SPECTRAL FUNCTION IN NEUTRINO GENERATORS: STRUCTURE FUNCTION MEASUREMENT of $^{40}\text{Ar}$ via $(E,E'p)$ and ITS IMPACT ON NEUTRINO OSCILLATING PARAMETERS

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NuINT 14’
May 20th, 2014
Introduction to Structure (Spectral) Functions

The Role of Spectral Function in Lepton-Nucleus Cross Section Cal.

Electron Cross-section Validation ($^{12}$C and $^{40}$Ca)

Neutrino Cross-section Comparison (GENIE, GiBUU, NuWro)

Impact on Oscillation Parameters

Conclusions
THROUGH ELECTRON SCATTERING, STRUCTURE FUNCTION IS EXTRACTED

- initial-state nucleon’s momentum distribution, $k_n$, is assumed to be flat (no structure) for all shells below the Fermi sea ($k_F$)

- the spectral function theory considers the difference in the structure function for each shell and thus is capable of yielding a much better prediction while being compared to the measured electron cross-section than RFGM

$$n^D = \sigma_{(e,e'p)}/K\sigma_{cc1}$$

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SEPARATION ENERGY OF EACH NUCLEON WITH ONE SPECIFIC QUANTUM NUMBER IS INDIVIDUAL

- absence of position correlation in the ground state wave function between each pair of nucleons
- the energy distribution of each shell is a delta function (discontinuous)
- all nucleons feel the same force acting on them as a result of a mean field potential

- $N$-$N$ (position) correlations are formed by the charge density distribution
- $N$-$N$ (position) correlations cause the overlap of energy distributions (continuous)
- $N$-$N$ (position) correlations lead to the strong short-range repulsion (back-to-back)
- $N$-$N$ (position) correlations affect the momentum distribution of each shell

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- **RFG model**

$$P(|\vec{p}|, E) = \frac{3}{4\pi k_F^3} \theta(k_F - |\vec{p}|) \delta(\sqrt{m^2 + |\vec{p}|^2} - m - E_B + E)$$

*step function (no structure) energy-momentum conservation*

- **SF model (1p-1h : the simplest case)**

$$P(|\vec{p}|, E) = \left| <^{40}_{\text{Ar}}|^{39}_{\text{Ar}}, p > \right|^2 \delta(E_{^{40}_{\text{Ar}}} - E_{^{39}_{\text{Ar}}} - m_n + E)$$

*probability amplitude of knocking out one nucleon energy-momentum conservation*

**Math Form of Spectral Functions**

telling you the “probability” of knocking out one nucleon with one specific quantum number

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in RFGM, nucleons sitting near the Fermi sea are most likely to interact with beam particles

the spectral function theory, however, demonstrates the non-zero probability of interacting with nucleons with translational momentum above the Fermi sea as a result of N-N interactions

probability of interacting with one nucleon with a quantum number

\[ \sigma_{eA} = K \sigma_{ep} P(p - q, \omega - T_p) \]

integrating over the entire phase space \(\rightarrow dp^3 = p^2 dp\)
A more realistic initial-state nuclear shell model for $^{40}\text{Ar}$ is needed

why nuclear physicist cares?
- what is the spectroscopic factor for $^{40}\text{Ar}$?
- what is the separation energy of proton and neutron with the specific quantum number on each shell?
- what is the energy-dependence of each shell’s momentum?

why neutrino physicist cares?
- RFGM vs. SF (how different?)
- the energy transfer or $Q^2$ on the interaction vertex of neutrino-nucleus
- the probability of knocking out one nucleon with one specific quantum number determines the size of neutrino-nucleus cross-section
- non negligible effect on the determination of the neutrino energy either for just CCQE or CC-all
- oscillation parameters determination are affected by neutrino energy reconstruction uncertainties
- fundamental constants like $\alpha$ affected by nuclear model - new nuclear model validation

far beyond nuclear physics
- beam energy/flux are un-determined
- $Q^2$ at the interaction vertex is not constrained
- no way to detect final-state neutrinos
- hard to do final-state energy reconstruction

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We discuss the implementation of the nuclear model based on realistic nuclear spectral functions in the GENIE neutrino interaction generator. Besides improving on the Fermi gas description of the nuclear ground state, our scheme involves a new prescription for $Q^2$ selection, meant to efficiently enforce energy momentum conservation. The results of our simulations, validated through comparison to electron scattering data, have been obtained for a variety of target nuclei, ranging from carbon to argon, and cover the kinematical region in which quasi elastic scattering is the dominant reaction mechanism. We also analyze the influence of the adopted nuclear model on the determination of neutrino oscillation parameters.

PACS numbers: 14.60.Pq, 14.60.Lm

structure functions for $^{12}$C, $^{16}$O, $^{40}$Ca and $^{40}$Ar (not well-determined yet) are “temporarily” implemented in GENIE at present. all modifications which we made in our studies and generated results from GENIE are identified as GENIE v2.8.0+ “nuVT”
Final-State-Interactions (FSI)

an optical model used to describe FSI corporates within the spectral function approach

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Electron Validation $^{12}\text{C}$ and $^{40}\text{Ca}$

The FSI effects can be consistently described within the spectral function approach. Please see Artur Ankowski’s presentation.

The accuracy of neutrino energy reconstruction in charged-current quasielastic scattering off nuclear targets

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1Department of Physics, Okayama University, Okayama 700-8530, Japan
2INFN and Department of Physics, “Sapienza” Università di Roma, I-00185 Roma, Italy
(Dated: April 24, 2014)
Owing to the lack of electron scattering data, the energy-momentum dependence in structure functions of argon is not strongly constrained.

JLab Hall-A PAC 41 (Jul. 2014) electron scattering experiment on $^{40}\text{Ar}$

UVa / VT / Syracuse / LANL / INFN-Rome / JLab (Hall-A)
Neutrino Cross-Section

RFGM vs. SF

GENIE / Omar Benhar’s cal.
**Neutrino Cross-Section**

*FIG. 6.* (Color online). Comparison of the $d\sigma/dQ^2$ obtained from $2 \times 10^5$ CCQE events with $E_\nu = 1$ GeV in oxygen obtained using RFGM and SF.

- **RFG** (no PB)
- **RFG** (PB)
- **SF** (PB)

courtesy of O. Benhar, N. Farina, H. Nakamura, M. Sakuda, R. Seki
PRD 72, 053005 (2005)

very good agreement between simulation and theoretical calculation on $Q^2$
Neutrino Cross-Section

$\nu + O \rightarrow \mu + X$, GENIE 2.8.0 + $\nu T$, no Pauli Blocking, no FSI

courtesy of O. Benhar, N. Farina, H. Nakamura, M. Sakuda, R. Seki
PRD 72, 053005 (2005)
very good agreement between simulation and theoretical calculation. however, Ar is now an approximation and more calculations are needed

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Neutrino Cross-Section

GENIE vs. GiBUU vs. NuWro vs. O. Benhar’s cal.
Neutrino Cross-Section

\[ Q^2 \text{ on } {^{12}\text{C}} \text{ at } E_{\nu_{\mu}} = 0.2 \text{ GeV} - \text{SF (PB)} \]

\[ Q^2 \text{ on } {^{12}\text{C}} \text{ at } E_{\nu_{\mu}} = 0.4 \text{ GeV} - \text{SF (PB)} \]

\[ Q^2 \text{ on } {^{12}\text{C}} \text{ at } E_{\nu_{\mu}} = 0.3 \text{ GeV} - \text{SF (PB)} \]

true \[ Q^2(E_{\nu_{\mu}} = 0.225 \text{ GeV}) \]: GENIE-sold blue line, GiBUU-red dot

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Neutrino Cross-Section

\( Q^2 \) on \(^{12}\)C at \( E_{\nu_\mu} = 0.5 \, \text{GeV} \) - SF (PB)

\( Q^2 \) on \(^{12}\)C at \( E_{\nu_\mu} = 0.7 \, \text{GeV} \) - SF (PB)

\( Q^2 \) on \(^{12}\)C at \( E_{\nu_\mu} = 0.6 \, \text{GeV} \) - SF (PB)

\( Q^2 \) on \(^{12}\)C at \( E_{\nu_\mu} = 0.8 \, \text{GeV} \) - SF (PB)

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Neutrino Cross-Section

$Q^2$ on $^{12}$C at $E_{\nu_{\mu}} = 0.9$ GeV - SF (PB)

$Q^2$ on $^{12}$C at $E_{\nu_{\mu}} = 1.0$ GeV - SF (PB)

$E_{\mu}$ on $^{12}$C at $E_{\nu_{\mu}} = 0.8$ GeV - SF (no PB, no FSI)

besides silencing FSI, turning off PB, too. compare simulated results to the theoretical calculation

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$Q^2$ (with PB on) is quite similar in RFGM and SF. You see the similarity of $Q^2$ within GENIE, GiBUU and NuWro. But, this similarity cannot exactly tell you which nuclear model you used to compute $Q^2$.

However, if you look at $E_{\text{mu}}$. The difference in the nuclear model option reveals (because it influences the phase space, $(|q|,w)$, at the interaction vertex). That is, the shape of $E_{\text{mu}}$ reflects which nuclear model you apply to computing your cross-section. We have seen that $E_{\text{mu}}$ from GiBUU is very close to the one derived from RFGM in GENIE and NuWro.

That implies NO SF in GiBUU.
Does the nuclear model influence the extracted neutrino oscillating parameters?
Using GLoBES analysis to “quantify” the answer by feeding one toy model

this toy model is developed by adapting T2K flux and near detector’s efficiency. Note: this is just a toy model used to simplify the simulation loading. this simulation is NOT exclusively for T2K experiment.


P. Coloma and P. Huber
Experimental Setup - Simulation

- Ideal and perfect near detector ($^{12}$C or $^{16}$O), 1 km, 1kton
- Far detector at 295 km, 22.5 kton
  - Oxygen
  - Carbon (RFG and SF)
- Use T2K flux, peak at 0.6 GeV, 750kW, 5 years running
- Use SK reconstruction efficiency as function of energy
- Use migration matrices produced by GENIE 2.8.0+$\nu$T

Migration Matrix is reconstructed energies as a function of true energies

- Muon neutrino disappearance only -> fit to atmospheric parameters
Go beyond simple case

- Use different nuclear models - RFG and SF - and evaluate effects on neutrino oscillation parameters

- Use one neutrino generator (GiBUU) to simulate the nuclear effects and use another neutrino generator (GENIE) to extract the oscillation parameters

- In a real experiment the “real” effects from data will be used in the oscillation analysis together with “some” simulation of nuclear effects

- Neutrino generators are “enough” different to help understanding what will be the effect of different nuclear models on neutrino oscillation analyses

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Neglecting all FSI and multinucleon contributions, we can compute the number of events as:

\[ N_i^{QE} = \sigma_{QE}(E_i)\phi(E_i)P_{\mu\mu}(E_i) \]

However, in practice we will observe a different distribution at the detector, given by:

\[ N_i^{QE-like} = \sum_j M_{ij}^{QEQE} N_j^{QE} + \sum_{j} M_{ij}^{non-QE} N_j^{non-QE} \]

However, an intermediate situation would most likely take place:

\[ N_i^{test}(\alpha) = \alpha N_i^{QE} + (1 - \alpha) N_i^{QE-like} \]

- the event distribution is a convolution of cross-section, flux, detector efficiency, oscillation probability and migration matrix
- different nuclear models consider different nuclear effects in treating nucleon’s kinematics
- as a result, event distributions are different for different nuclear models
Summary of results

Input “true” Values

\[
\begin{align*}
\theta_{12} &= 33.2^\circ \\
\Delta m_{21}^2 &= 7.64 \times 10^{-5} \text{ eV}^2 \\
\theta_{13} &= 9^\circ \\
\Delta m_{31}^2 &= 2.45 \times 10^{-3} \text{ eV}^2 \\
\theta_{23} &= 45^\circ \\
\delta &= 0^\circ
\end{align*}
\]

Fitted Values

<table>
<thead>
<tr>
<th>True</th>
<th>Fitted</th>
<th>(\theta_{23,\text{min}})</th>
<th>(\Delta m_{31,\text{min}}^2) [eV^2]</th>
<th>(\chi^2_{\text{min}})</th>
<th>(\sigma_a)</th>
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</thead>
<tbody>
<tr>
<td>GENIE (^{16}\text{O})</td>
<td>GENIE (^{12}\text{C})</td>
<td>44°</td>
<td>2.49 \times 10^{-3}</td>
<td>2.28</td>
<td>–</td>
</tr>
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<td>GiBUU (^{16}\text{O})</td>
<td>GENIE (^{16}\text{O})</td>
<td>41.75°</td>
<td>2.69 \times 10^{-3}</td>
<td>47.64</td>
<td>–</td>
</tr>
<tr>
<td>GiBUU (^{16}\text{O})</td>
<td>GiBUU (^{16}\text{O}) w/o MEC</td>
<td>47°</td>
<td>2.55 \times 10^{-3}</td>
<td>20.95</td>
<td>5%</td>
</tr>
<tr>
<td>GENIE (^{16}\text{O})</td>
<td>GENIE (^{16}\text{O}) w/o MEC</td>
<td>42.5°</td>
<td>2.44 \times 10^{-3}</td>
<td>22.38</td>
<td>–</td>
</tr>
<tr>
<td>GENIE (^{16}\text{O})</td>
<td>GENIE (^{16}\text{O}) w/o MEC</td>
<td>44.5°</td>
<td>2.36 \times 10^{-3}</td>
<td>19.54</td>
<td>–</td>
</tr>
</tbody>
</table>

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Summary of results cont’d

Input “true” Values

\[
\begin{align*}
\theta_{12} &= 33.2^\circ \\
\theta_{13} &= 9^\circ \\
\theta_{23} &= 45^\circ \\
\Delta m^2_{21} &= 7.64 \times 10^{-5} \text{ eV}^2 \\
\Delta m^2_{31} &= 2.45 \times 10^{-3} \text{ eV}^2 \\
\delta &= 0^\circ
\end{align*}
\]

Fitted Values

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<th>True</th>
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<th>$\theta_{23,\text{min}}$</th>
<th>$\Delta m^2_{31,\text{min}}$ [eV$^2$]</th>
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</thead>
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<tr>
<td>RFGM$_{2.8.0+\nu T}$</td>
<td>RFGM$_{2.8.0}$</td>
<td>45.7 deg</td>
<td>2.45 $\times 10^{-3}$</td>
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<tr>
<td>SF$_{2.8.0+\nu T}$</td>
<td>RFGM$_{2.8.0+\nu T}$</td>
<td>44 deg</td>
<td>2.41 $\times 10^{-3}$</td>
</tr>
<tr>
<td>SF$_{2.8.0+\nu T}$</td>
<td>RFGM$_{2.8.0}$</td>
<td>44.5 deg</td>
<td>2.41 $\times 10^{-3}$</td>
</tr>
</tbody>
</table>

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Conclusions

□ due to the lack of knowledge in Argon structure functions, taking electron data at JLab is needed.

□ the systematics uncertainty of nuclear models for extracted oscillating parameters can be more exactly determined.

Acknowledgement:
A. Ankowski, O. Benhar, P. Coloma, D. Day, S. Dytman, D. Higinbotham, P. Huber, C. Mariani
NUCLEAR MODEL

STANDARD SHELL MODEL

INDEPENDENT PARTICLE SHELL MODEL (IPSM)

0th-order (the most naive model - Fermi Gas Model):
no N-N correlation; all nucleons sitting on bound states are regarded as quasi-free particles

- all nucleons are bounded
- the energy of each shell is a delta function
- all nucleons feel the same force acting on them as a result of a mean field potential
- absence of position correlation in the ground state wave function between each pair of nucleons

the well-depth is determined by being adjusted to re-produce the measured $E_{\text{binding}}$ or $E_{\text{miss}}$

average volume (number) density = 0.16 GeV/fm$^3$

courtesy of D. Day

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momentum density $n(k) \leftrightarrow$ charge ($\sim$ proton) density

$1P_{1/2}$ parallel kinematics

experimental momentum distribution is dramatically different from the one predicted by RFGM.

strong evidence proves the existence of N-N correlations in the ground state wave function.

The difference in charge density between two nuclei shows that one single shell state is absent in one nucleus but is present in the other.

Figure 1-10: $^{16}\text{O}(e,e'p)$ distorted momentum distribution for the $1p_{1/2}$ state measured at Mainz [15]. The kinetic energy is $\sim 93$ MeV. The curve is a DWIA calculation which uses a Woods-Saxon potential with parameters fit from the NIKHEF data [16], and the Schmiedt optical potential [10] for final state interactions.

density difference between $^{206}\text{Pb}$ and $^{205}\text{Tl}$: differ by a single $3s^{1/2}$ proton

courtesy of D. Day
NUCLEAR MODEL
STANDARD ——> MODIFIED SHELL MODEL
THE NATURE OF N-N CORRELATIONS
(QUALITATIVE)

1. N-N (position) correlation is formed by the nucleon charge density distribution and leads to the individual shell structure for each specific nucleus (different A, different Z)
2. N-N correlation causes the overlap between adjacent shells and thus produces the strong short-range repulsive force
3. N-N correlation force is universal for all nuclei (how do we know? - see the next slide)
4. N-N correlation force is quite similar to the type of force acting on quasi-free nucleons
WHAT IS SPECTROSCOPIC FACTOR? (QUANTITATIVE DESCRIPTION OF N-N INTERACTIONS)

The spectroscopic factor is extracted by taking the ratio of the experimental cross-section to the theoretical one:

\[ Z_n = \int \frac{d^3 k}{(2\pi)^3} |\hat{\chi}_n(k)|^2 \]

\[ \hat{\chi}_n(r_1) = \int d^3 r_2 \ldots d^3 r_A \, \Psi_{n-1}^A(r_2 \ldots r_A)^\dagger \Psi_0^A(r_1 \ldots r_A) \]

- **spectroscopic factor** tells you the number of bound-state (valence orbital - near Fermi sea) nucleons interacting with the incoming beam particles.
- **spectroscopic factor** reflects the evidence of presence of short-range N-N correlations; the stronger N-N correlations; the more reduction the spectroscopic factor is (no N-N, \( Z_n = 1 \)).
- **spectroscopic factor** also manifests the existence of long-range N-N correlations, leading to 2p2h.
- **spectroscopic factor** is not the same as the occupation number, for the spectroscopic factor is determined by integrating the probability of knocking out "one" bound-state nucleon associated with one quantum number and meanwhile leaving the residual system with an excitation energy over the \( E_{\text{miss}} \) of bound states. The occupation number instead tells you the total number of nucleons from both bound and continuous states, so it's always larger than the spectroscopic factor.

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shapes of tail structures above Fermi sea are nearly the same for different nuclei \(\leftrightarrow\) N-N short-range correlation effect is universal for all nuclei (spectroscopic factor is similar for light/medium/heavy nuclei)

courtesy of D. Day
Take $^{40}\text{Ca}$ - proton separation energy

![Graph showing proton removal energy distribution in the $^{40}\text{Ca}$ ground state.](image.png)

$f(E) = 4\pi \int dk \, k^2 P(k, E)$

FIG. 2: Proton removal energy distribution in the $^{40}\text{Ca}$ ground state, obtained from Eq. (18) using the model spectral function of Ref. [13]. **courtesy of O. Benhar**
**Take $^{40}\text{Ca}$ Structure Functions** - proton’s momentum distribution (bound states only)

**FIG. 3:** Momentum distributions of the shell model orbits occupied by protons in the $^{40}\text{Ca}$ ground state. The results have been obtained using the Woods-Saxon potential reported in Ref. [16].

courtesy of O. Benhar
Improving the accuracy of neutrino energy reconstruction in charged-current quasielastic scattering off nuclear targets

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(Dated: April 24, 2014)

FIG. 2. (color online). Double differential electron-carbon cross sections, $d\sigma/d\omega d\Omega$. The results obtained with Pauli blocking accounted for in the local-density (solid lines) and step-function (short-dashed lines) approximations are compared to the experimental data of Refs. [13–17]. The IA (long-dashed lines) and RFG calculations (dotted lines) are also shown, for reference. The panels are labeled according to beam energy, scattering angle, and values of $|q|$ and $Q^2$ at the quasielastic peak.