THE RELATIVISTIC GREEN'S FUNCTION MODEL FOR CCQE and NCE SCATTERING

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nuclear response to the electroweak probe

\[ \omega \approx \frac{q^2}{2m} \]
nuclear response to the electroweak probe

\[ \omega \approx \frac{q^2}{2m} \]

QE-peak dominated by one-nucleon knockout
$e + A \iff e' + N + (A - 1)$
QE e-nucleus scattering

\[ e + A \rightarrow e' + N + (A - 1) \]

- both \( e' \) and \( N \) detected \((A-1)\) discrete eigenstate n exclusive \((e,e'p)\)
QE e-nucleus scattering

\[ e + A \rightarrow e' + N + (A - 1) \]

- both \(e'\) and \(N\) detected (A-1) discrete eigenstate n exclusive \((e,e'p)\)
- only \(e'\) detected, all final nuclear states included inclusive \((e,e')\)
IMPULSE APPROXIMATION
EXCLUSIVE SCATTERING: interaction through a 1-body current on a quasi-free nucleon, direct 1NKO
**IMPULSE APPROXIMATION**

**EXCLUSIVE SCATTERING:** interaction through a 1-body current on a quasi-free nucleon, direct 1NKO

**INCLUSIVE SCATTERING:** c.s given by the sum of integrated direct 1NKO over all the nucleons

\[ \sum_{i} \]
EXCLUSIVE SCATTERING: interaction through a 1-body current on a quasi-free nucleon, direct 1NKO

INCLUSIVE SCATTERING: c.s given by the sum of integrated direct 1NKO over all the nucleons

FINAL-STATE INTERACTION between the emitted nucleon and the residual nucleus
FSI described by a complex OP with an imaginary absorptive part. The imaginary part gives a reduction of the calculated c.s. which is essential to reproduce data.
INCLUSIVE SCATTERING: FSI

RDWIA

sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux
INCLUSIVE SCATTERING: FSI

RDWIA: sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

RPWIA: FSI neglected
INCLUSIVE SCATTERING: FSI

RDWIA
sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

RPWIA
FSI neglected

REAL POTENTIAL
INCLUSIVE SCATTERING: FSI

RDWIA: sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux.

RPWIA: FSI neglected

REAL POTENTIAL:

rROP: only the real part of the OP: conserves the flux but it is conceptually wrong.
INCLUSIVE SCATTERING: FSI

RDWIA: sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

RPWIA: FSI neglected

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RMF: RELATIVISTIC MEAN FIELD: same real energy-independent potential of bound states
INCLUSIVE SCATTERING: FSI

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RELATIVISTIC MEAN FIELD: same real energy-independent potential of bound states

RGF
GREEN’S FUNCTION complex OP conserves the flux consistent description of FSI in exclusive and inclusive QE electron scattering
with suitable approximations (basically related to the IA) the components of the inclusive response can be written in terms of the s.p. optical model Green's function.

The explicit calculation of the s.p. Green's function can be avoided by its spectral representation which is based on a biorthogonal expansion in terms of the eigenfunctions of the non-Herm optical potential \( V \) and \( V^+ \).

Matrix elements similar to RDWIA

Scattering states eigenfunctions of \( V \) and \( V^+ \) (absorption and gain of flux): the imaginary part redistributes the flux and the total flux is conserved.
FSI for the inclusive scattering:
Green's Function Model

\[ W^{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} T_n^{\mu\mu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M \text{d}\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T_n^{\mu\mu}(\varepsilon, E_f - \varepsilon_n) \right] \]
FSI for the inclusive scattering: Green's Function Model

\[ W^{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} T_n^{\mu\mu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^\infty d\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T_n^{\mu\mu}(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T_n^{\mu\mu}(\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - \mathcal{V}(E)} | \chi^{(-)}_\varepsilon(E) \rangle \langle \chi^{(-)}_\varepsilon(E) | \sqrt{1 - \mathcal{V}(E)} j^{\mu}(q) | \varphi_n \rangle \]
FSI for the inclusive scattering: Green's Function Model

\[ W^{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} \left( T^{\mu\mu}_n(E_f - \varepsilon_n, E_f - \varepsilon_n) \right) - \frac{1}{\pi} \mathcal{P} \int_M^\infty d\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T^{\mu\mu}_n(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T^{\mu\mu}_n(\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - \mathcal{V}(E)} \chi_\varepsilon^{-}(E) \rangle \langle \chi_\varepsilon^{-}(E) | \sqrt{1 - \mathcal{V}(E)} j^\mu(q) | \varphi_n \rangle \]
FSI for the inclusive scattering: Green's Function Model

\[ W_{\mu\nu}(\omega, q) = \sum_n \left[ \text{Re} T_{n\mu\nu}^{\mu\nu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^\infty d\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T_{n\mu\nu}^{\mu\nu}(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T_{n\mu\nu}^{\mu\nu}(\varepsilon, E) = \lambda_n \langle \varphi_n \mid j^{\mu\dagger}(q) \sqrt{1 - V'(E)} \mid \bar{\chi}_{\varepsilon}^{(-)}(E)\rangle \langle \chi_{\varepsilon}^{(-)}(E) \mid \sqrt{1 - V'(E)} j^{\mu}(q) \mid \varphi_n \rangle \]

\[ \langle \chi^{(-)}(E) \mid j^{\mu}(q) \mid \varphi_n \rangle \]
DWIA exclusive (e,e'p)

\[ \langle \chi^{(-)}(E) \mid j^\mu(q) \mid \varphi_n \rangle \]

- \( j^\mu \) one-body nuclear current
- \( \varphi_n \) s.p. bound state overlap function
- \( \chi^{(-)} \) s.p. scattering w.f. eigenfunction of an OP
FSI for the inclusive scattering: Green's Function Model

\[ W^{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} T_{\mu n}^{\mu\mu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_{M}^{\infty} \frac{dE}{E_f - \varepsilon_n - E} \text{Im} T_{\mu n}^{\mu\mu}(E, E_f - \varepsilon_n) \right] \]

\[ T_{\mu n}^{\mu\mu}(E, E) = \lambda_n \langle \varphi_n | j^{\mu\uparrow}(q) \sqrt{1 - \mathcal{V}_n(E)} | \chi^{(-)}_E(E) \rangle \langle \chi^{(-)}_E(E) | \sqrt{1 - \mathcal{V}_n(E)} j^{\mu}(q) | \varphi_n \rangle \]
FSI for the inclusive scattering: Green's Function Model

\[ W^{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} T_n^{\mu\mu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^\infty d\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T_n^{\mu\mu}(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T_n^{\mu\mu}(\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - V'(E)} | \chi^{(-)}_\varepsilon(E) \rangle \langle \chi^{(-)}_\varepsilon(E) | \sqrt{1 - V'(E)} j^{\mu}(q) | \varphi_n \rangle \]

**eigenfunctions of V and V^+**
FSI for the inclusive scattering: Green's Function Model

\[ W^{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} T_n^{\mu\mu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^\infty d\varepsilon \frac{1}{E_f - \varepsilon - \varepsilon_n} \text{Im} T_n^{\mu\mu}(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T_n^{\mu\mu}(\varepsilon, E) = \lambda_n \langle \varphi_n \mid j^{\mu\dagger}(q) \sqrt{1 - \mathcal{V}(E)} \mid \chi_\varepsilon^{(-)}(E) \rangle \langle \chi_\varepsilon^{(-)}(E) \mid \sqrt{1 - \mathcal{V}(E)} j^{\mu}(q) \mid \varphi_n \rangle \]

loss of flux
FSI for the inclusive scattering: 
Green's Function Model

\[ W^\mu_\mu(\omega, q) = \sum_n \left[ \text{Re} T_n^{\mu\mu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^\infty \text{d}\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T_n^{\mu\mu}(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T_n^{\mu\mu}(\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - \mathcal{V}'(E)} | \chi^{(-)}_\varepsilon(E) \rangle \langle \chi^{(-)}_\varepsilon(E) | \sqrt{1 - \mathcal{V}'(E)} j^\mu(q) | \varphi_n \rangle \]

gain of flux

loss of flux
FSI for the inclusive scattering: Green’s Function Model

\[ W^{\mu\nu}(\omega, q) = \sum_n \left[ \text{Re} T^{\mu\nu}_n (E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_{M}^{\infty} d\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T^{\mu\nu}_n (\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T^{\mu\nu}_n (\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - \mathcal{V}'(E)} | \chi^{(-)}_{\varepsilon}(E) \rangle \langle \chi^{(-)}_{\varepsilon}(E) | \sqrt{1 - \mathcal{V}'(E)} j^{\mu}(q) | \varphi_n \rangle \]

Flux redistributed and conserved

The imaginary part of the optical potential is responsible for the redistribution of the flux among the different channels.
consistent treatment of FSI in the exclusive and in the inclusive scattering
the imaginary part of the ROP includes inelastic channels
with a complex ROP the model can include contributions not included in other models based on the IA
the use of a phen. ROP does not allow us to disentangle specific contributions
different phen ROP’s available, theoretical uncertainties in the numerical predictions of the model
RGF: comparison with QE (e, e') data
\((e,e')\)

\[ E_0 = 1080 \text{ MeV} \quad \vartheta = 32^\circ \]

\[ E_0 = 841 \text{ MeV} \quad \vartheta = 45.5^\circ \]

\[ E_0 = 2020 \text{ MeV} \quad \vartheta = 20^\circ \]
RGF: comparison CCQE data

\[ \nu_l (\bar{\nu}_l) + A \rightarrow l^- (l^+) + N + (A - 1) \]
RGF: comparison CCQE data

\[ \nu_i (\bar{\nu}_i) + A \rightarrow l^- (l^+) + N + (A - 1) \]

- only final lepton detected inclusive CC
Differences between Electron and Neutrino Scattering

- **Electron scattering:**
  - Beam energy known, cross section as a function of $\omega$

- **Neutrino scattering:**
  - Beam energy and $\omega$ not known
  - Calculations over the energy range relevant for the neutrino flux

The flux-average procedure can include contributions from different kinematic regions where the neutrino flux has significant strength. Contributions other than direct 1-nucleon emission are included.
Comparison with MiniBooNe CCQE data

\[ 12 \, C(\nu_\mu, \mu^-) \quad 0.4 < \cos \theta_\mu < 0.5 \]

\[ \frac{d^2 \sigma}{d \cos \theta dT_\mu} (10^{-39} \, \text{cm}^2/\text{GeV}) \]

A. Meucci et al. PRL 107 (2011) 172501
Comparison with MiniBooNe CCQE data

$0.4 < \cos \theta_\mu < 0.5$

$^{12}C(\nu_\mu, \mu^-)$

$0.7 < \cos \theta_\mu < 0.8$

RGF-EDAI

RGF-EDAD1

RMF

$0.8 < \cos \theta_\mu < 0.9$

A. Meucci et al. PRL 107 (2011) 172501
Comparison with MiniBooNe CCQE data

$^{12}C(\bar{\nu}_\mu, \mu^+)$

A. Meucci and C. Giusti PRD 85 (2012) 093002
Comparison MiniBooNE CCQE neutrino-antineutrino scattering

$^{12}C(\nu_\mu, \mu^-)$

$^{12}C(\bar{\nu}_\mu, \mu^+)$

$v$ MiniBooNE

$\cos^2\theta_\mu = 0.75$

- data
- $\text{RGF-EDA1}$
- $\text{RGF-EDAD1}$
- $\text{RGF-DEM}$

$\frac{d^2\sigma}{dE_d d\cos^2\theta_\mu}$

$T_\mu$ [GeV]
Comparison MiniBooNE CCQE neutrino-antineutrino scattering

$^{12}C(\nu_\mu, \mu^-)$

$^{12}C(\bar{\nu}_\mu, \mu^+)$
Comparison MINERvA CCQE neutrino-antineutrino scattering

- higher energy (energy range 1.5-10 GeV)
- models based on the IA underpredict the MiniBooNE data but in general provide a good description of the MINERvA data
- RGF...?
Comparison MINERvA CCQE neutrino-antineutrino scattering
only the outgoing nucleon is detected: semi-inclusive scattering

FSI?

RDWIA: sum of all integrated exclusive 1NKO channels with absorptive imaginary part of the ROP. The imaginary part accounts for the flux lost in each channel towards other inelastic channels. Some of these reaction channels are not included in the experimental cross section when one nucleon is detected. For these channels RDWIA is correct, but there are channels excluded by the RDWIA and included in the experimental c.s.

RGF recovers the flux lost to these channels but can include also contributions of channels not included in the semi-inclusive cross section

we can expect RDWIA smaller and RGF larger than the experimental cross sections

relevance of contributions neglected in RDWIA and added in RGF depends on kinematics
Comparison with MiniBooNE NCE data

A. Meucci and C. Giusti PRD 89 (2014) 057302
RGF describes FSI in the inclusive lepton-nucleus scattering developed for inclusive QE electron-nucleus scattering, successfully tested in comparison with (e,e') data and then applied to QE neutrino-nucleus scattering.

RGF describes CCQE and NCE MiniBooNE data, MINERvA CCQE data.

The imaginary part of the ROP includes the overall effect of inelastic channels (rescattering, non-nucleonic, multi-nucleon,...).

The role of different inelastic processes cannot be disentangled.

The use of different phenomenological ROP’s may introduce theoretical uncertainties on the RGF results.

Better determination of the ROP desirable.

2p-2h MEC....?