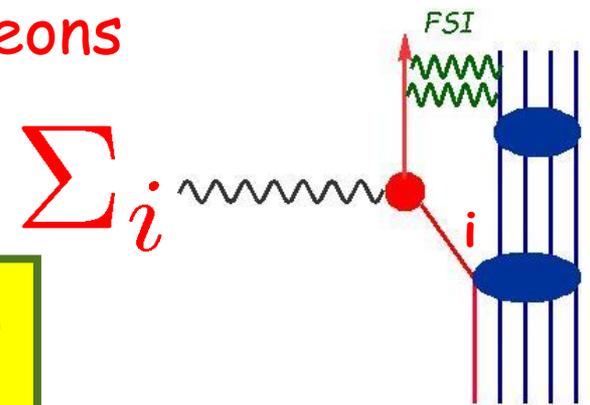


IMPULSE APPROXIMATION

- ✱ **EXCLUSIVE SCATTERING:** interaction through a 1-body current on a quasi-free nucleon, direct 1NKO



- ✱ **INCLUSIVE SCATTERING:** c.s given by the sum of integrated direct 1NKO over all the nucleons



FINAL-STATE INTERACTION between the emitted nucleon and the residual nucleus

EXCLUSIVE SCATTERING: FSI

RDWIA

FSI described by a complex OP with an imaginary absorptive part. The imaginary part gives a reduction of the calculated c.s. which is essential to reproduce data

INCLUSIVE SCATTERING: FSI

RDWIA

sum of $1NKO$ where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

INCLUSIVE SCATTERING: FSI

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sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

RPWIA

FSI neglected

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REAL POTENTIAL

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rROP

only the real part of the OP: conserves the flux but it is conceptually wrong

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RMF

RELATIVISTIC MEAN FIELD: same real energy-independent potential of bound states

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RGF

GREEN'S FUNCTION complex OP conserves the flux
consistent description of FSI in exclusive and inclusive QE
electron scattering

FSI for the inclusive scattering : Green's Function Model

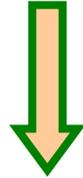
- with suitable approximations (basically related to the IA) the components of the inclusive response can be written in terms of the s.p. optical model Green's function
- the explicit calculation of the s.p. Green's function can be avoided by its spectral representation which is based on a biorthogonal expansion in terms of the eigenfunctions of the non Herm optical potential V and V^+
- matrix elements similar to RDWIA
- scattering states eigenfunctions of V and V^+ (absorption and gain of flux): the imaginary part redistributes the flux and the total flux is conserved

FSI for the inclusive scattering : Green's Function Model

$$W^{\mu\mu}(\omega, q) = \sum_n \left[\mathbf{Re}T_n^{\mu\mu}(E_{\mathbf{f}} - \varepsilon_n, E_{\mathbf{f}} - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^\infty d\mathcal{E} \frac{1}{E_{\mathbf{f}} - \varepsilon_n - \mathcal{E}} \mathbf{Im}T_n^{\mu\mu}(\mathcal{E}, E_{\mathbf{f}} - \varepsilon_n) \right]$$

FSI for the inclusive scattering : Green's Function Model

$$W^{\mu\mu}(\omega, q) = \sum_n \left[\text{Re} \Gamma_n^{\mu\mu}(E_{\mathbf{f}} - \varepsilon_n, E_{\mathbf{f}} - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^\infty d\varepsilon \frac{1}{E_{\mathbf{f}} - \varepsilon_n - \varepsilon} \text{Im} \Gamma_n^{\mu\mu}(\varepsilon, E_{\mathbf{f}} - \varepsilon_n) \right]$$



$$T_n^{\mu\mu}(\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(\mathbf{q}) \sqrt{1 - \mathcal{V}'(E)} | \tilde{\chi}_\varepsilon^{(-)}(E) \rangle \langle \chi_\varepsilon^{(-)}(E) | \sqrt{1 - \mathcal{V}'(E)} j^\mu(\mathbf{q}) | \varphi_n \rangle$$

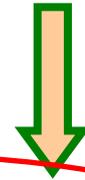
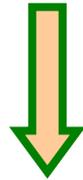
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$$\langle \chi^{(-)}(E) | j^\mu(\mathbf{q}) | \varphi_n \rangle$$

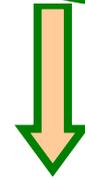
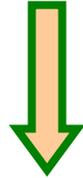
DWIA exclusive ($e, e'p$)

$$\langle \chi^{(-)}(E) | j^\mu(q) | \varphi_n \rangle$$

- j^μ one-body nuclear current
- φ_n s.p. bound state overlap function
- $\chi^{(-)}$ s.p. scattering w.f. eigenfunction of an OP

FSI for the inclusive scattering : Green's Function Model

$$W^{\mu\mu}(\omega, q) = \sum_n \left[\text{Re} \Gamma_n^{\mu\mu}(E_{\mathbf{f}} - \varepsilon_n, E_{\mathbf{f}} - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^\infty d\varepsilon \frac{1}{E_{\mathbf{f}} - \varepsilon_n - \varepsilon} \text{Im} \Gamma_n^{\mu\mu}(\varepsilon, E_{\mathbf{f}} - \varepsilon_n) \right]$$



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eigenfunctions of V
and V^+

FSI for the inclusive scattering : Green's Function Model

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loss of flux

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gain of flux

loss of flux

FSI for the inclusive scattering : Green's Function Model

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gain of flux

loss of flux

Flux redistributed and conserved

The imaginary part of the optical potential is responsible for the redistribution of the flux among the different channels

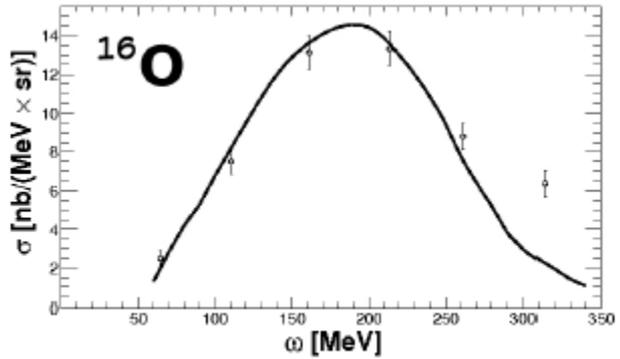
Relativistic Green's Function Model

- consistent treatment of FSI in the exclusive and in the inclusive scattering
- the imaginary part of the ROP includes inelastic channels
- with a complex ROP the model can include contributions not included in other models based on the IA
- the use of a phen. ROP does not allow us to disentangle specific contributions
- different phen ROP's available , theoretical uncertainties in the numerical predictions of the model

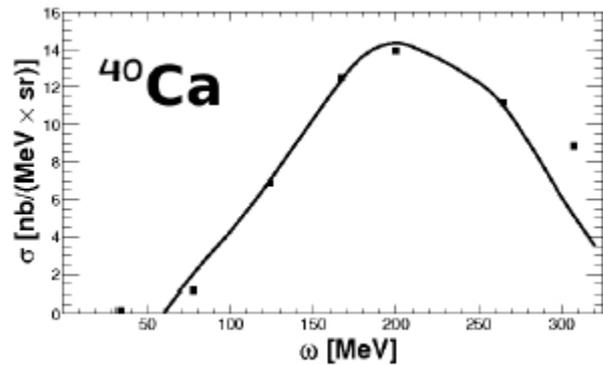
RGF: comparison with QE (e, e') data

(e, e')

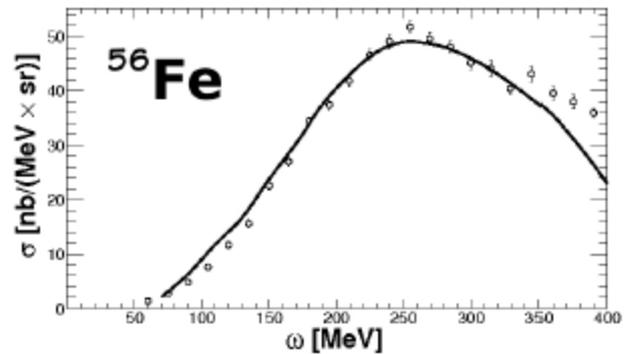
RGF



$$E_0 = 1080 \text{ MeV} \quad \vartheta = 32^\circ$$



$$E_0 = 841 \text{ MeV} \quad \vartheta = 45.5^\circ$$



$$E_0 = 2020 \text{ MeV} \quad \vartheta = 20^\circ$$

RGF: comparison CCQE data

$$\nu_l(\bar{\nu}_l) + A \longrightarrow l^- (l^+) + N + (A - 1)$$

RGF: comparison CCQE data

$$\nu_l(\bar{\nu}_l) + A \longrightarrow l^-(l^+) + N + (A - 1)$$

- only final lepton detected *inclusive CC*

Differences between Electron and Neutrino Scattering

- **electron scattering :**

beam energy known, cross section as a function of ω

- **neutrino scattering:**

beam energy and ω not known

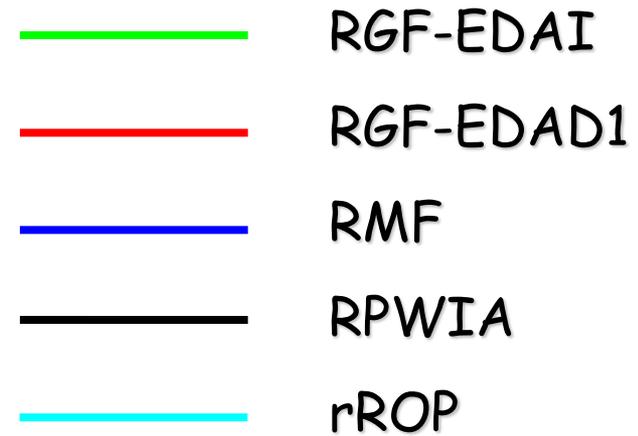
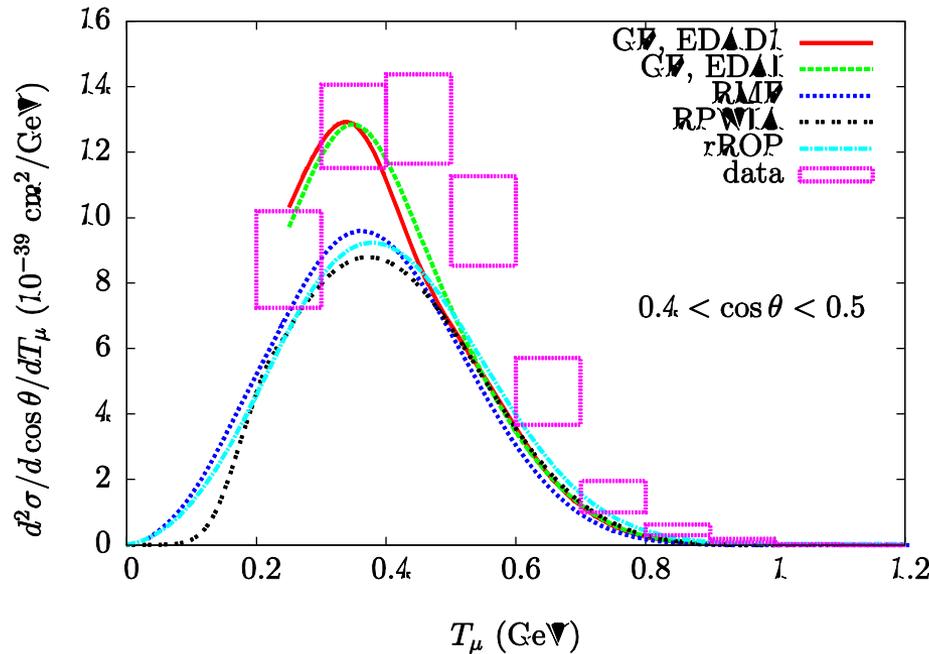
calculations over the energy range relevant for the neutrino flux

the flux-average procedure can include contributions from different kinematic regions where the neutrino flux has significant strength, contributions other than direct 1-nucleon emission

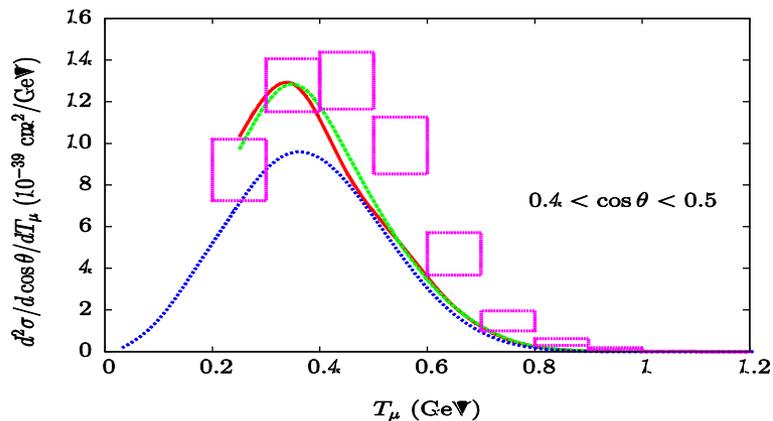
Comparison with MiniBooNe CCQE data

$$^{12}C(\nu_{\mu}, \mu^{-})$$

$$0.4 < \cos\theta_{\mu} < 0.5$$

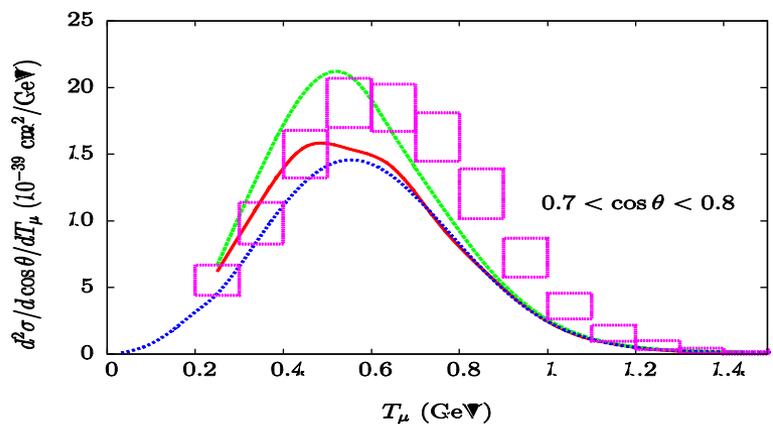


Comparison with MiniBooNe CCQE data



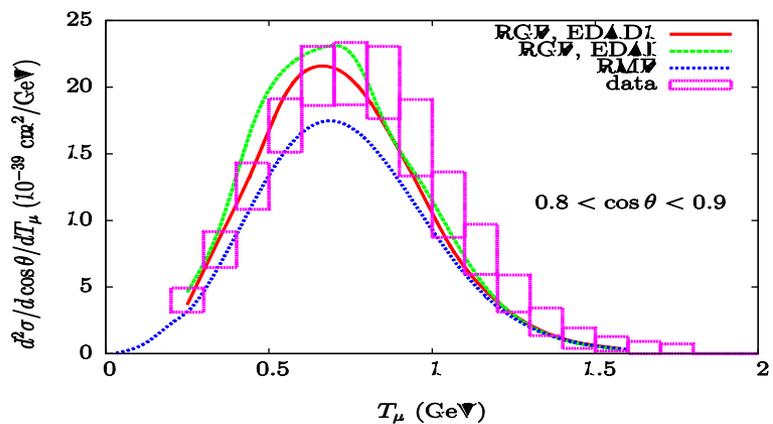
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$^{12}\text{C}(\nu_\mu, \mu^-)$



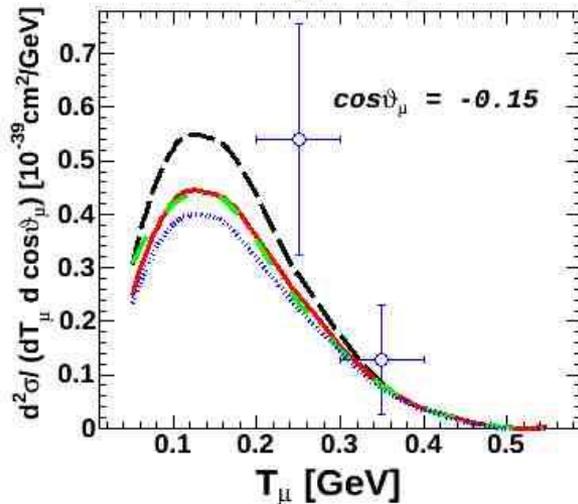
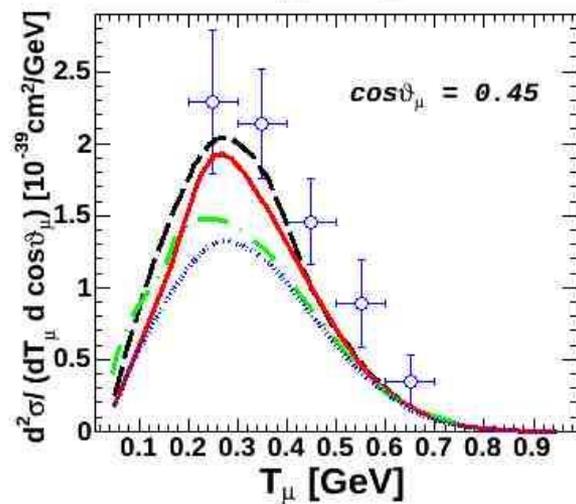
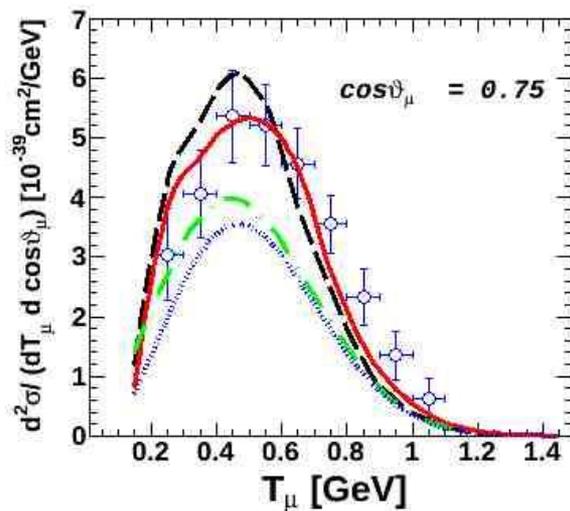
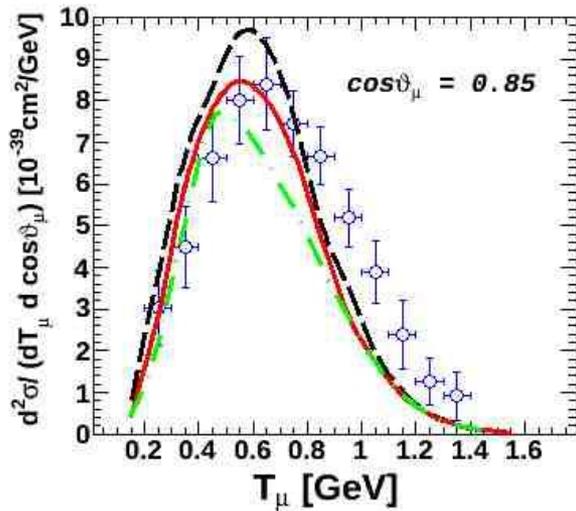
$0.7 < \cos \theta_\mu < 0.8$

- RGF-EDAI
- RGF-EDAD1
- RMF



$0.8 < \cos \theta_\mu < 0.9$

Comparison with MiniBooNe CCQE data

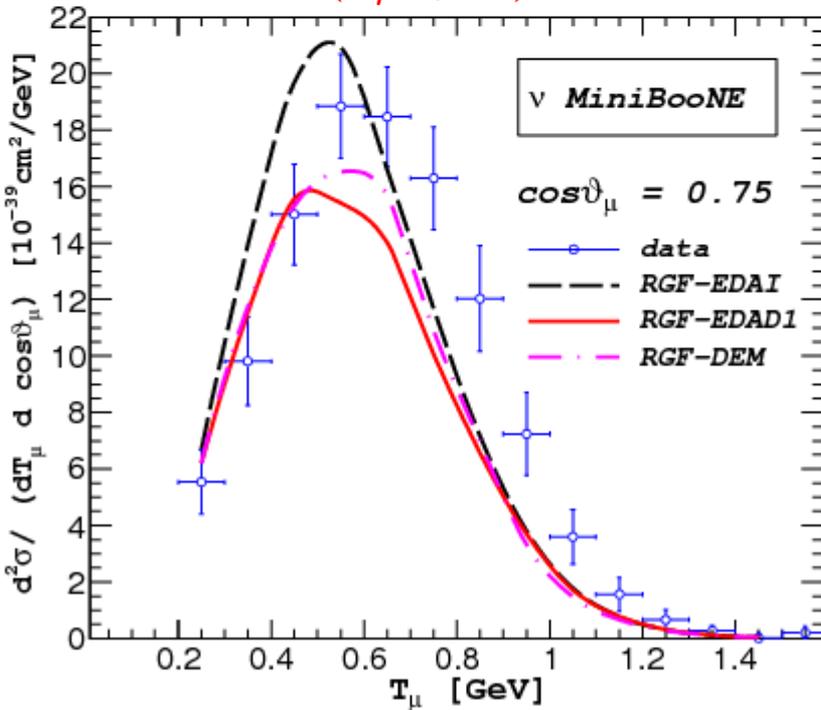


$$^{12}\text{C}(\bar{\nu}_\mu, \mu^+)$$

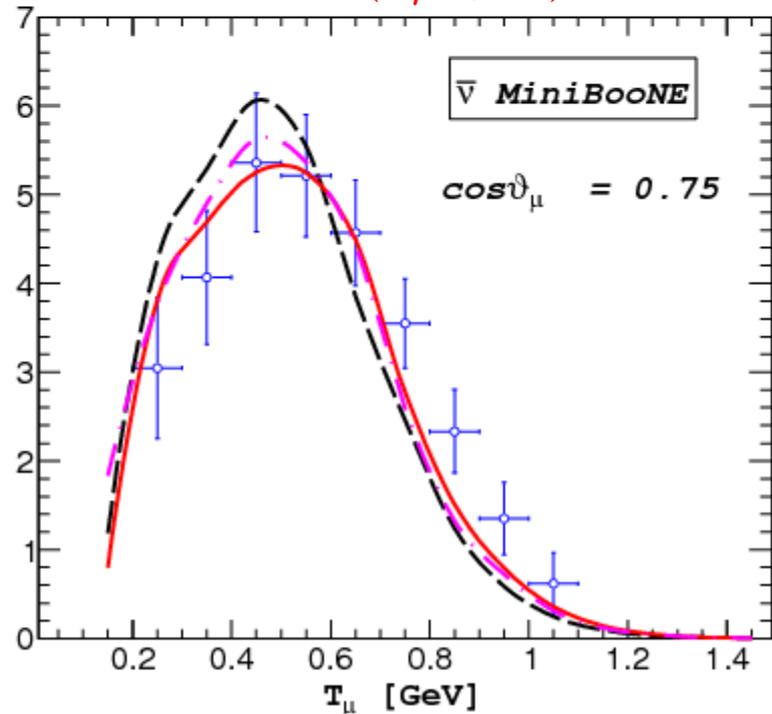
- - - RPWIA
- - - rROP
- - - RGF EDAI
- - - RGF-EDAD1

Comparison MiniBooNE CCQE neutrino-antineutrino scattering

$^{12}\text{C}(\nu_\mu, \mu^-)$



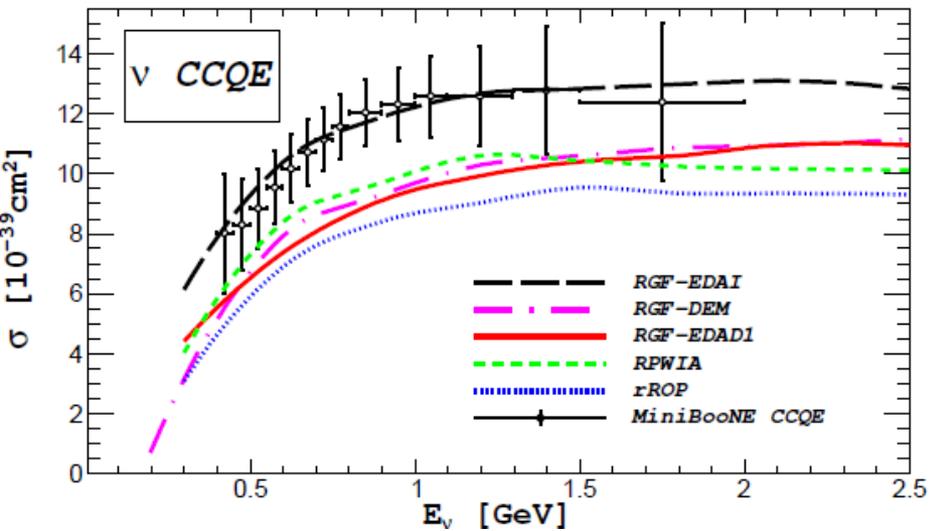
$^{12}\text{C}(\bar{\nu}_\mu, \mu^+)$



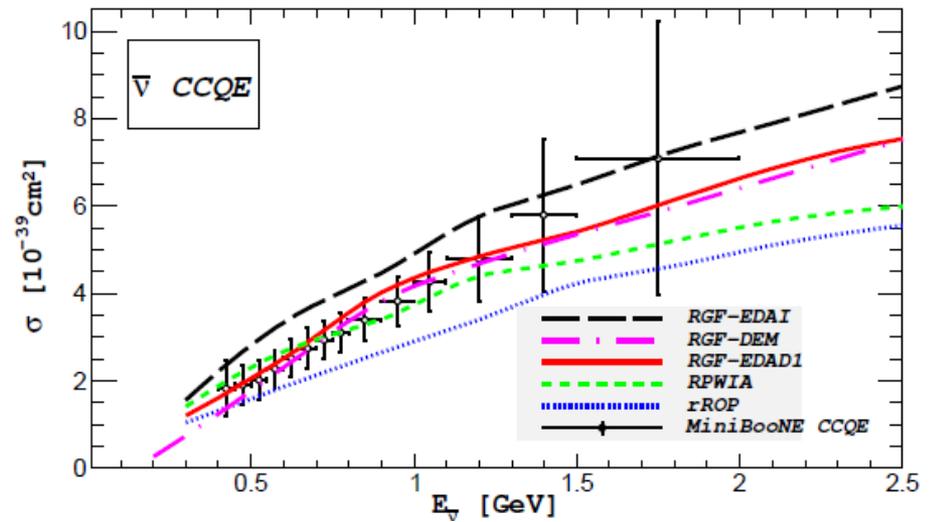
- RGF EDAI
- RGF-EDAD1
- RGF-DEM ←

Comparison MiniBooNE CCQE neutrino-antineutrino scattering

$^{12}\text{C}(\nu_{\mu}, \mu^{-})$



$^{12}\text{C}(\bar{\nu}_{\mu}, \mu^{+})$

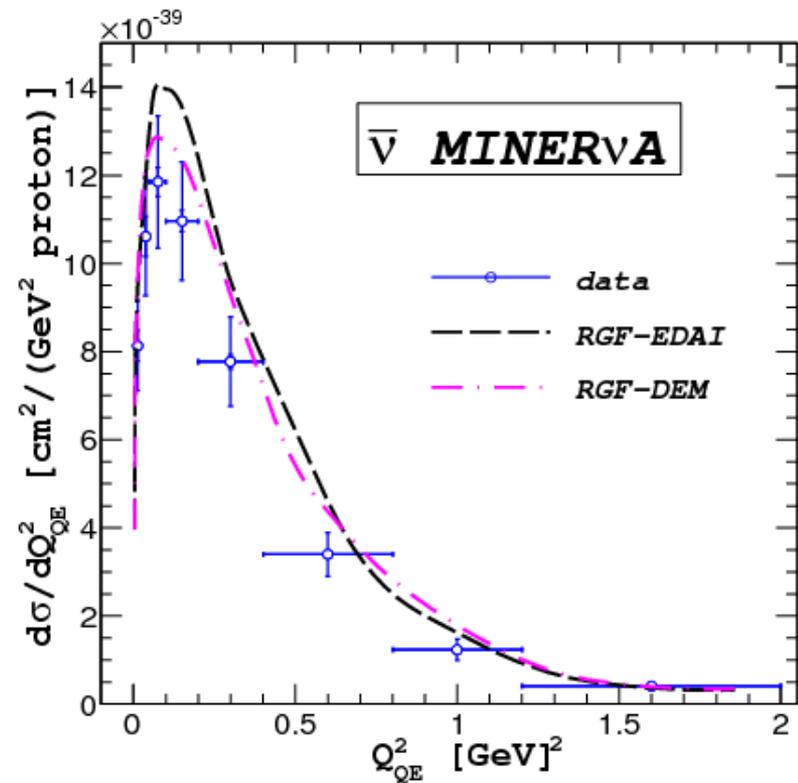
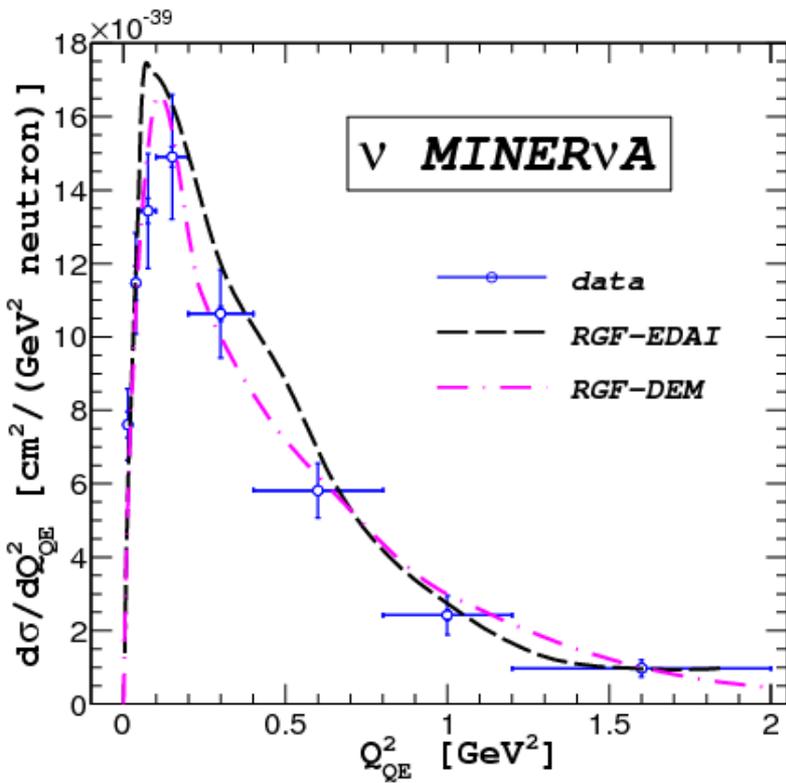


- RGF EDAI
- RGF-EDAD1
- · - RGF-DEM
- rROP
- RPWIA

Comparison MINERvA CCQE neutrino-antineutrino scattering

- higher energy (energy range 1.5-10 GeV)
- models based on the IA underpredict the MiniBooNE data but in general provide a good description of the MINERvA data
- RGF...?

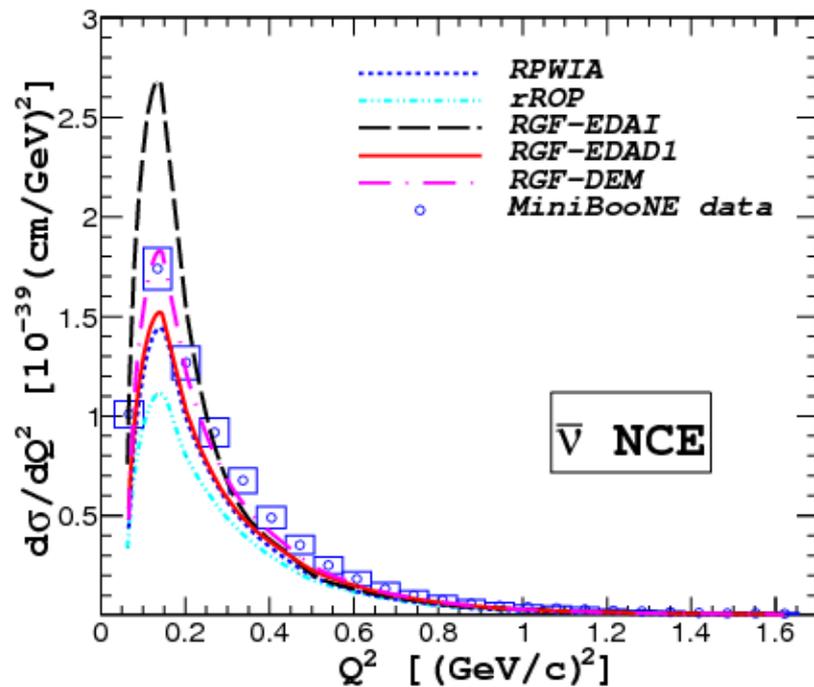
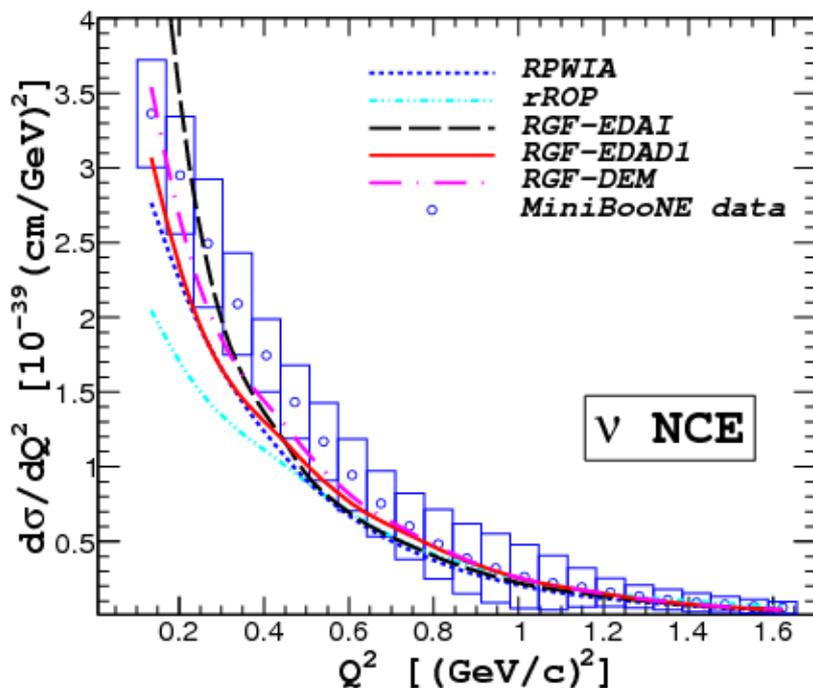
Comparison MINERvA CCQE neutrino-antineutrino scattering



NC ν -nucleus scattering

- only the outgoing nucleon is detected: semi-inclusive scattering
- FSI ?
- RDWIA : sum of all integrated exclusive 1NKO channels with absorptive imaginary part of the ROP. The imaginary part accounts for the flux lost in each channel towards other inelastic channels. Some of these reaction channels are not included in the experimental cross section when one nucleon is detected. For these channels RDWIA is correct, but there are channels excluded by the RDWIA and included in the experimental c.s.
- RGF recovers the flux lost to these channels but can include also contributions of channels not included in the semi-inclusive cross section
- we can expect RDWIA smaller and RGF larger than the experimental cross sections
- relevance of contributions neglected in RDWIA and added in RGF depends on kinematics

Comparison with MiniBooNE NCE data



conclusions

- RGF
- describes FSI in the inclusive lepton-nucleus scattering
- developed for inclusive QE electron-nucleus scattering, successfully tested in comparison with (e,e') data and then applied to QE neutrino-nucleus scattering
- describes CCQE and NCE MiniBooNE data, MINERvA CCQE data
- the imaginary part of the ROP includes the overall effect of inelastic channels (rescattering, non-nucleonic, multi-nucleon,....)
- the role of different inelastic processes cannot be disentangled
- the use of different phenomenological ROP's may introduce theoretical uncertainties on the RGF results
- better determination of the ROP desirable
- 2p-2h MEC....?