9th International Workshop on Neutrino-Nucleus Interactions in the Few-GeV Region

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Inclusion of MEC in the SuSA-based calculations:

Status and perspectives

Jose Enrique Amaro

Departamento de Física Atómica, Molecular y Nuclear

Universidad de Granada

(Spain)

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MEC in QE neutrino scattering

Collaborators:

- Ignacio Ruiz Simo (Granada)
- Conrado Albertus (Granada)
- Maria Barbaro (Turin)
- Juan Antonio Caballero (Sevilla)
- Guillermo Megias (Sevilla)
- Bill Donnelly (MIT)

MEC in QE neutrino scattering

Results from the papers:

1. Meson-exchange currents and quasielastic Neutrino cross sections in the superscaling approximation model.
   J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, C.F. Williamson.
   Physics Letters B 696 (2011) 151.
2. Meson-Exchange Currents and Quasielastic Antineutrino Cross Sections in the Superscaling Approximation
   J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly.
   Physical Review Letters 108, 152501 (2012)
3. Relativistic effects in two-particle emission for electron and neutrino reactions.
   I. Ruiz Simo, C. Albertus, J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly
   arXiv:1405.4280 [nucl-th]

Outline

1. Formalism of (e, e') and (ν_l, l^-)
   - The relativistic Fermi gas
   - Super-scaling approach (SuSA)
   - 2p-2h Meson exchange currents
2. Status of MEC in SuSA
3. Perspectives: new approach to 2p-2h
   - Phase space function
   - Relativistic problems at high q
   - Angular distribution: new integration method
   - Properties of the 2p-2h phase space
   - The frozen approximation
   - Application to ν reactions

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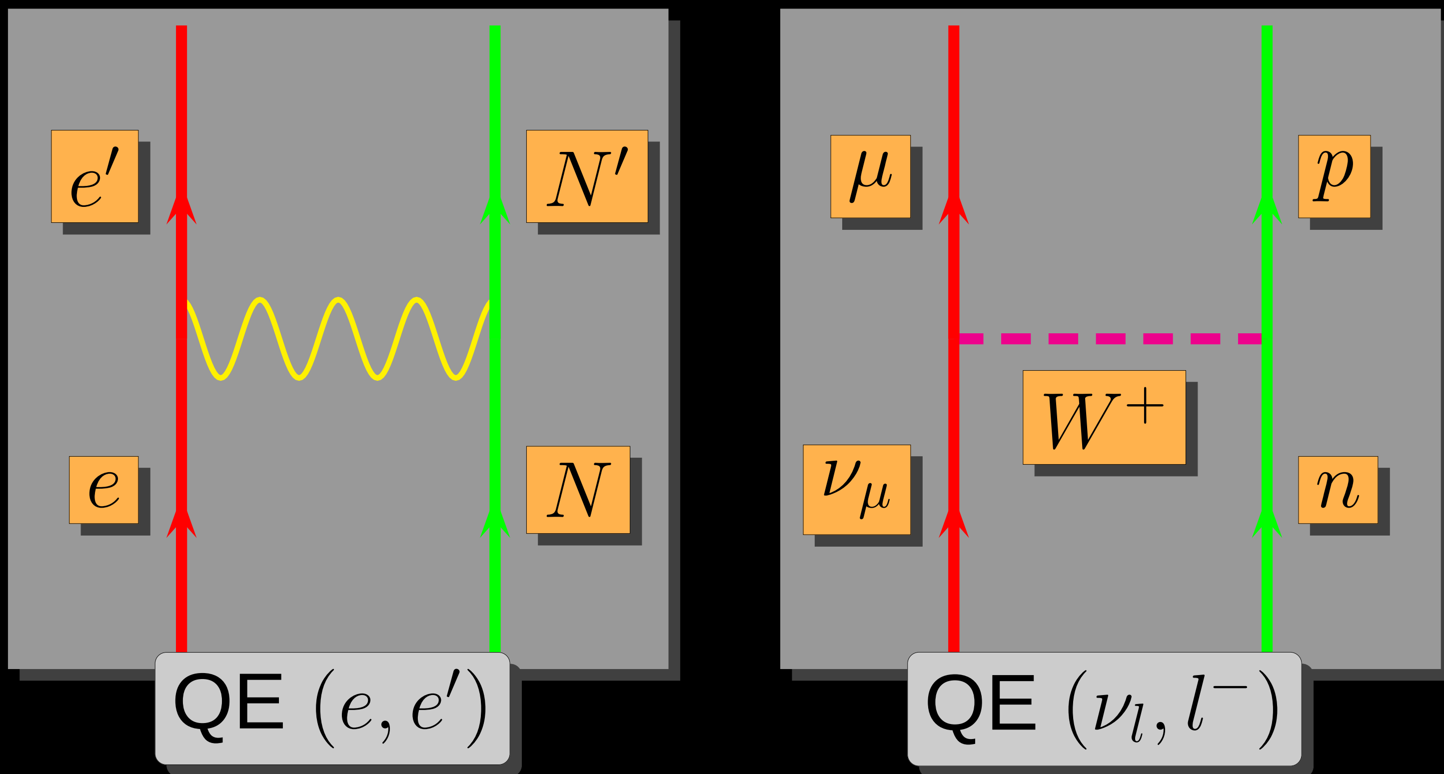
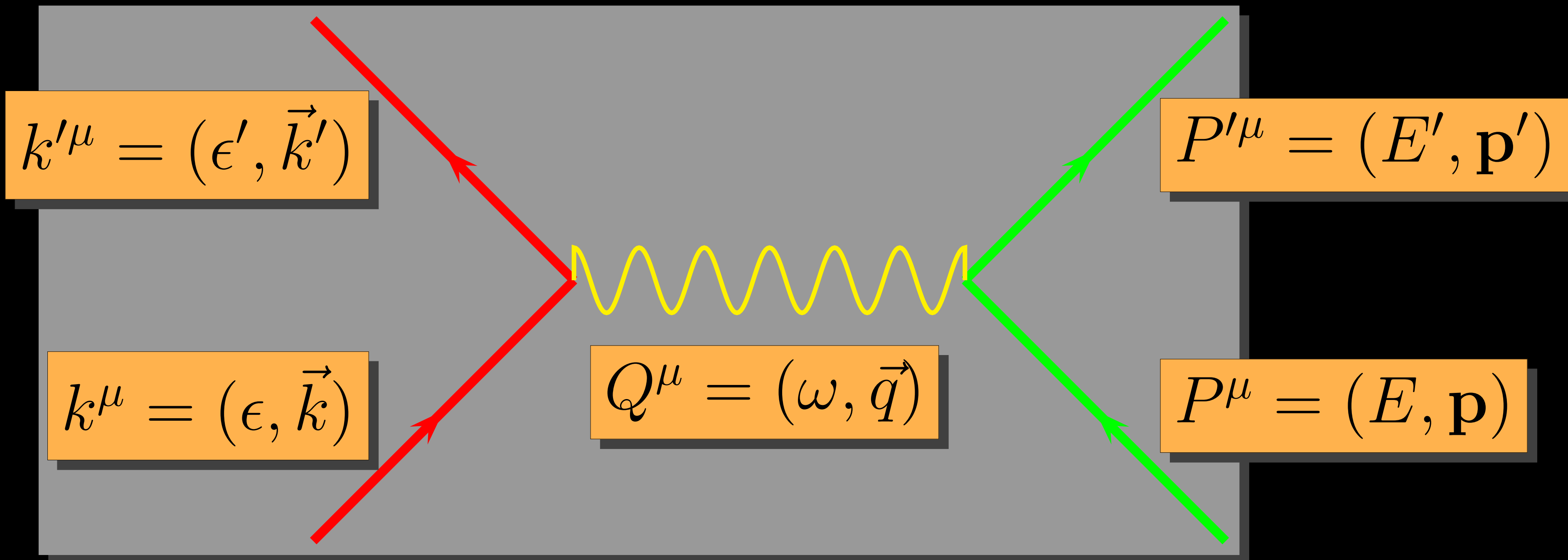
1 General formalism



Kinematics



$$Q^2 = \omega^2 - q^2 < 0$$

(e, e') *formalism*

1

$$\frac{d\sigma}{d\epsilon' d\Omega'} = \sigma_{Mott} \left(v_L R_L + v_T R_T \right)$$

Electron kinematical factors

$$v_L = \rho^2, \qquad v_T = \frac{1}{2}\rho + \tan^2\frac{\theta}{2}, \qquad \rho \equiv \frac{|Q^2|}{q^2}$$

Response functions:

$$\begin{aligned} R_L &= W^{00} \\ R_T &= W^{11} + W^{22} \end{aligned}$$

Hadronic tensor for (e, e')

$$W^{\mu\nu}(q,\omega) = \overline{\sum_{fi}} \delta(E_f - E_i - \omega) \langle f|J^\mu(Q)|i\rangle^* \langle f|J^\nu(Q)|i\rangle$$

$J^\mu(Q)$ is the electromagnetic nuclear current

(ν_l, l^-) *formalism*

Cross section:	$\frac{d\sigma}{d\Omega' d\epsilon'} = \sigma_0 \mathcal{F}_+^2$
Similar to σ_{Mott}:	$\sigma_0 = \frac{G^2 \cos^2\theta_c}{2\pi^2} k'\epsilon' \cos^2\frac{\tilde{\theta}}{2}$
Fermi constant:	$G = 1.166 \times 10^{-11} \quad \mathrm{MeV}^{-2}$
Cabibbo angle:	$\cos\theta_c = 0.975$
Generalized scattering angle:	$\tan^2\frac{\tilde{\theta}}{2} = \frac{\lvert Q^2 \rvert}{(\epsilon+\epsilon')^2 - q^2}$

(ν_l, l^-) formalism (II)

1

Nuclear structure information:

$$\mathcal{F}_+^2 = \widehat{V}_{CC} R_{CC} + 2\widehat{V}_{CL} R_{CL} + \widehat{V}_{LL} R_{LL} + \widehat{V}_T R_T + 2\widehat{V}_{T'} R_{T'}$$

kinematical factors $\widehat{V}_K$ from the leptonic tensor

$$\begin{aligned}
\widehat{V}_{CC} &= 1 - \delta^2 \tan^2 \frac{\tilde{\theta}}{2} \\
\widehat{V}_{CL} &= \frac{\omega}{q} + \frac{\delta^2}{\rho'} \tan^2 \frac{\tilde{\theta}}{2} \\
\widehat{V}_{LL} &= \frac{\omega^2}{q^2} + \left(1 + \frac{2\omega}{q\rho'} + \rho\delta^2\right) \delta^2 \tan^2 \frac{\tilde{\theta}}{2} \\
\widehat{V}_{T} &= \tan^2 \frac{\tilde{\theta}}{2} + \frac{\rho}{2} - \frac{\delta^2}{\rho'} \left(\frac{\omega}{q} + \frac{1}{2}\rho\rho'\delta^2\right) \tan^2 \frac{\tilde{\theta}}{2} \\
\widehat{V}_{T'} &= \frac{1}{\rho'} \left(1 - \frac{\omega\rho'}{q}\delta^2\right) \tan^2 \frac{\tilde{\theta}}{2}
\end{aligned}$$

Adimensional variables:

$$\begin{aligned}
\delta &= \frac{m'}{\sqrt{|Q^2|}} \\
\rho &= \frac{|Q^2|}{q^2} \\
\rho' &= \frac{q}{\epsilon + \epsilon'} \,.
\end{aligned}$$

(ν_l, l^-) formalism (III)

Weak response functions

$$
\begin{aligned}
R_{CC} &= W^{00} \\
R_{CL} &= -\frac{1}{2}\left(W^{03} + W^{30}\right) \\
R_{LL} &= W^{33} \\
R_{T} &= W^{11} + W^{22} \\
R_{T'} &= -\frac{i}{2}\left(W^{12} - W^{21}\right)
\end{aligned}
$$

Weak CC hadronic tensor:

$$W^{\mu\nu}(q,\omega) = \overline{\sum_{fi}} \delta(E_f - E_i - \omega)\langle f|J^{\mu}(Q)|i\rangle^{*}\langle f|J^{\nu}(Q)|i\rangle \ .$$

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or

Single-nucleon current

Electromagnetic current

$$j^\mu(\mathbf{p}',\mathbf{p}) = \overline{u}(\mathbf{p}')\left[F_1\gamma^\mu + i\frac{F_2}{2m_N}\sigma^{\mu\nu}Q_\nu\right]u(\mathbf{p})$$

Weak CC current $j^\mu = j^\mu_V - j^\mu_A$.

$$j^\mu_V(\mathbf{p}',\mathbf{p}) = \overline{u}(\mathbf{p}')\left[2F_1^V\gamma^\mu + i\frac{F_2^V}{m_N}\sigma^{\mu\nu}Q_\nu\right]u(\mathbf{p})$$ ← Vector

$$j^\mu_A(\mathbf{p}',\mathbf{p}) = \overline{u}(\mathbf{p}')\left[G_A\gamma^\mu + G_P\frac{Q^\mu}{2m_N}\right]\gamma^5 u(\mathbf{p})$$ ← Axial-Vector

The relativistic Fermi gas (RFG)

Nuclear response functions for (ν_μ, μ^-) reactions

$$R_K = N\Lambda_0 U_K f_{RFG}(\psi), \quad K = CC, CL, LL, T, T',$$

- N is the neutron number,
- $\Lambda_0 = \frac{\xi_F}{m_N \eta_F^3 \kappa}, \quad \eta_F = k_F/m_N, \quad \xi_F = \sqrt{1+\eta_F^2} - 1.$
- Scaling function $f_{RFG}(\psi) = \frac{3}{4}(1-\psi^2)\theta(1-\psi^2)$
- Scaling variable

$$\psi = \frac{1}{\sqrt{\xi_F}} \frac{\lambda - \tau}{\sqrt{(1+\lambda)\tau + \kappa\sqrt{\tau(1+\tau)}}}$$

- single-nucleon responses U_K

Single-nucleon responses, $K = CC$

$$U_{CC} = U^V_{CC} + \left(U^A_{CC}\right)_{\rm c.} + \left(U^A_{CC}\right)_{\rm n.c.}$$

$$U^V_{CC} = \frac{\kappa^2}{\tau}\left[(2G^V_E)^2 + \frac{(2G^V_E)^2 + \tau(2G^V_M)^2}{1+\tau}\Delta\right] ,$$

$$\Delta = \frac{\tau}{\kappa^2}\xi_F(1-\psi^2)\left[\kappa\sqrt{1+\frac{1}{\tau}} + \frac{\xi_F}{3}(1-\psi^2)\right]$$

The axial-vector response is the sum of conserved (c.) plus non conserved (n.c.) parts,

$$\left(U^A_{CC}\right)_{\rm c.} = \frac{\kappa^2}{\tau}G_A^2\Delta \quad , \quad \left(U^A_{CC}\right)_{\rm n.c.} = \frac{\lambda^2}{\tau}{G'_A}^2.$$

Single-nucleon responses, $K = CL, LL$

$$U_{CL} = U^V_{CL} + \left(U^A_{CL}\right)_{\mathrm{c.}} + \left(U^A_{CL}\right)_{\mathrm{n.c.}}$$
$$U_{LL} = U^V_{LL} + \left(U^A_{LL}\right)_{\mathrm{c.}} + \left(U^A_{LL}\right)_{\mathrm{n.c.}} ,$$

The vector and conserved axial-vector parts are determined by current conservation

$$U^V_{CL} = -\frac{\lambda}{\kappa} U^V_{CC} \qquad \left(U^A_{CL}\right)_{\mathrm{c.}} = -\frac{\lambda}{\kappa} \left(U^A_{CC}\right)_{\mathrm{c.}}$$
$$U^V_{LL} = \frac{\lambda^2}{\kappa^2} U^V_{CC} \qquad \left(U^A_{LL}\right)_{\mathrm{c.}} = \frac{\lambda^2}{\kappa^2} \left(U^A_{CC}\right)_{\mathrm{c.}} ,$$

Non-conserved n.c. parts:

$$\left(U^A_{CL}\right)_{\mathrm{n.c.}} = -\frac{\lambda\kappa}{\tau} {G'_A}^2 , \quad \left(U^A_{LL}\right)_{\mathrm{n.c.}} = \frac{\kappa^2}{\tau} {G'_A}^2 .$$

Single-nucleon responses, $K = T, T'$

$$U_T = U_T^V + U_T^A$$

$$U_T^V = 2\tau(2G_M^V)^2 + \frac{(2G_E^V)^2 + \tau(2G_M^V)^2}{1+\tau}\Delta$$

$$U_T^A = 2(1+\tau)G_A^2 + G_A^2\Delta$$

$$U_{T'} = 2G_A(2G_M^V)\sqrt{\tau(1+\tau)}[1+\tilde{\Delta}]$$

with

$$\tilde{\Delta} = \sqrt{\frac{\tau}{1+\tau}}\frac{\xi_F(1-\psi^2)}{2\kappa}\ .$$

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Super-Scaling Analysis (SuSA)

Scaling in the RFG (Relativistic Fermi gas)

$$R_K = G_K f_{RFG}(\psi)$$

Functions G_K from the RFG for electrons ($K = L, T$) and neutrinos $K = CC, CL, LL, T, T'$.
Scaling function in the RFG

$$f_{RFG}(\psi) = \frac{3}{4}(1-\psi^2)\theta(1-\psi^2)$$

Scaling variable:

$$\psi = \frac{1}{\sqrt{\xi_F}} \frac{\lambda - \tau}{\sqrt{(1+\lambda)\tau + \kappa\sqrt{\tau(1+\tau)}}}$$

Experimental scaling function from (e, e')

$$f(\psi') = \frac{\left(\dfrac{d\sigma}{d\Omega' d\epsilon'}\right)_{exp}}{\sigma_{Mott}(v_L G_L + v_T G_T)}$$

shifted $\longrightarrow \psi' = \dfrac{1}{\sqrt{\xi_F}} \dfrac{\lambda' - \tau'}{\sqrt{(1+\lambda')\tau' + \kappa\sqrt{\tau'(1+\tau')}}}$

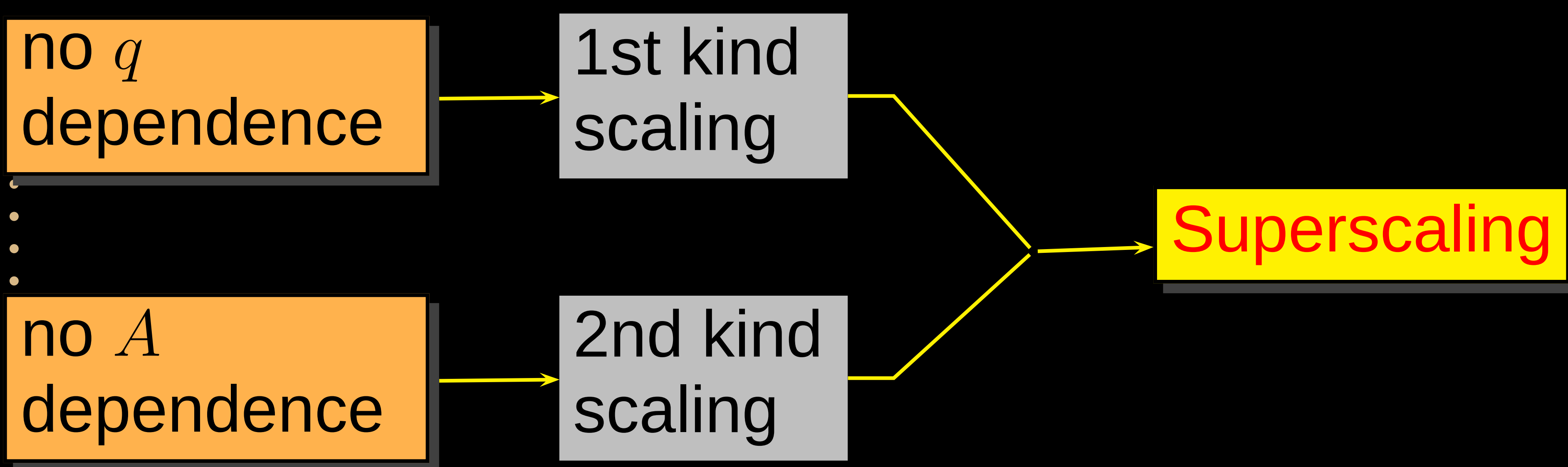
$$\lambda' = (\omega - E_s)/2m_N, \quad \tau' = \kappa^2 - \lambda'^2$$

k_F y E_s are fitted to the data

$f_L = \dfrac{R_L}{G_L}$ Longitudinal $\qquad f_T = \dfrac{R_T}{G_T}$ Transverse

Superscaling

- Plot the experimental $f(\psi')$ versus ψ' for different kinematics and nuclei
- Fit E_s and k_F to get scaling (one universal scaling function)



Scaling in the QE peak

Scaling of R_L [Donnelly & Sick PRC 60 (1999)]

SUPERSCALING OF INCLUSIVE ELECTRON . . . PHYSICAL REVIEW C **60** 065502

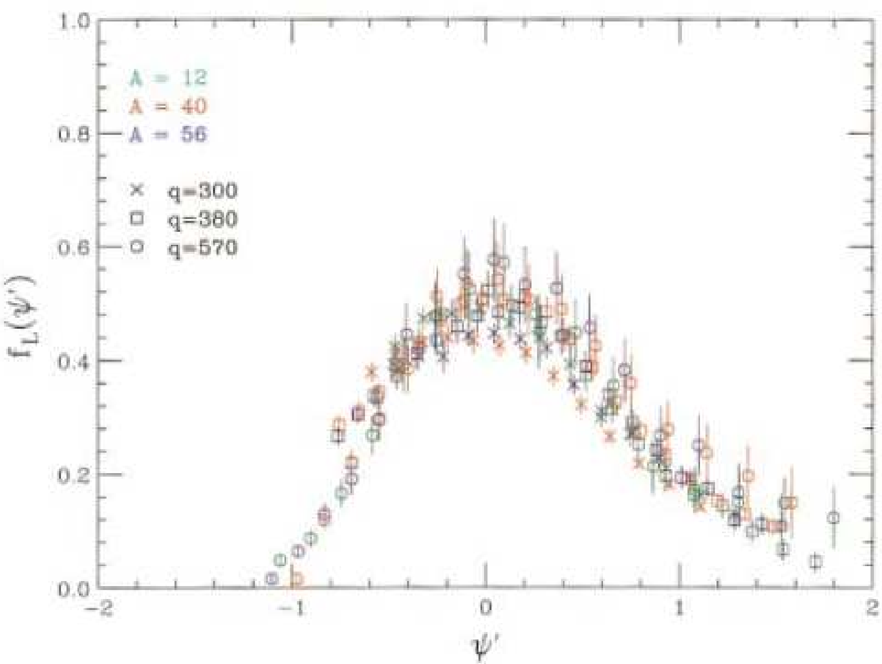
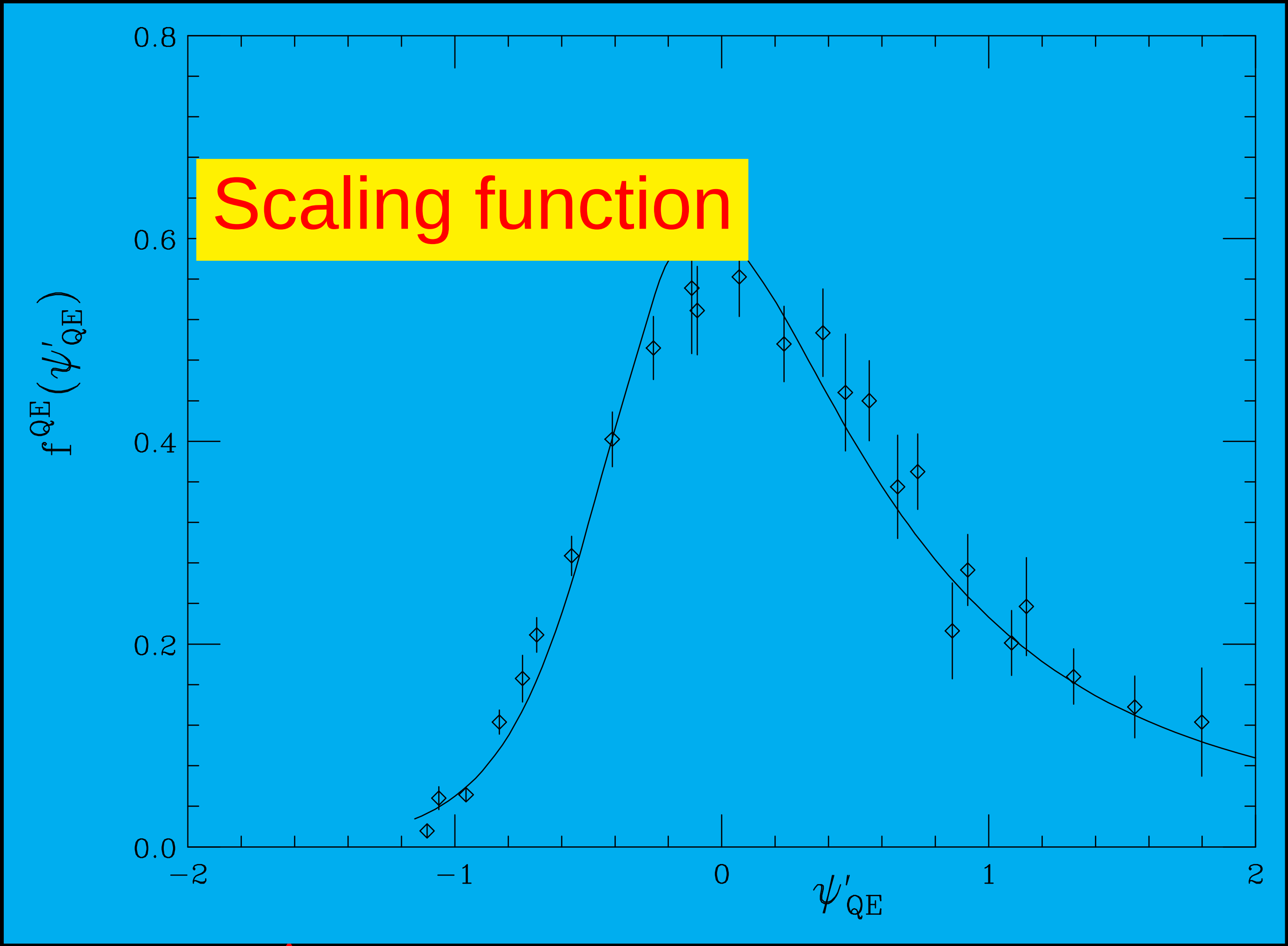


FIG. 10. (Color) Scaling function $f_L(\psi')$ from the longitudinal response.

Fit in the Quasi-elastic peak



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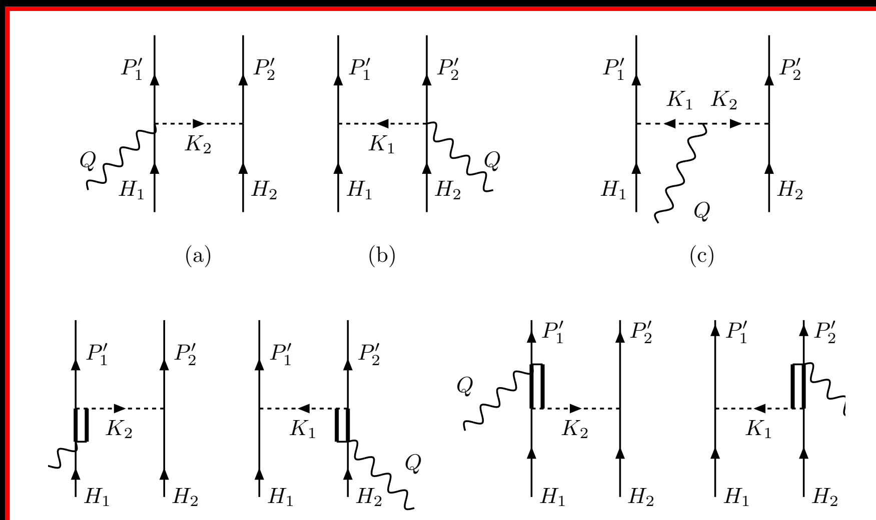
SuSA (Super Scaling Analysis)

- Using the experimental (e, e') scaling function to predict neutrino cross sections
- Use the RFG equations to compute the (ν_l, l^-) response functions with the substitution $f_{RFG}(\psi) \longrightarrow f_{exp}(\psi)$

2p-2h Meson-Exchange Currents

Two-particle two-hole Meson Exchange Currents (MEC)

- Relativistic Fermi Gas two-nucleon emission channel
- Added to the SuSA results
- A. De Pace *et al.* NPA 726, 303 (2003)
- J.E. Amaro *et al.* PRC 82, 0444601 (2010)



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2N emission in Fermi gas

- Initial state $|i\rangle = |F\rangle$,
- Sum over final states

$$\sum_f = \sum_{1p-1h} + \sum_{2p-2h} + \sum_{otherchannels}$$

- 2p-2h channel final states

$$|f\rangle = |2p-2h\rangle = |1', 2', 1^{-1}, 2^{-1}\rangle$$

$$|1\rangle = |\mathbf{h}_1 s_1 t_1\rangle \qquad |2\rangle = |\mathbf{h}_2 s_2 t_2\rangle$$
$$|1'\rangle = |\mathbf{p}'_1 s'_1 t'_1\rangle \qquad |2'\rangle = |\mathbf{p}'_2 s'_2 t'_2\rangle$$

- Particles $P'_i = (E'_i, \mathbf{p}'_i)$, Pauli blocking $p'_i > k_F$
- Holes $H_i = (E_i, \mathbf{h}_i)$, with $h_i < k_F$.

2p-2h Hadronic tensor

$$W^{\mu\nu} = W^{\mu\nu}_{1p1h} + W^{\mu\nu}_{2p2h} + \cdots$$

$$W^{\mu\nu}_{2p2h} = \frac{V}{(2\pi)^9} \int d^3p'_1 d^3p'_2 d^3h_1 d^3h_2 \frac{m_N^4}{E_1 E_2 E'_1 E'_2} r^{\mu\nu}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2)$$
$$\times \delta^3(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{h}_1 - \mathbf{h}_2 - \mathbf{q}) \delta(E'_1 + E'_2 - \omega - E_1 - E_2)$$
$$\times \Theta(p'_1, p'_2, h_1, h_2)$$

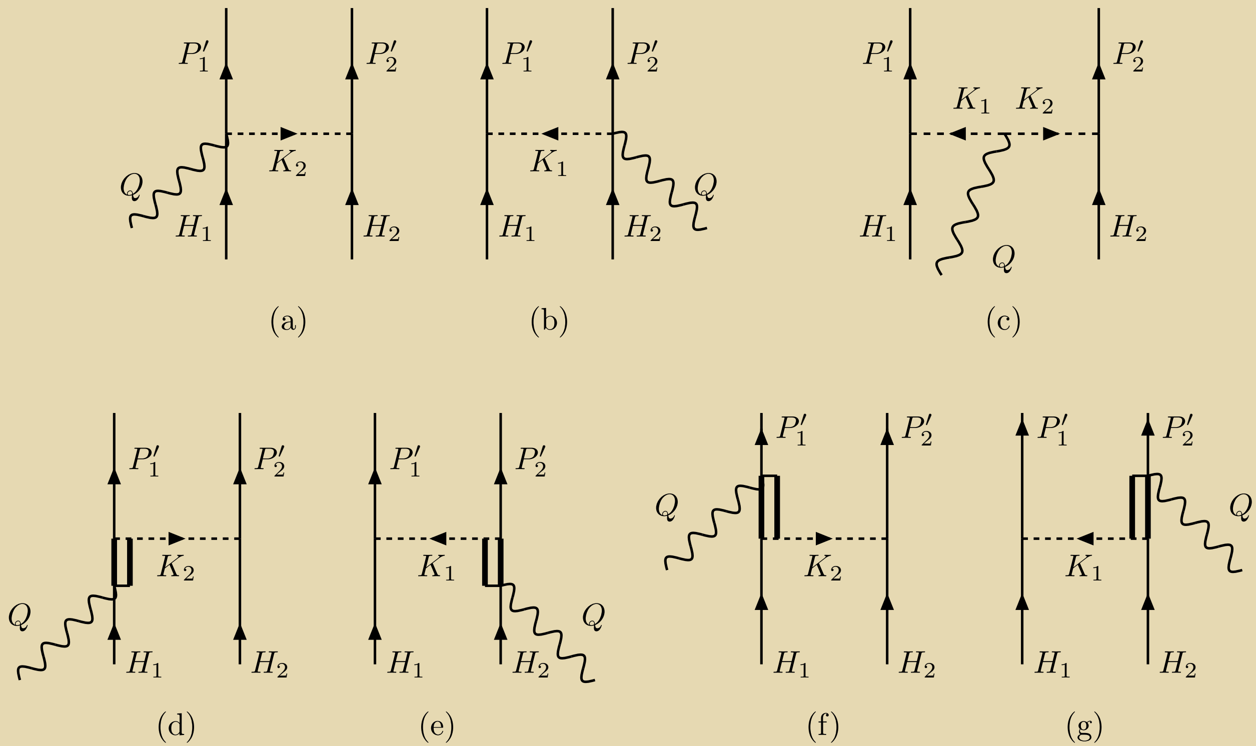
$$\Theta(p'_1, p'_2, h_1, h_2) = \theta(p'_1 - k_F)\theta(p'_2 - k_F)\theta(k_F - h_1)\theta(k_F - h_2)$$

Elementary hadronic tensor for a nucleon pair transition:

$$r^{\mu\nu}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2) = \frac{1}{4} \sum_{s_1 s_2 s'_1 s'_2} \sum_{t_1 t_2 t'_1 t'_2} j^\mu(1', 2', 1, 2)^*_A j^\nu(1', 2', 1, 2)_A$$

$V = 3\pi^2 N / k_F^3$.

Meson-Exchange Currents



Feynman diagrams:
Seagull (a,b), pionic (c), and Δ current (d-g)
Pionic four-momenta $K_i^\mu = P_i'^\mu - H_i^\mu$

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Seagull and pionic

Seagull:

$$j_s^\mu(\mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_1, \mathbf{p}_2) = \frac{f^2}{m_\pi^2} i\epsilon_{3ab}\overline{u}(\mathbf{p}_1')\tau_a\gamma_5 \not{K}_1 u(\mathbf{p}_1) \times \frac{F_1^V}{K_1^2 - m_\pi^2}\overline{u}(\mathbf{p}_2')\tau_b\gamma_5\gamma^\mu u(\mathbf{p}_2) + (1 \leftrightarrow 2)\,. \quad (1)$$

Pion in flight:

$$j_p^\mu(\mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_1, \mathbf{p}_2) = \frac{f^2}{m_\pi^2} i\epsilon_{3ab} \frac{F_\pi(K_1 - K_2)^\mu}{(K_1^2 - m_\pi^2)(K_2^2 - m_\pi^2)} \times \overline{u}(\mathbf{p}_1')\tau_a\gamma_5 \not{K}_1 u(\mathbf{p}_1)\overline{u}(\mathbf{p}_2')\tau_b\gamma_5 \not{K}_2 u(\mathbf{p}_2)\,. \quad (2)$$

F_1^V and F_π: the electromagnetic form factors
pion-nucleon coupling constant: $f^2/4\pi = 0.08$.

Δ *Current*

$$
\begin{aligned}
j^{\mu}_{\Delta}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}_1, \mathbf{p}_2) &= \frac{f_{\pi N \Delta} f}{m_\pi^2} \frac{1}{K_2^2 - m_\pi^2} \overline{u}(\mathbf{p}'_1) T^{\mu}_a(1) u(\mathbf{p}_1) \\
&\times \overline{u}(\mathbf{p}'_2) \tau_a \gamma_5 \not{K}_2 u(\mathbf{p}_2) + (1 \leftrightarrow 2)\,. \qquad (3)
\end{aligned}
$$

$T^{\mu}_a(1)$ is related to the pion electroproduction amplitude

$$
\begin{aligned}
T^{\mu}_a(1) &= K_{2,\alpha} \Theta^{\alpha\beta} G^{\Delta}_{\beta\rho}(H_1 + Q) S^{\rho\mu}_f(H_1) T_a T^{\dagger}_3 \\
&+ T_3 T^{\dagger}_a S^{\mu\rho}_b(P'_1) G^{\Delta}_{\rho\beta}(P'_1 - Q) \Theta^{\beta\alpha} K_{2,\alpha}\,. \qquad (4)
\end{aligned}
$$

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Δ *electromagnetic tensor*

Forward

$$
\begin{aligned}
S_f^{\rho\mu}(H_1) &= \Theta^{\rho\mu}\left[g_1\, \not{Q} - g_2 H_1\cdot Q + g_3 Q^2\right]\gamma_5 \\
&- \Theta^{\rho\nu}Q_\nu\left[g_1\gamma^\mu - g_2 H_1^\mu + g_3 Q^\mu\right]\gamma_5
\end{aligned} \tag{5}
$$

Backward

$$
\begin{aligned}
S_b^{\rho\mu}(P_1') &= \gamma_5\left[g_1\, \not{Q} - g_2 P_1'\cdot Q - g_3 Q^2\right]\Theta^{\mu\rho} \\
&- \gamma_5\left[g_1\gamma^\mu - g_2 P_1'^\mu - g_3 Q^\mu\right]Q_\nu\Theta^{\nu\rho}\,.
\end{aligned} \tag{6}
$$

The tensor $\Theta_{\mu\nu}$

$$
\Theta_{\mu\nu} = g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu\,. \tag{7}
$$

Δ *propagator*

Rarita-Schwinger tensor

$$G^{\Delta}_{\beta\rho}(P) = -\frac{\not{P} + m_\Delta}{P^2 - m_\Delta^2} \times \left[g_{\beta\rho} - \frac{1}{3}\gamma_\beta\gamma_\rho - \frac{2}{3}\frac{P_\beta P_\rho}{m_\Delta^2} - \frac{\gamma_\beta P_\rho - \gamma_\rho P_\beta}{3m_\Delta} \right] . \quad (8)$$

Δ width: $m_\Delta \to m_\Delta + \frac{i}{2}\Gamma(P)$ in the denominator of the propagator to account for the Δ decay probability

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Integration of the energy delta function

9-D integral for the 2p-2h response functions

$$\int d^3p'_1 d^3h_1 d^3h_2 \delta(E_1 + E_2 + \omega - E'_1 - E'_2) f(\mathbf{h}_1, \mathbf{h}_2, \mathbf{p}'_1, \mathbf{p}'_2) \,, \qquad (9)$$

Momentum conservation $\mathbf{p}'_2 = \mathbf{h}_1 + \mathbf{h}_2 + \mathbf{q} - \mathbf{p}'_1$.
We integrate over the momentum p'_1 using the delta function:

- For fixed $\mathbf{h}_1, \mathbf{h}_2$, θ'_1, ϕ'_1
- Change variables $p'_1 \to E' = E'_1 + E'_2$.
- compute the Jacobian of the transformation

$$dp'_1 = \frac{dE'}{\left| \frac{p'_1}{E'_1} - \frac{\mathbf{p}'_2 \cdot \mathbf{p}'_1}{E'_2 p'_1} \right|} \,, \qquad (10)$$

Momentum of the final nucleon

- Compute p'_1 for fixed angles θ'_1, ϕ'_1, by solving the energy conservation equation.
- Second degree equation with two solutions

$$p'_1 = \frac{a}{b}\left(v \pm v_0\sqrt{1 - \frac{bm_N^2}{a^2}}\right) , \quad (11)$$

where

$$a = \frac{1}{2}p'^2 \qquad b = E'^2 - p'^2\cos^2\beta'_1 \quad (12)$$

$$v_0 = E' \qquad v = p'\cos\beta'_1 , \quad (13)$$

Final total energy: $E' = E_1 + E_2 + \omega$
Final total momentum: $\mathbf{p}' = \mathbf{h}_1 + \mathbf{h}_2 + \mathbf{q}$
$\beta'_1 =$ angle between $\mathbf{p}'_1$ and $\mathbf{p}'$.

Results for 2p-2h responses

- Compute transverse response functions in the 2p-2h channel.
- Relativistic Fermi gas
- 7D integrals $\int d^3h_1 d^3h_2 d\theta'_1$
- We choose $\phi'_1 = 0$ and multiply by 2π.
- p'_1 is fixed from energy conservation

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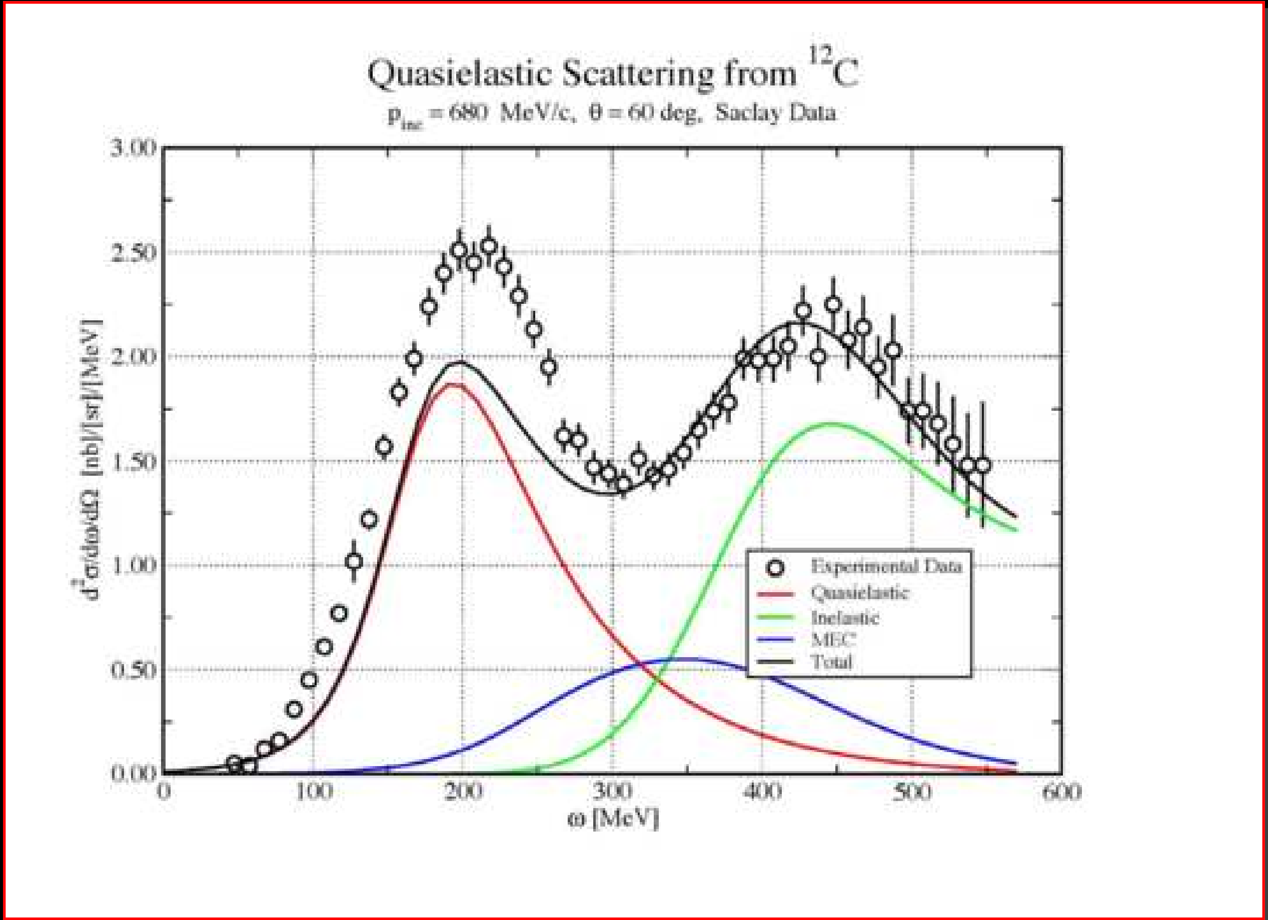
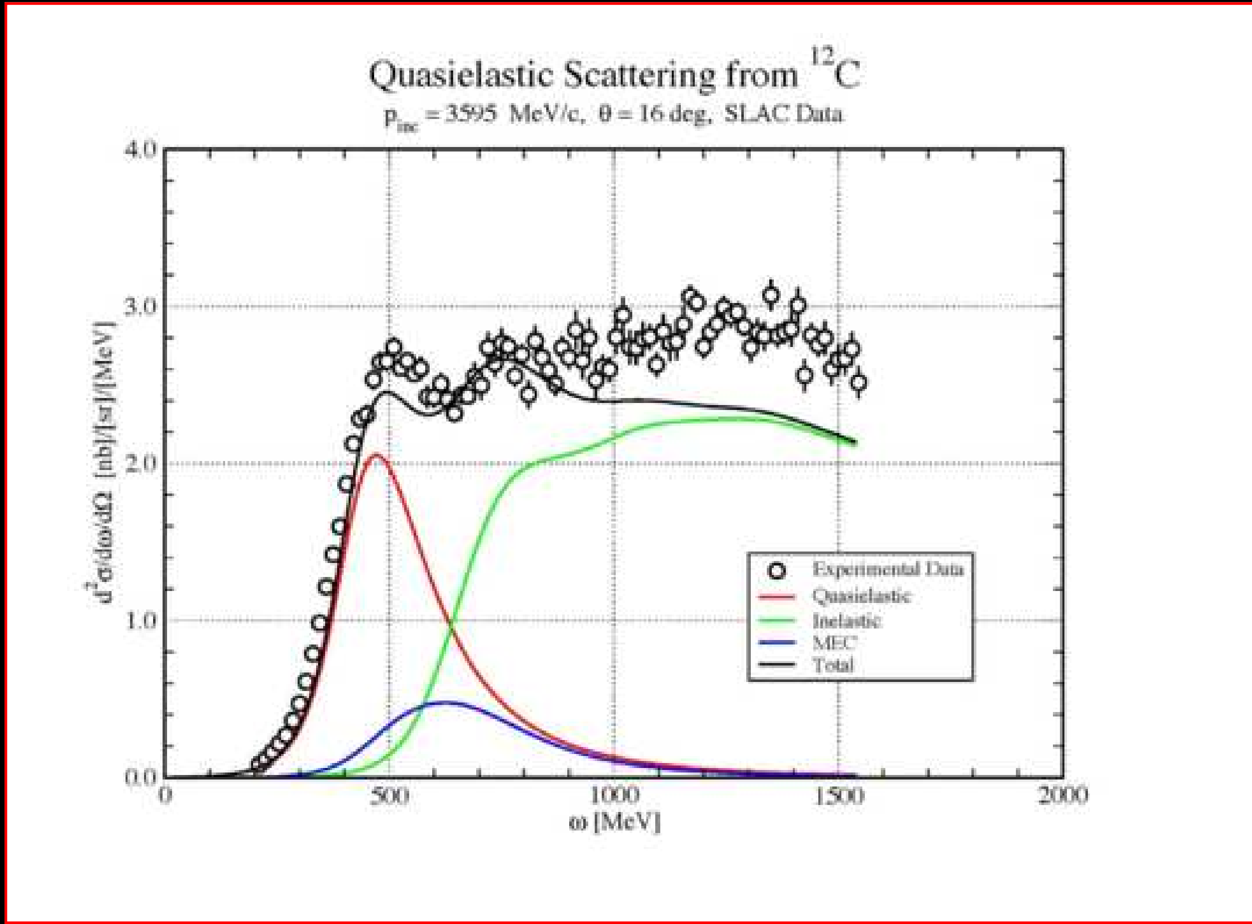
(e, e') *results with MEC 2p-2h*

- Quasielastic + MEC + Inelastic cross section
- $\epsilon_e = 680$ MeV
- $\theta = 60^o$
- Data from Saclay



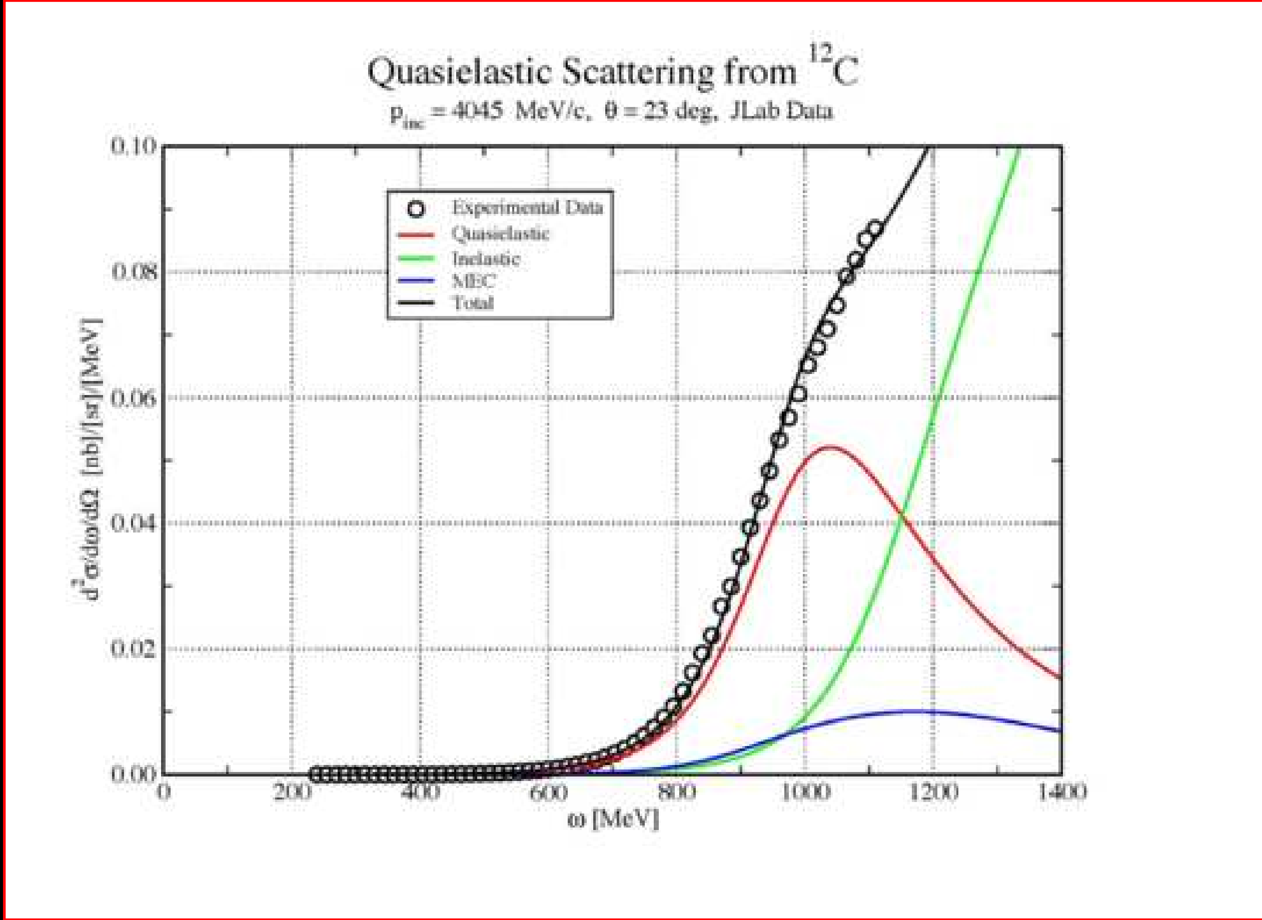
(e, e') results with MEC 2p-2h

- Quasielastic + MEC + Inelastic cross section
- $\epsilon_e = 3595$ MeV
- $\theta = 16^o$
- Data from SLAC



(e, e') *results with MEC 2p-2h*

- Quasielastic + MEC + Inelastic cross section
- $\epsilon_e = 4045$ MeV
- $\theta = 60^o$
- Data JLab



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2 Status of MEC in SuSA for (ν_μ, μ^-)

J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, C.F. Williamson, Physics Letters B (2011)

- Double differential neutrino cross sections from ^{12}C
- Integrated over the neutrino flux
- Contribution of vector meson-exchange currents in the 2p-2h sector

The R_T^{VV} response with MEC is computed assuming that the weak CC MEC contribution is the same as the MEC contribution to the isovector part of the electromagnetic R_T

$$R_T^{VV} = [R_T^{VV}]^{OB} \frac{[R_T^{em}]^{OB}_{\text{isovector}} + [R_T^{em}]^{\text{2p2hMEC}}}{[R_T^{em}]^{OB}_{\text{isovector}}}$$

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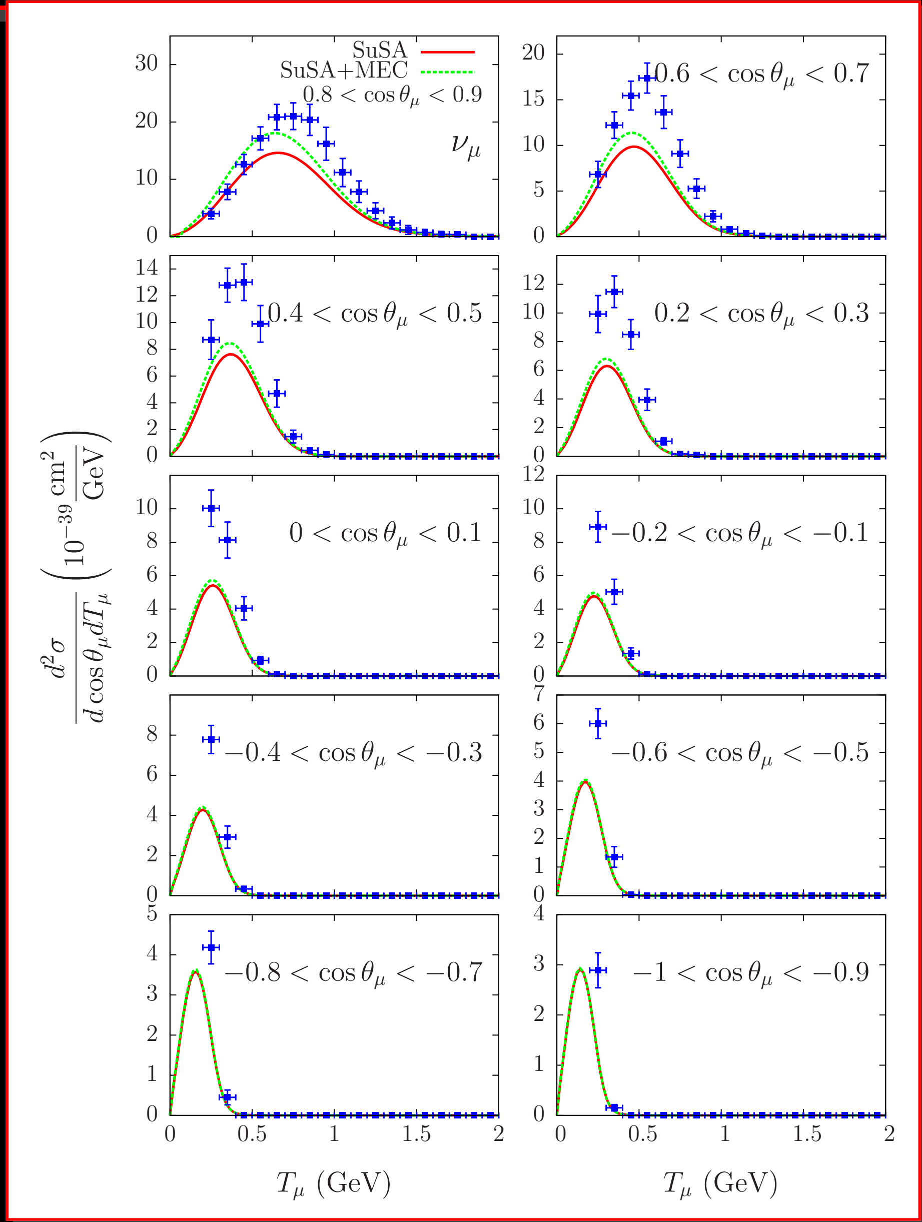
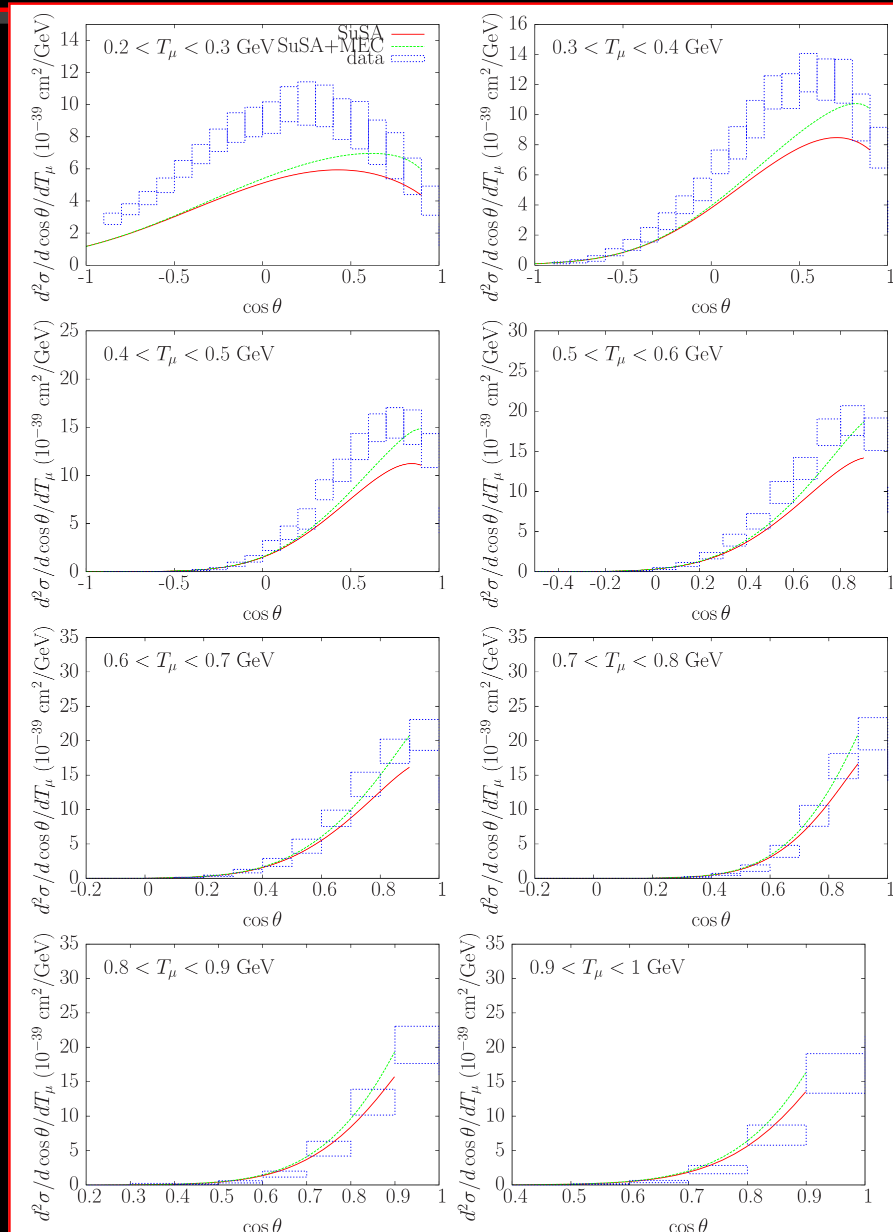
Neutrino results with MEC 2p-2h

- Calculations from Amaro, Barbaro, Caballero, Donnelly, Williamson, PLB 696 (2011) 151.
- The MEC increase the cross section less than 10%
- Data from A.A. Aguilar-Arevalo *et al.*, (MiniBooNE Collaboration), PRD 81, 092005 (2010)



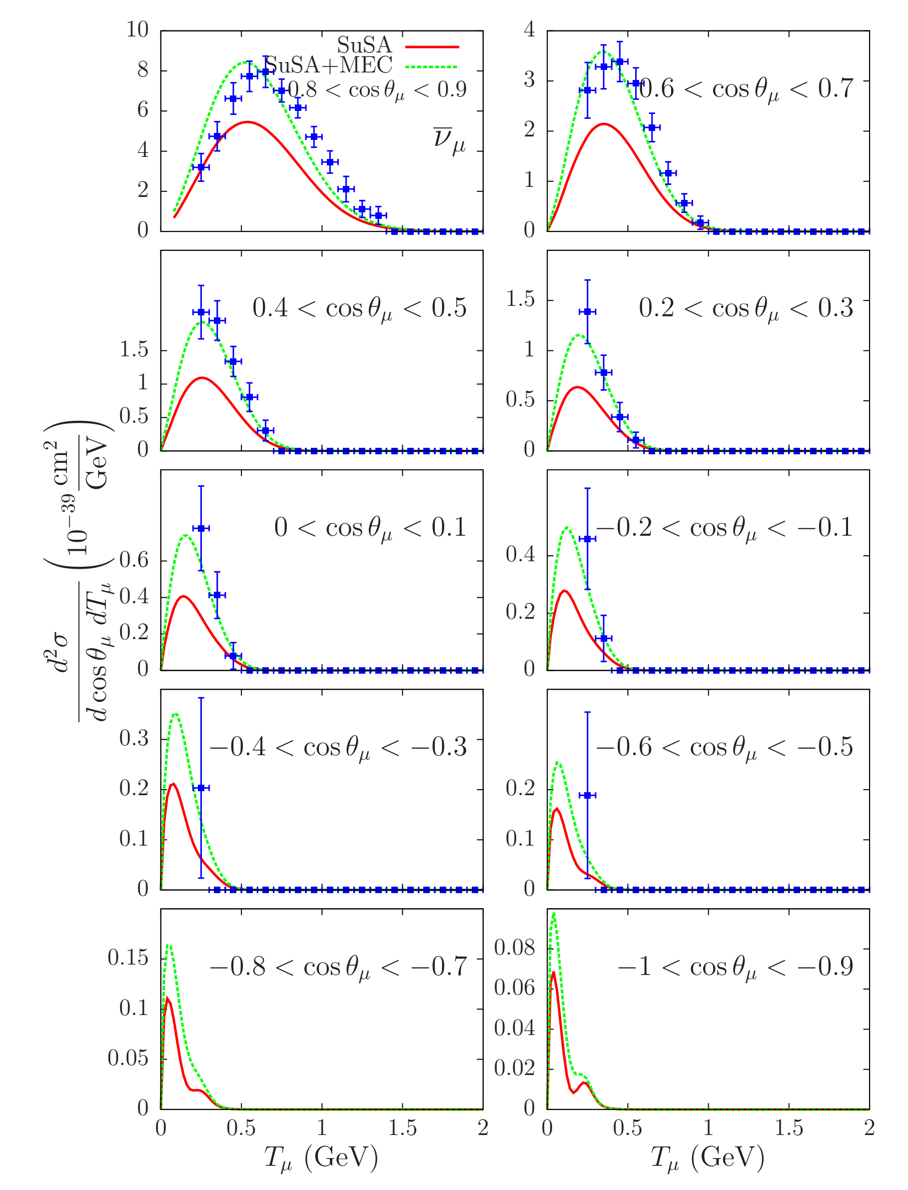
Neutrino results. Angle projection

- Calculations from Amaro, Barbaro, Caballero, Donnelly, Williamson, PLB 696 (2011) 151.
- The MEC tend to increase the cross section about 5-10%
- Data from Aguilar-Arevalo et. al. (MiniBooNE Collaboration)



Antineutrino results with MEC 2p-2h

- Calculations from Amaro, Barbaro, Caballero, Donnelly, PRL 108 (2012).
- The MEC tend to increase the cross section more than for neutrinos.
- Data from Aguilar-Arevalo et. al. (MiniBooNE Collaboration) PRD 88 (2013)



Further modifications of SuSA

- Estimation of the axial 2p-2h MEC:

$$
\begin{aligned}
[R_T^{AA}]^{\rm 2p2hMEC} &\simeq [R_T^{VV}]^{\rm 2p2hMEC} \\
[R_{T'}^{VA}]^{\rm 2p2hMEC} &\simeq [R_T^{VV}]^{\rm 2p2hMEC}
\end{aligned}
$$

- SuSA v2: Two different scaling functions f_L and f_T (from the Relativistic Mean Field)
- Extend SuSA for low q and ω with Pauli blocking

$$
f_{PB}[\psi(\omega,q)] = f[\psi(\omega,q)] - f[\psi(-\omega,q)]
$$

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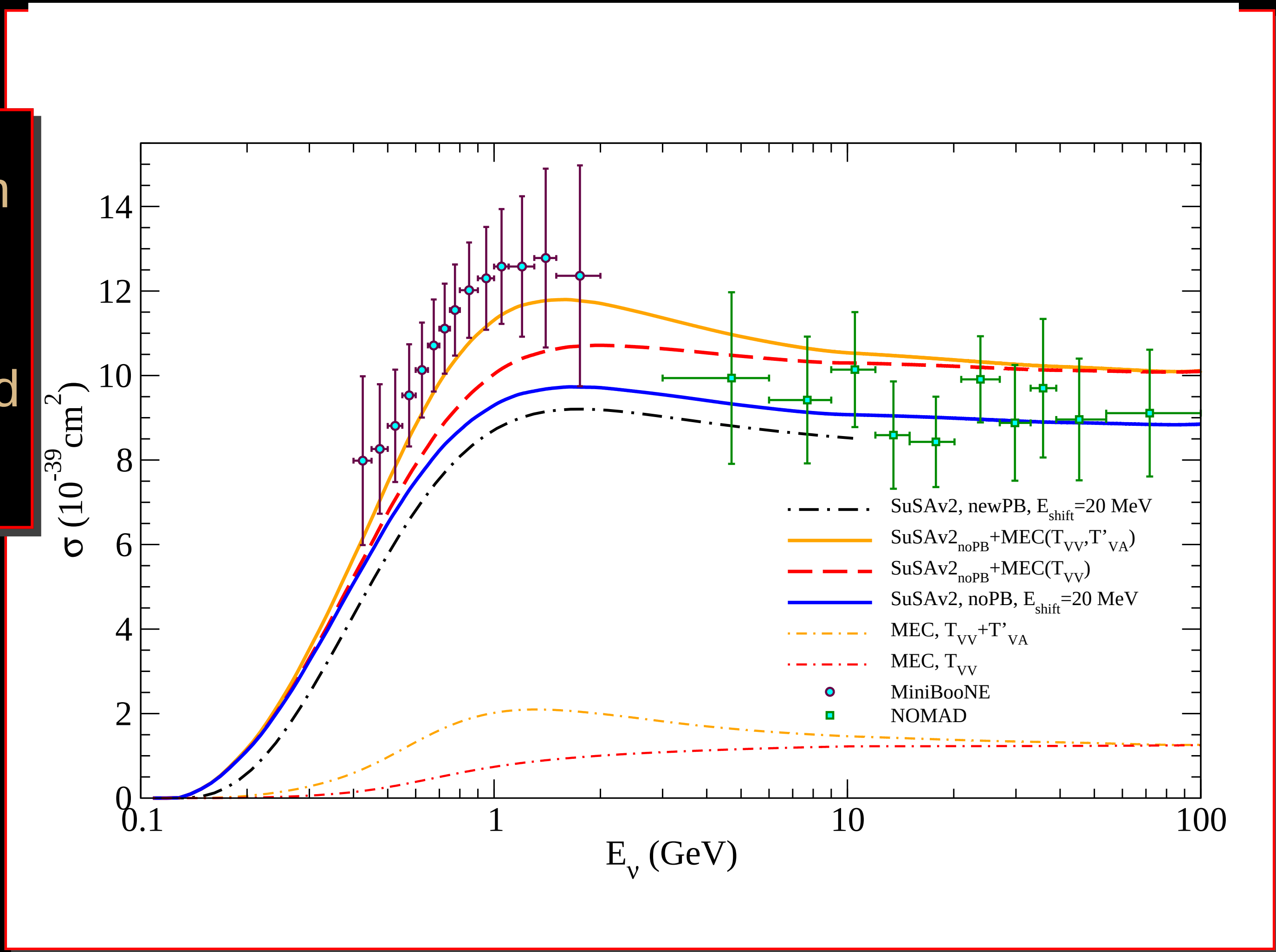
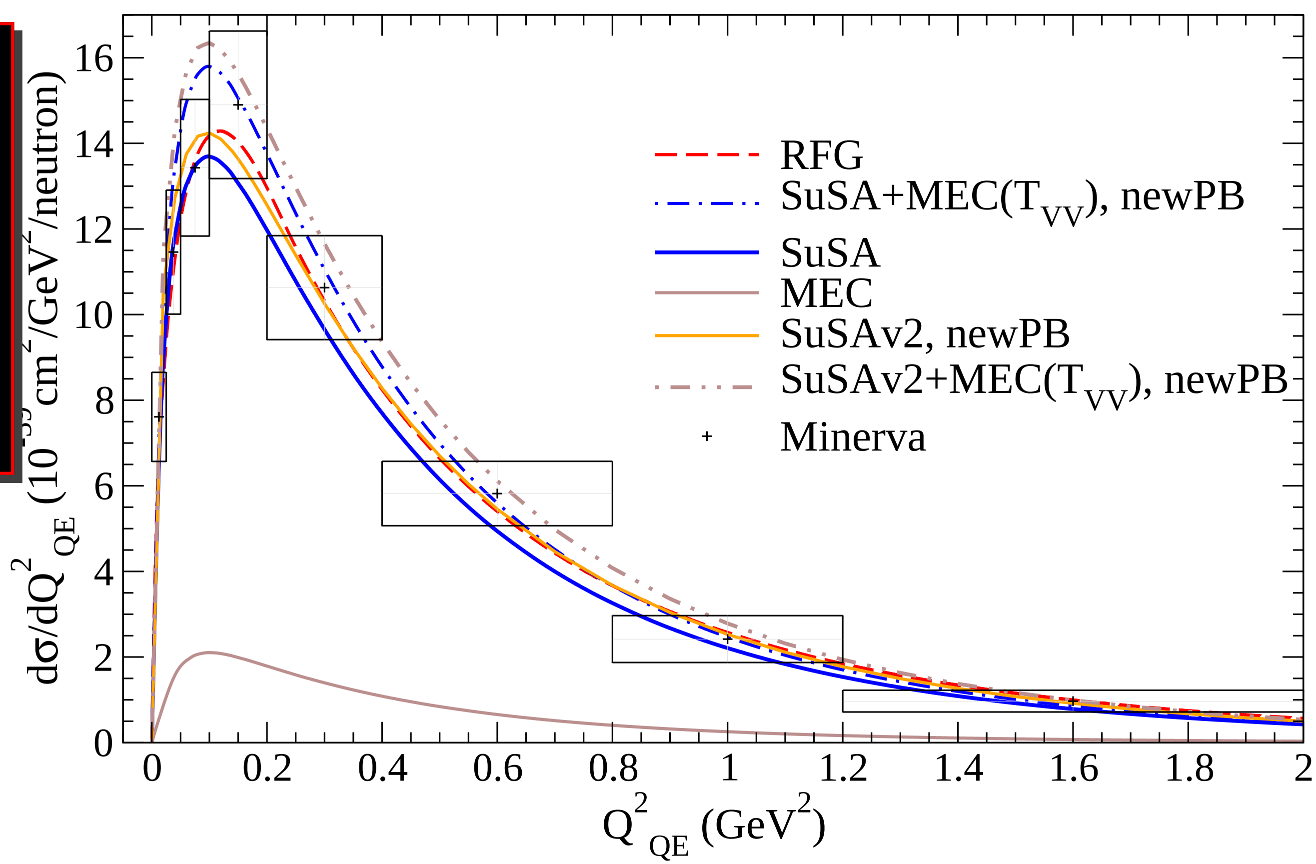
SuSA New results (preliminary)

- Total cross section versus neutrino energy.
- SuSA with and without MEC



SuSA new results for MINERVA

- MINERVA neutrino differential cross section
- SuSA with and without MEC



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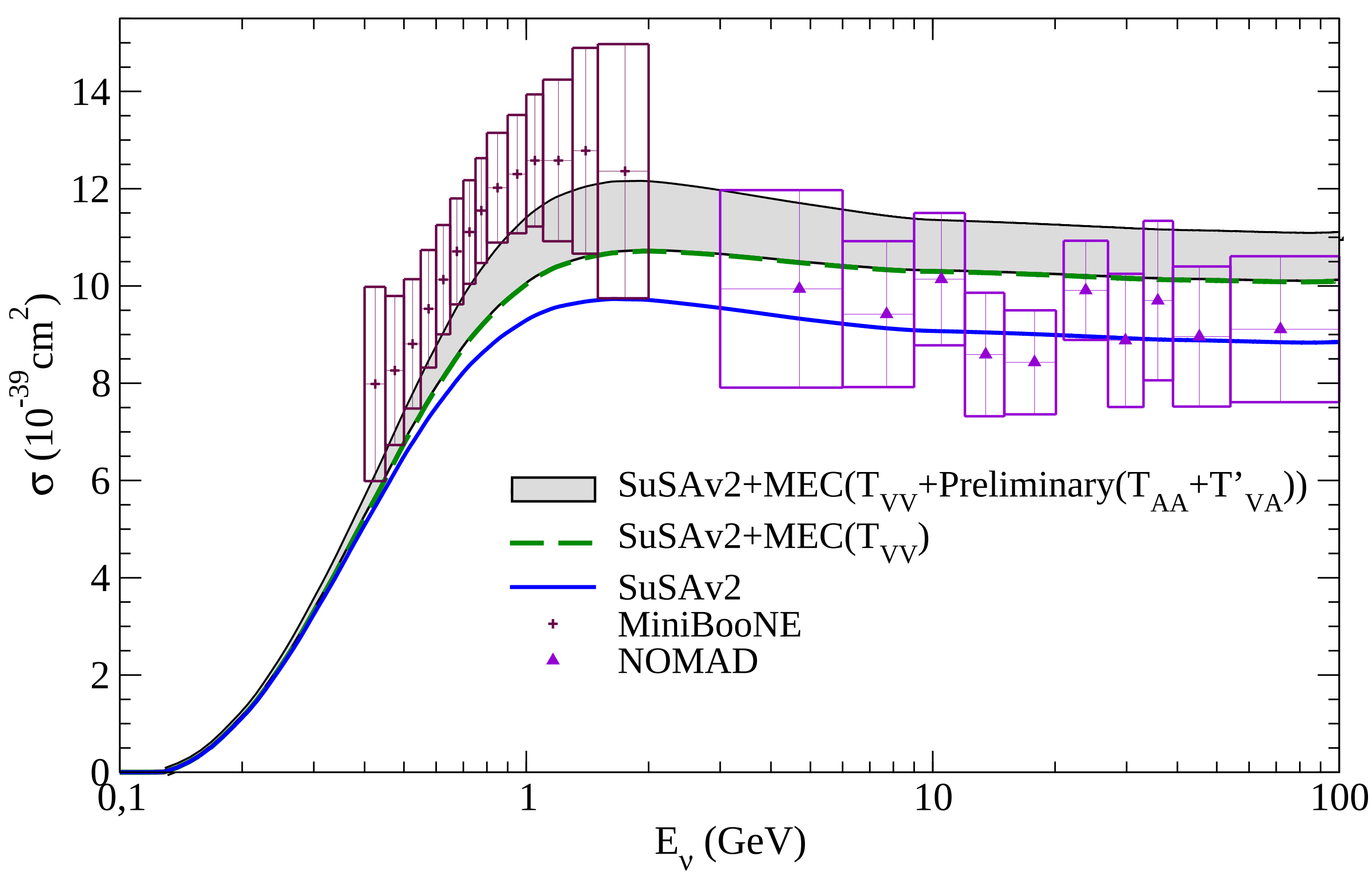
SuSA estimation of axial MEC

- ν cross section.
- Preliminary results.
- For the 2p-2h,

$$R^T_{AA} = R^{T'}_{VA} = R^T_{VV}$$

- SuSA with and without MEC

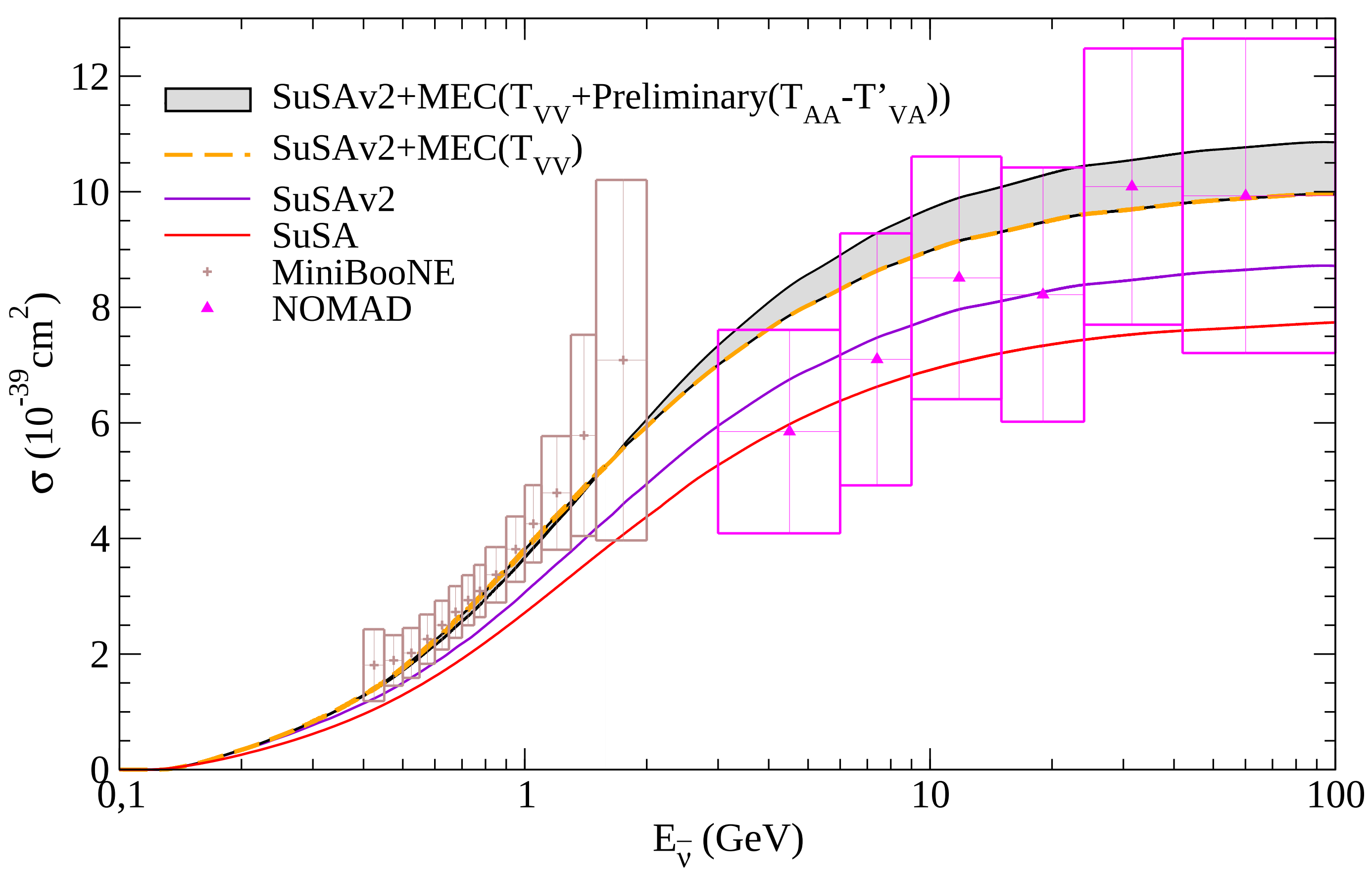


SuSA estimation of axial MEC

- $\overline{\nu}$ cross section.
- Preliminary results.
- For the 2p-2h,

$$R^T_{AA} = R^{T'}_{VA} = R^T_{VV}$$

- SuSA with and without MEC



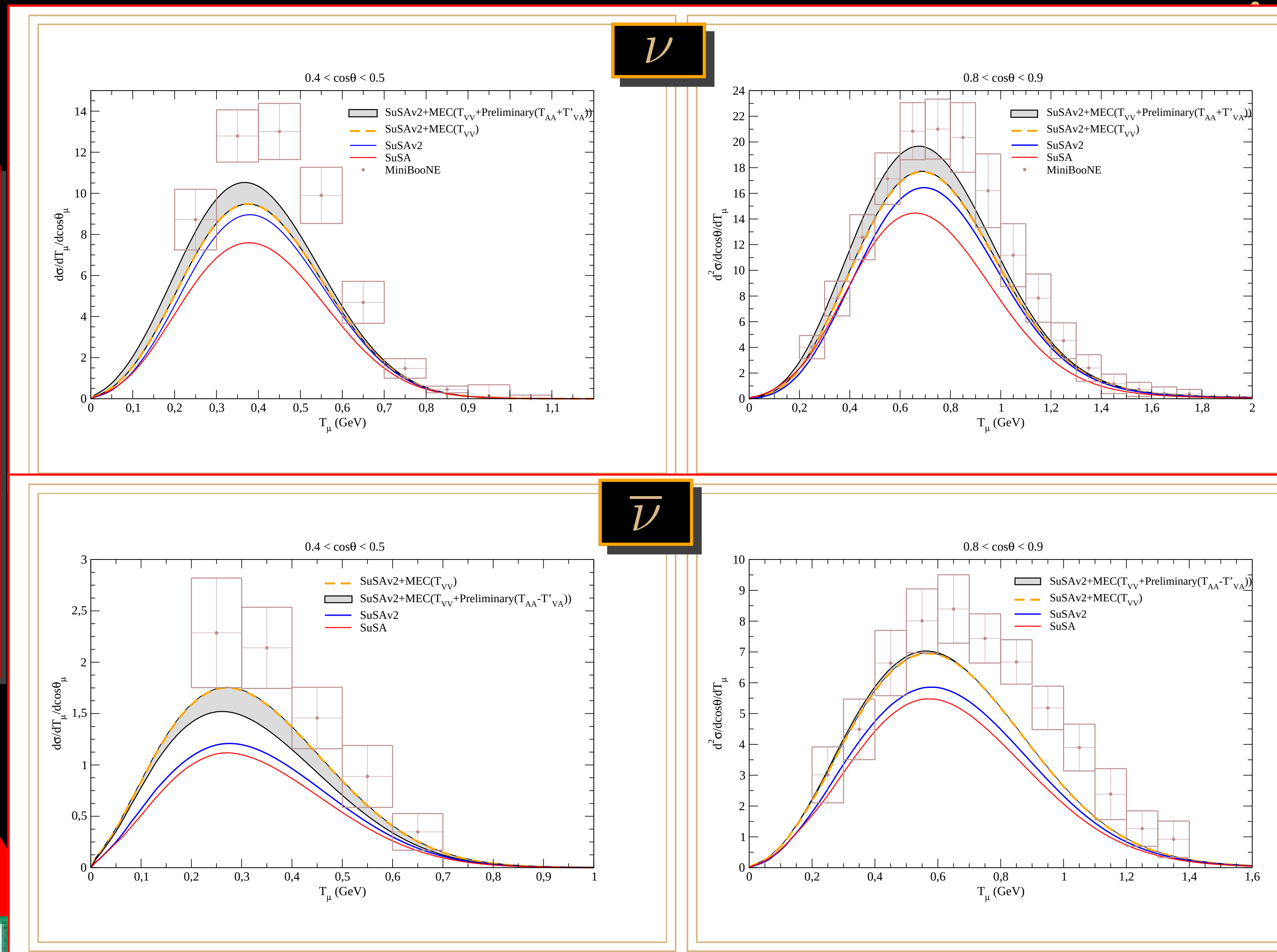
NuInt14

SuSA estimation of axial MEC

- MiniBooNE double-differential cross section.
- Preliminary results for the 2p-2h MEC,

$$R^T_{AA} = R^{T'}_{VA} = R^T_{VV}$$

- SuSA with and without MEC



3 Perspectives: new approach to 2p-2h

- I. Ruiz Simo, C. Albertus, J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly,
  Relativistic effects in two-particle emission for electron and neutrino reactions
  arXiv:1405.4280 [nucl-th] (16 May 2014)

See also:

- C. Albertus, et al.,
  2p-2h distribution in phase space for neutrino and electron scattering
  Nuint14 Poster P:01
- I. Ruiz Simo, et al.,
  Relativistic effects and meson exchange currents in two-particle emission with neutrinos.
  Nuint14 Poster P:15

New MEC calculation

- Including the axial current
- Codes more efficient and faster
- Relativistic effects
- Angular distribution of final nucleons: singularities for high q
- Analytical integration around the singularities

Study of the 7D integral in 2p-2h hadronic tensor

$$W^{\mu\nu}_{2p-2h} = \frac{V}{(2\pi)^9}\int d^3p'_1 d^3h_1 d^3h_2 \frac{m_N^4}{E_1 E_2 E'_1 E'_2}\Theta(p'_1,p'_2,h_1,h_2)$$
$$r^{\mu\nu}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2)\delta(E'_1+E'_2-E_1-E_2-\omega)$$

$\mathbf{p'_2} = \mathbf{h_1} + \mathbf{h_2} + \mathbf{q} - \mathbf{p'_1}.$
$r^{\mu\nu}$ = elementary hadronic tensor for 2p-2h

The 2p-2h phase space function

Simplest case: $r^{\mu\nu} = 1$.
2p-2h phase-space function

$$F(q,\omega) \equiv \int d^3p'_1 d^3h_1 d^3h_2 \frac{m_N^4}{E_1 E_2 E'_1 E'_2} \delta(E'_1 + E'_2 - E_1 - E_2 - \omega)\Theta(p'_1, p'_2, h_1, h_2)\,.$$

with $\mathbf{p}'_2 = \mathbf{h}_1 + \mathbf{h}_2 + \mathbf{q} - \mathbf{p}'_1$.
All 2p-2h models should agree about $F(q,\omega)$

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Non-relativistic phase space

Semi-analytical

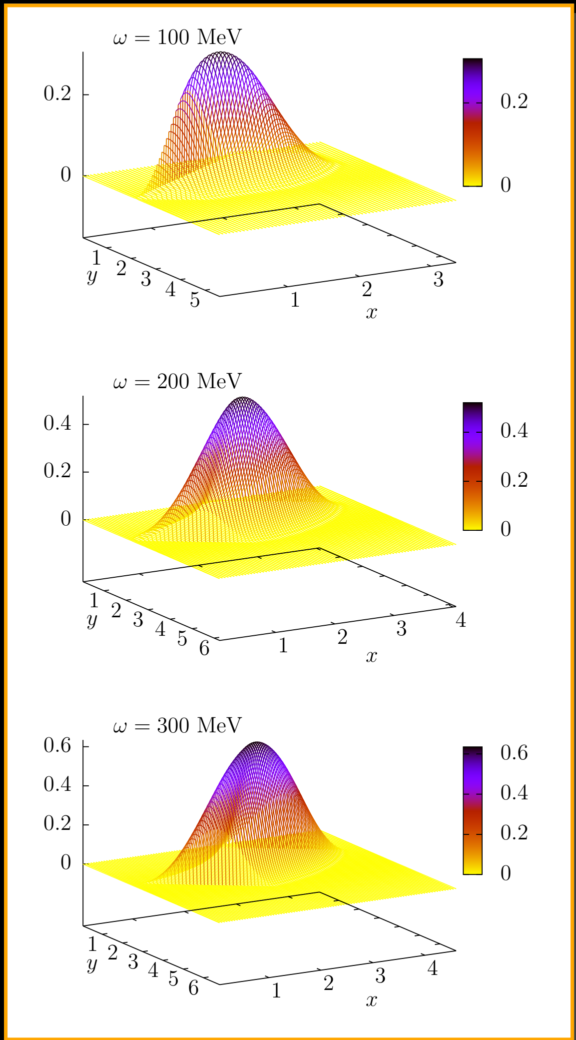
$$F(q,\omega) = (2\pi)^3 \frac{k_F^7 m_N}{q_F} \int_0^{x_{\max}} \frac{dx}{x^2} \int_{|q_F-x|}^{q_F+x} \frac{dy}{y^2} A(x,y,\nu)\,,$$

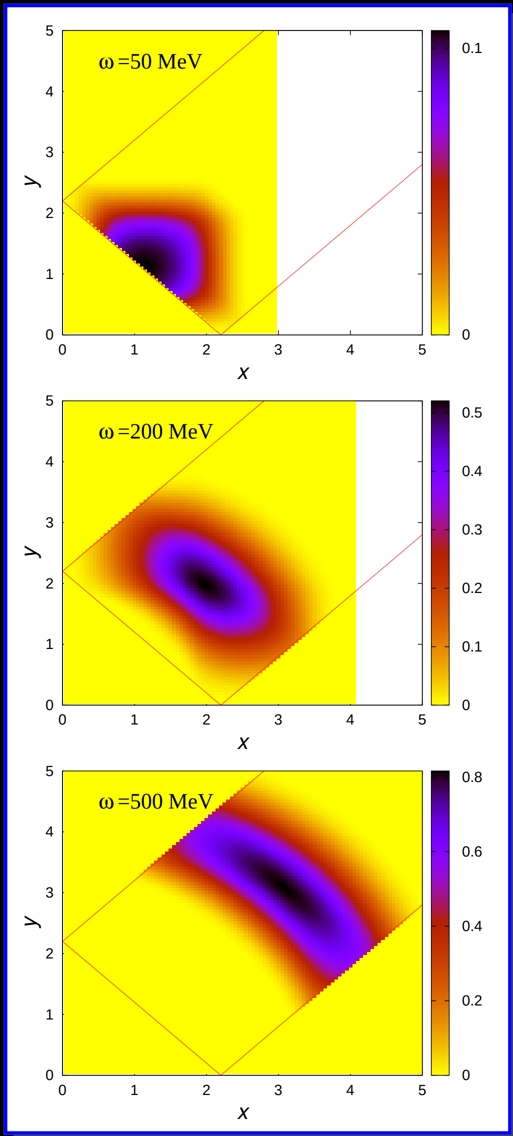
$$x_{\max} = 1+\sqrt{2(1+\nu)}, \qquad \mathbf{q}_F = \frac{\mathbf{q}}{k_F} \qquad \nu = \frac{m_N\omega}{k_F^2}$$

Van Orden-Donnelly function:

$$\begin{aligned} A(l_1,l_2,\nu) &= \frac{l_1^3 l_2^3}{(2\pi)^2} \int d^3x_1 d^3x_2 \delta(\nu - \mathbf{l}_1\cdot\mathbf{x}_1 - \mathbf{l}_2\cdot\mathbf{x}_2) \\ & \theta\left(1-\left|\mathbf{x}_1 - \frac{\mathbf{l}_1}{2}\right|\right)\theta\left(1-\left|\mathbf{x}_2 - \frac{\mathbf{l}_2}{2}\right|\right) \\ & \theta\left(\left|\mathbf{x}_1 + \frac{\mathbf{l}_1}{2}\right|-1\right)\theta\left(\left|\mathbf{x}_2 + \frac{\mathbf{l}_2}{2}\right|-1\right)\,. \end{aligned}$$

Van Orden function $A(x, y, \nu)$





- The function $A(x, y, \nu)$ is analytical (J.W. Van Orden, T.W. Donnelly, Ann. Phys. 131 (1981) 451)
- 3D plot for $q = 500$ MeV/c

Non-relativistic. Numerical

- $\phi_1' = 0$
- Integrate over p_1' for $\mathbf{h}_1$, $\mathbf{h}_2$ and θ_1' fixed.
- Sum over two solutions $p_1'^{(\pm)}$ of the energy conservation equation.
- 7D integral

$$F(q,\omega) = 2\pi \int d^3h_1 d^3h_2 d\cos\theta_1' \sum_{\alpha=\pm} \frac{p_1'^2 m_N}{|p_1' - \mathbf{p}_2' \cdot \hat{\mathbf{p}}_1'|} \Theta(p_1', p_2', h_1, h_2)\Big|_{p_1'=p_1'^{(\alpha)}}$$

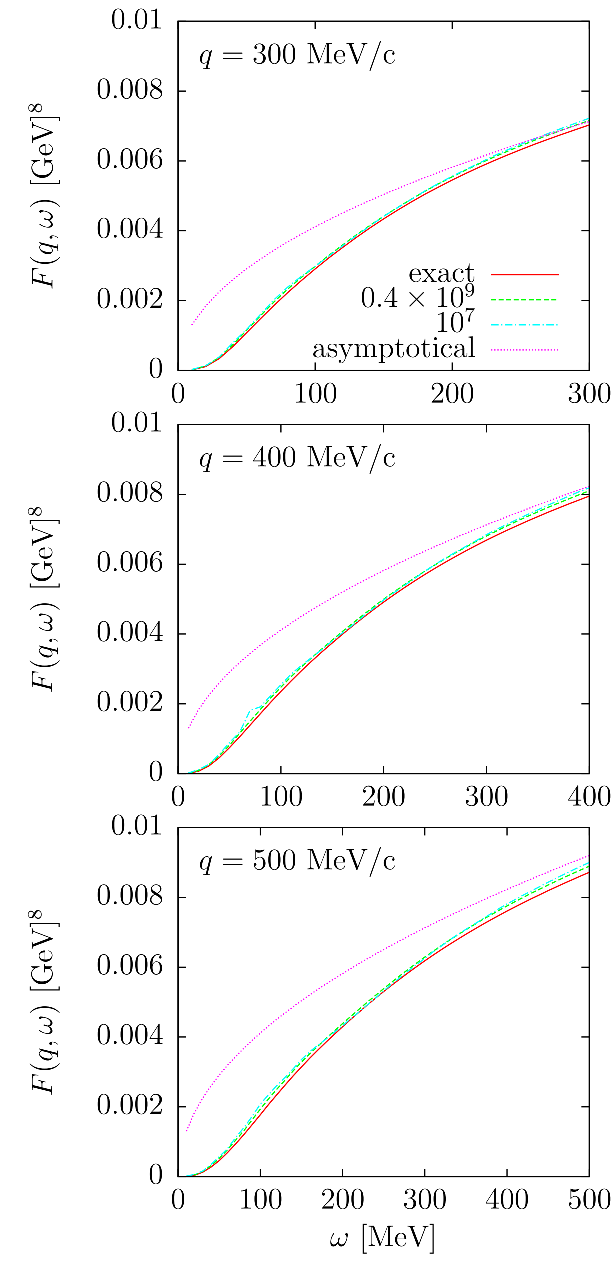
- Asymptotic expansion ($\sim \sqrt{m_N \omega}$)

$$F(q,\omega) \xrightarrow{\omega\to\infty} 4\pi \left(\frac{4}{3}\pi k_F^3\right)^2 \frac{m_N}{2} \sqrt{m_N\omega}\,.$$

Non-relativistic results

Comparison of phase space $F(q,\omega)$ calculations:

- Exact (semi-analytical)
- Numerical 7D integration with 10^7 points
- Asymptotic



Relativistic phase-space

- 7D integral ($\phi'_1 = 0$)

$$F(q,\omega) = 2\pi \int d^3h_1 d^3h_2 d\theta'_1 \sin\theta'_1 \frac{m_N^4}{E_1 E_2 E'_1 E'_2} \times \sum_{\alpha=\pm} \frac{{p'_1}^2}{\left|\frac{p'_1}{E'_1} - \frac{\mathbf{p}'_2 \cdot \hat{\mathbf{p}}'_1}{E'_2}\right|} \Theta(p'_1, p'_2, h_1, h_2)\Big|_{p'_1 = {p'_1}^{(\alpha)}},$$

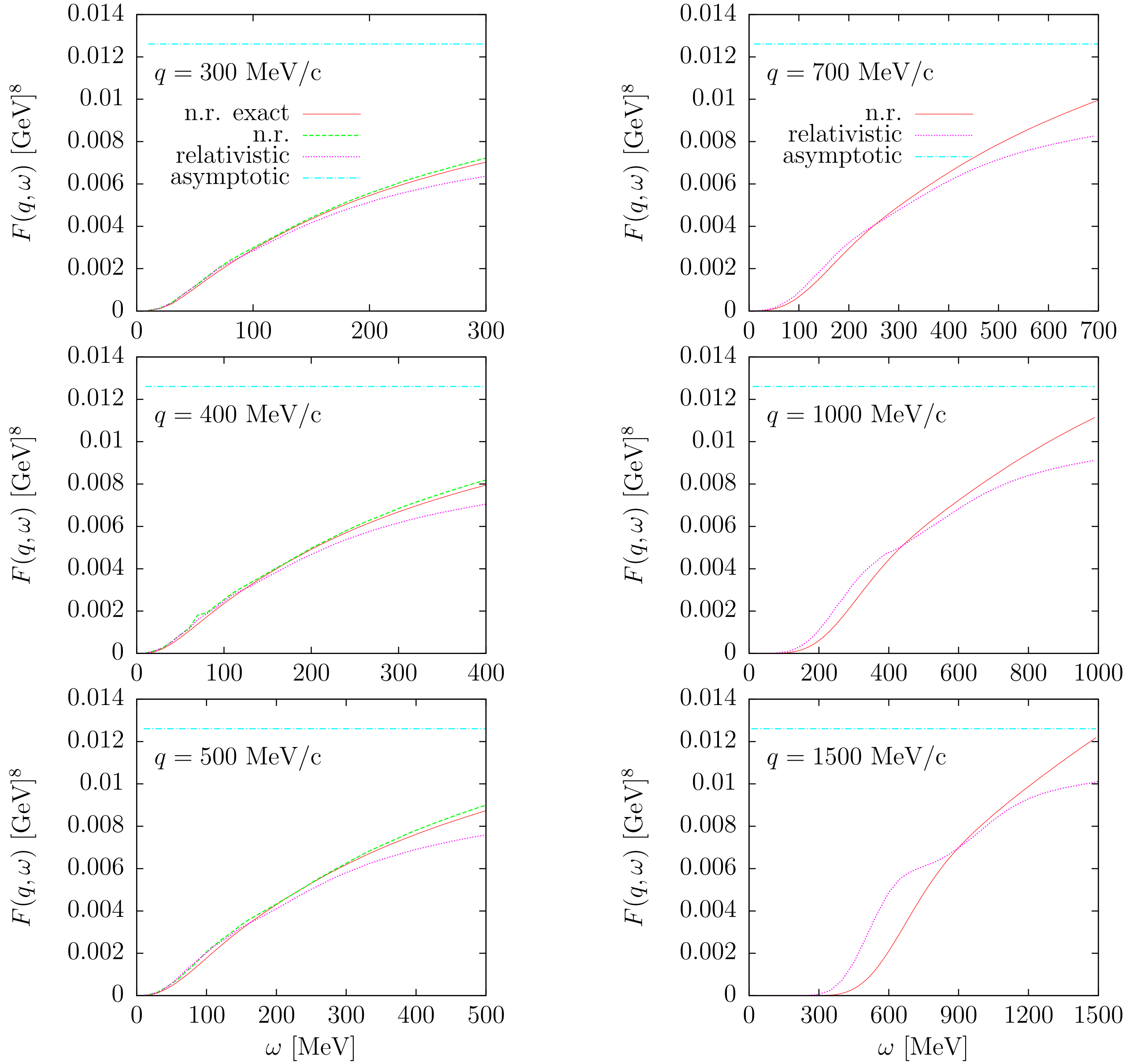
  the sum runs over the two solutions ${p'_1}^{(\pm)}$ of the relativistic energy conservation equation

- Asymptotic expansion $\sim m_N$ (constant!)

$$F(q,\omega) \stackrel{\omega\to\infty}{\longrightarrow} = 4\pi \left(\frac{4}{3}\pi k_F^3\right)^2 \frac{m_N^2}{2}.$$

Relativistic results.

- Straightforward integration.
- For low q and ω the relativistic $F(q,\omega)$ converge to the non relativistic phase space
- Numerical problems for high q



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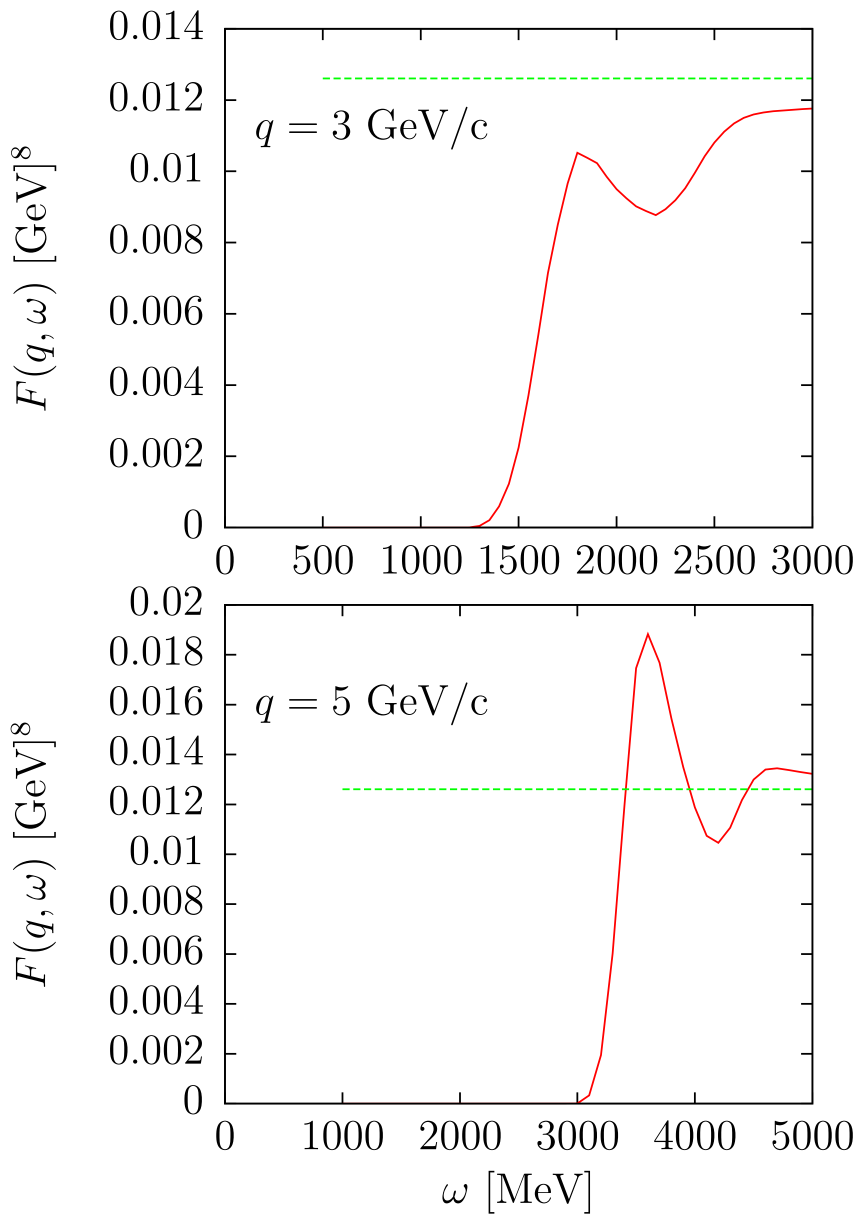
2p-2h problems at high q

- For high q a spurius peak appears at low ω as a result of numerical error in the straightforward 7D integration
- We study the specific case $q = 3$ GeV/c.



Frozen nucleon approximation

- For high $q \gg k_F$, the hole momenta can be neglected, $\mathbf{h}_1 = \mathbf{h}_2 = 0$
- A 6-D integration can be performed analytically

$$\begin{aligned} \overline{F}(q,\omega) &= \int d^3h_1 d^3h_2 d^3p'_1 \delta(E'_1 + E'_2 - \omega - 2m_N)\Theta(p'_1, p'_2, 0, 0)\frac{m_N^2}{E'_1 E'_2} \\ &= \left(\frac{4}{3}\pi k_F^3\right)^2 \int d^3p'_1 \delta(E'_1 + E'_2 - \omega - 2m_N)\Theta(p'_1, p'_2, 0, 0)\frac{m_N^2}{E'_1 E'_2} \,. \end{aligned}$$

- The integral over ϕ'_1 gives a factor 2π, with $\phi'_1 = 0$
- The integral over p'_1 is done analytically
- In the frozen nucleon approximation, the phase-space function is reduced to a 1D integral over θ'_1

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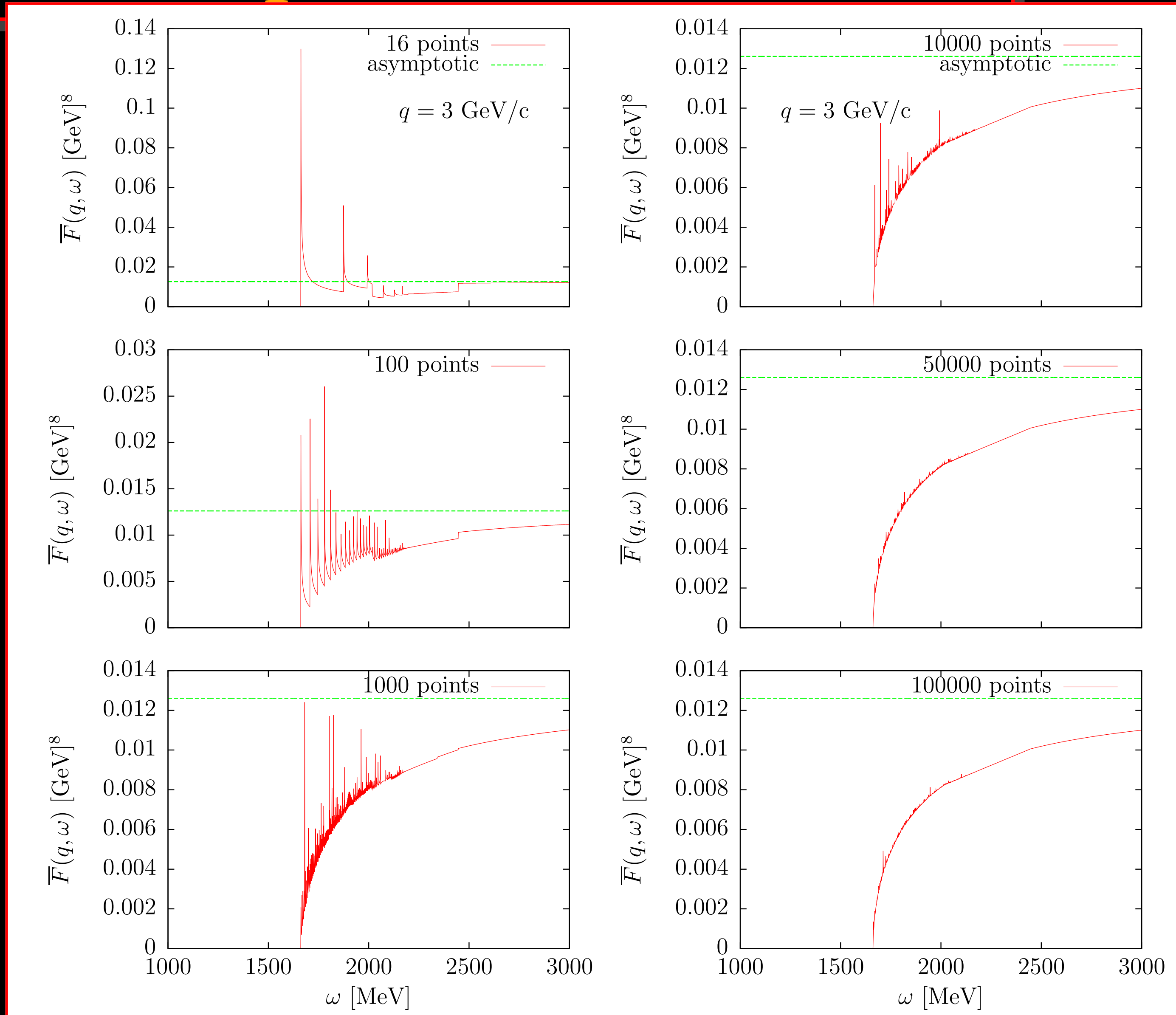
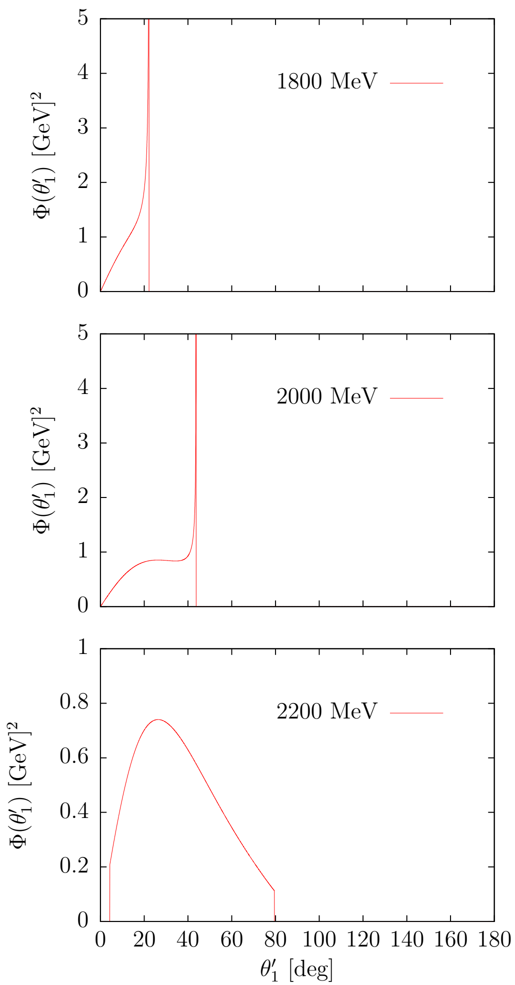
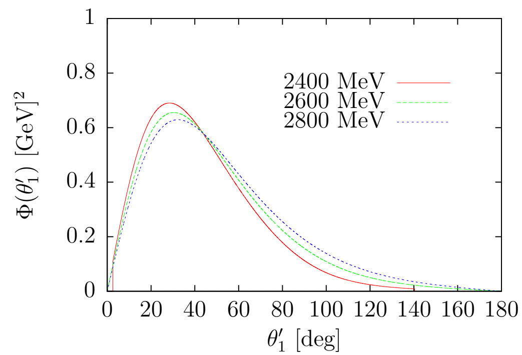
Frozen nucleon problems

- Numerical error shown as discontinuities
- $F(q,\omega)$ includes contributions from different $(\mathbf{h}_1, \mathbf{h}_2)$, with discontinuities at different ω-points
- The statistical distribution of millions of discontinuities appears as a smooth function $F(q,\omega)$ with a bump in the region where there are more discontinuities



Angular distribution of ejected nucleons

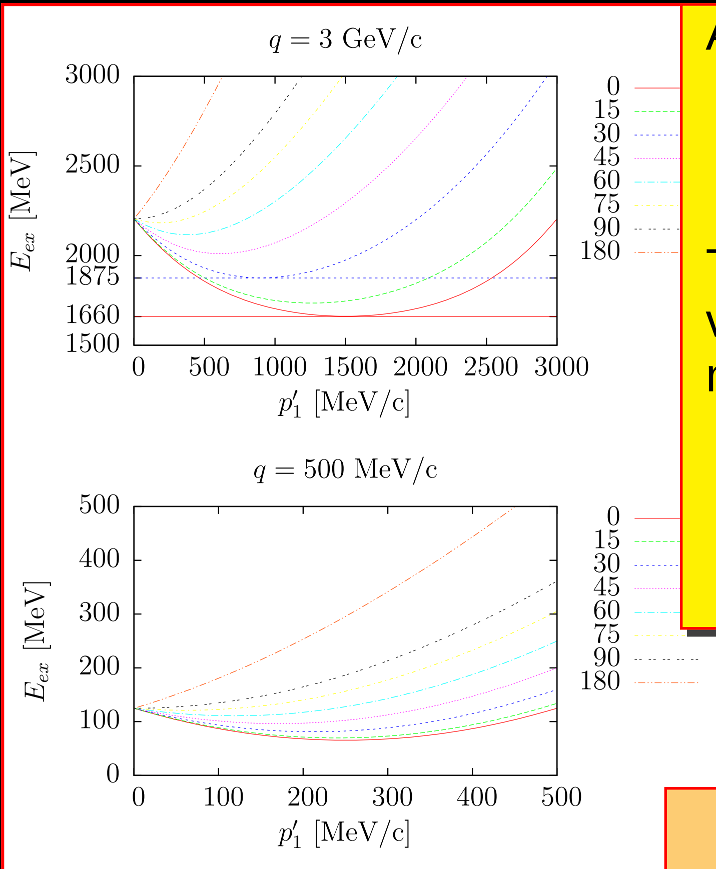




$q = 3$ GeV/c
Quasielastic peak at $\omega = 2200$ MeV
The denominator is zero for some angles for each energy

$$
\begin{aligned}
\Phi(\theta'_1) &= \sin\theta'_1 \int p'^2_1 dp'_1 \delta(E_1 + E_2 + \omega - E'_1 - E'_2) \\
&\quad \times \Theta(p'_1, p'_2, h_1, h_2) \frac{m_N^4}{E_1 E_2 E'_1 E'_2} \\
&= \sum_{\alpha=\pm} \frac{\sin\theta'_1 m_N^4}{E_1 E_2 E'_1 E'_2} \frac{p'^2_1}{\left| \frac{p'_1}{E'_1} - \frac{\mathbf{p}'_2 \cdot \hat{\mathbf{p}}'_1}{E'_2} \right|} \Theta(p'_1, p'_2, h_1, h_2) \Bigg|_{p'_1 = p'_1{}^{(\alpha)}}
\end{aligned}
$$

Kinematical analysis

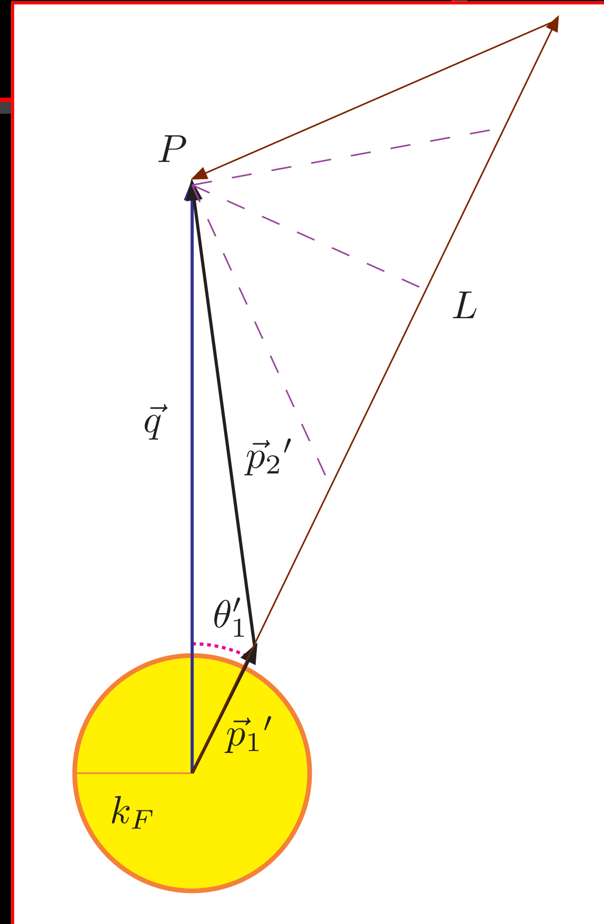


At the minimum

$$\frac{dE_{ex}}{dp'_1} = 0.$$

The Jacobian diverges at the minimum

$$dp'_1 = \frac{dE_{ex}}{\left|\frac{dE_{ex}}{dp'_1}\right|},$$



$$E_{ex} = E'_1 + E'_2 - E_1 - E_2.$$

In the frozen nucleon limit

$$E_{ex} = \sqrt{p'_1{}^2 + m_N^2} + \sqrt{p'_1{}^2 + m_N^2 + q^2 - 2p'_1 q\cos\theta'_1} - 2m_N^2,$$

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Analysis of the angular distribution

General form of integral over angles

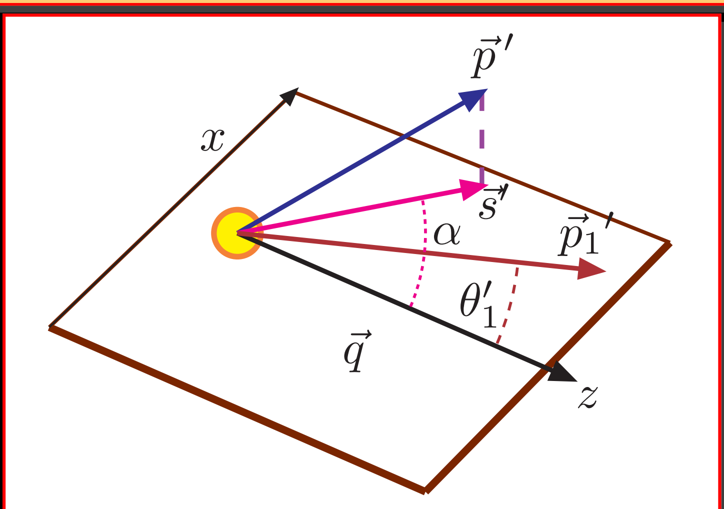
$$I = \int_0^{\pi} d\theta_1' \frac{f(\theta_1')}{\sqrt{g(\theta_1')}} \theta(g(\theta_1'))\,,$$

for positive values of the function

$$g(\theta_1') \equiv \cos^2(\theta_1' - \alpha) - w_0.$$

Non-dimensional variable

$$w_0 = \frac{E'^2}{s'^2}\left(1 - \frac{(E'^2 - p'^2)^2}{4m_N^2 E'^2}\right)$$



The integration region is determined by $g(\theta_1') > 0$.
Three cases depending on w_0:

- $w_0 > 1$. The angular distribuion is zero.
- $w_0 < 0$. All angles allowed
- $0 \le w_0 \le 1$. The angular distribution is different from zero only in one or two angular intervals.
  It is infinite for $\cos^2(\theta_1' - \alpha) = w_0$
  $\Rightarrow \cos(\theta_1' - \alpha) = \pm\sqrt{w_0}$
  Position of the divergence:

$$\theta_1' - \alpha = \varphi_1 \pm \pi, \varphi_2 \pm \pi\,.$$

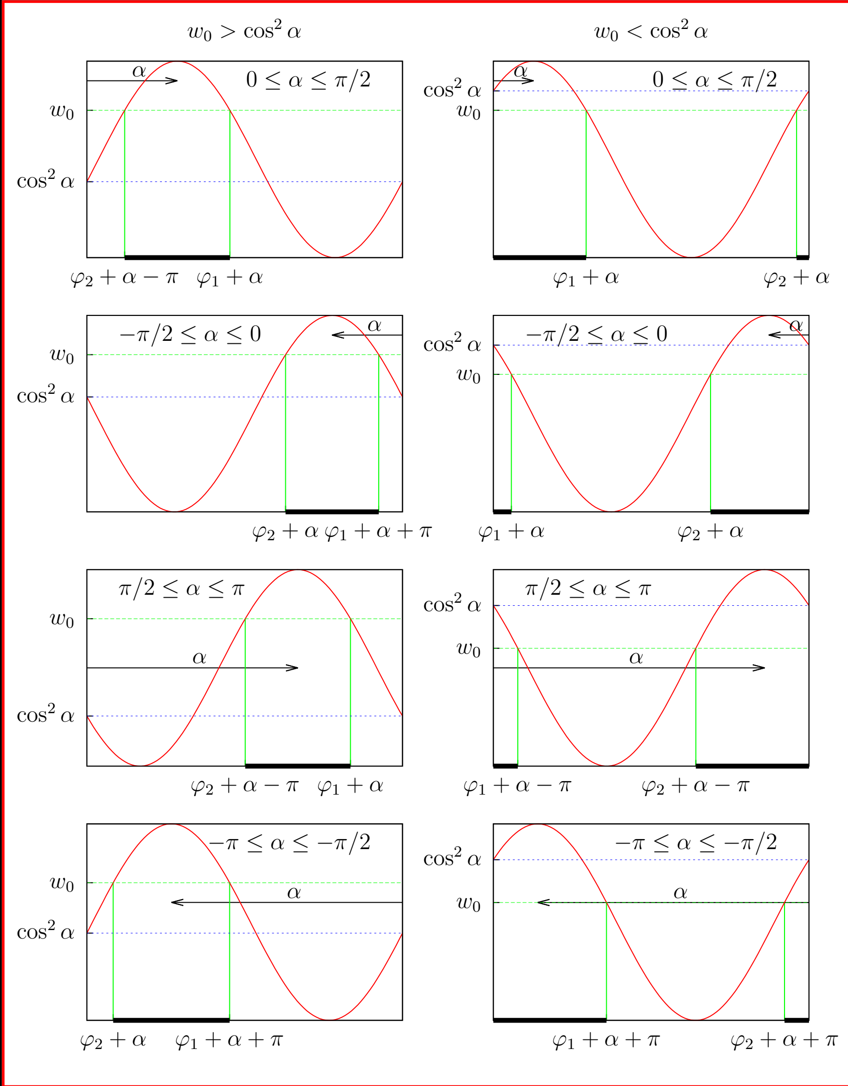
with

$$\varphi_1 \equiv \cos^{-1}\sqrt{w_0}, \qquad \varphi_2 \equiv \cos^{-1}(-\sqrt{w_0}),$$

$0 \le \varphi_1, \varphi_2 < \pi$.

Allowed angular intervals $0 \leq w_0 \leq 1$

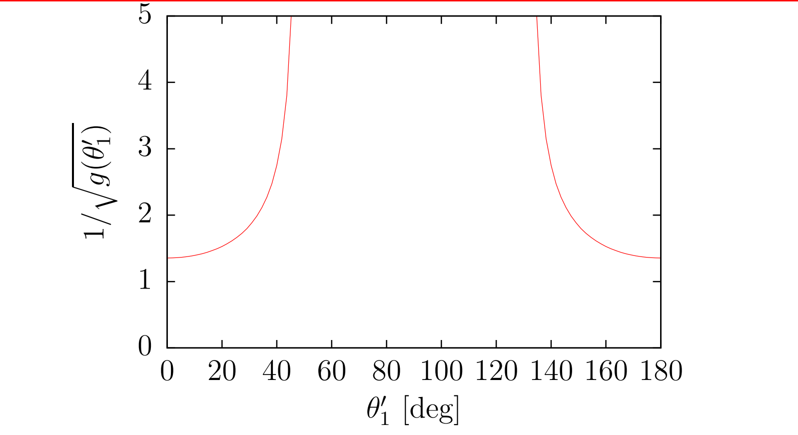
- Exact position of the divergence and the intervals.
- Eight possible cases
- Classified according to the values of α and w_0.



Integration of divergences

- The divergence is integrable.
- Similar to

$$\int_0^{\epsilon} \frac{dx}{\sqrt{x}} = 2\sqrt{x}\Big|_0^{\epsilon} = 2\sqrt{\epsilon}$$



$$\begin{aligned} I(\theta_1,\theta_2) &\equiv \int_{\theta_1}^{\theta_2} \frac{f(\theta)d\theta}{\sqrt{g(\theta)}} \\ &= I(\theta_1,\theta_1+\epsilon) + I(\theta_1+\epsilon,\theta_2-\epsilon) + I(\theta_2-\epsilon,\theta_2)\,. \end{aligned}$$

- Semi-analytical integration

$$\begin{aligned} I(\theta_1,\theta_1+\epsilon) &= \int_{\theta_1}^{\theta_1+\epsilon} \frac{f(\theta)d\theta}{\sqrt{g(\theta)}} \simeq 2\frac{f(\theta_1)}{g'(\theta_1)} \int_{\theta_1}^{\theta_1+\epsilon} \frac{d\sqrt{g(\theta)}}{d\theta} d\theta \\ &= 2\frac{f(\theta_1)}{g'(\theta_1)}\sqrt{g(\theta_1+\epsilon)} \simeq \frac{f(\theta_1)\sqrt{2\epsilon}}{[w_0(1-w_0)]^{1/4}} \end{aligned}$$

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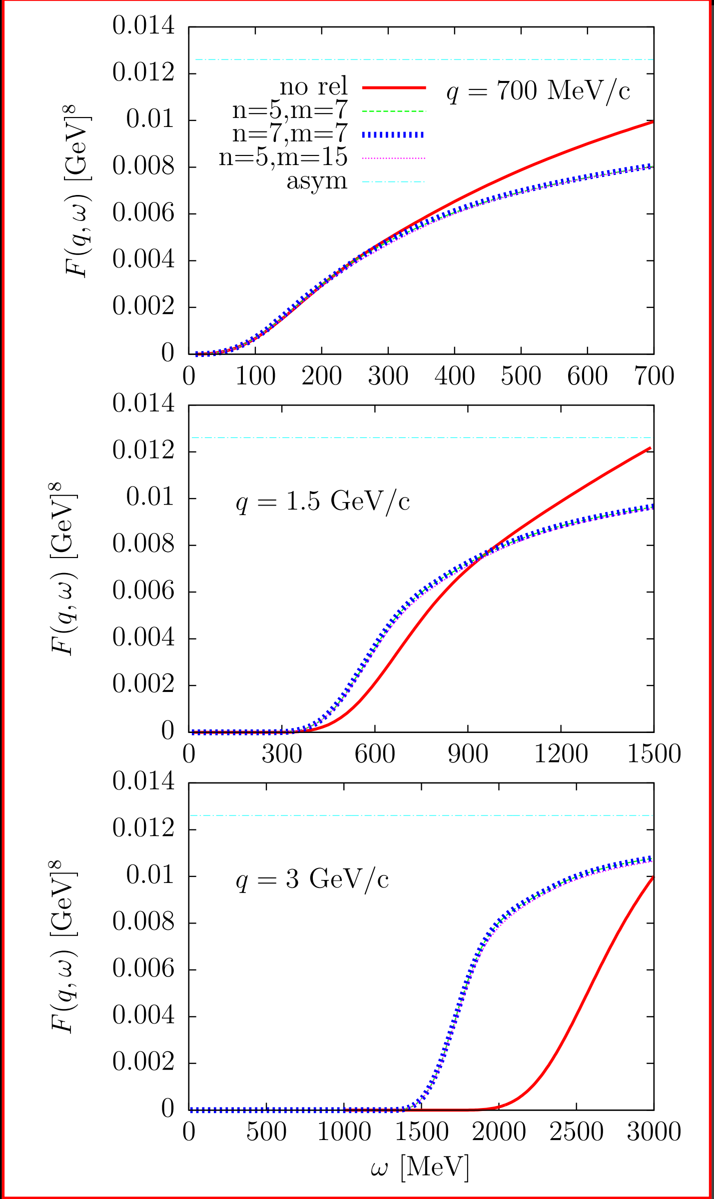
Phase space. New integration method.

- Results do not depend on ϵ
- Number of θ_1' points: $n = 7$
- Number of $\mathbf{h}_1, \mathbf{h}_2$ points: 5^6
- Total number of 7D points: $5^6 \times 7 \simeq 10^5$



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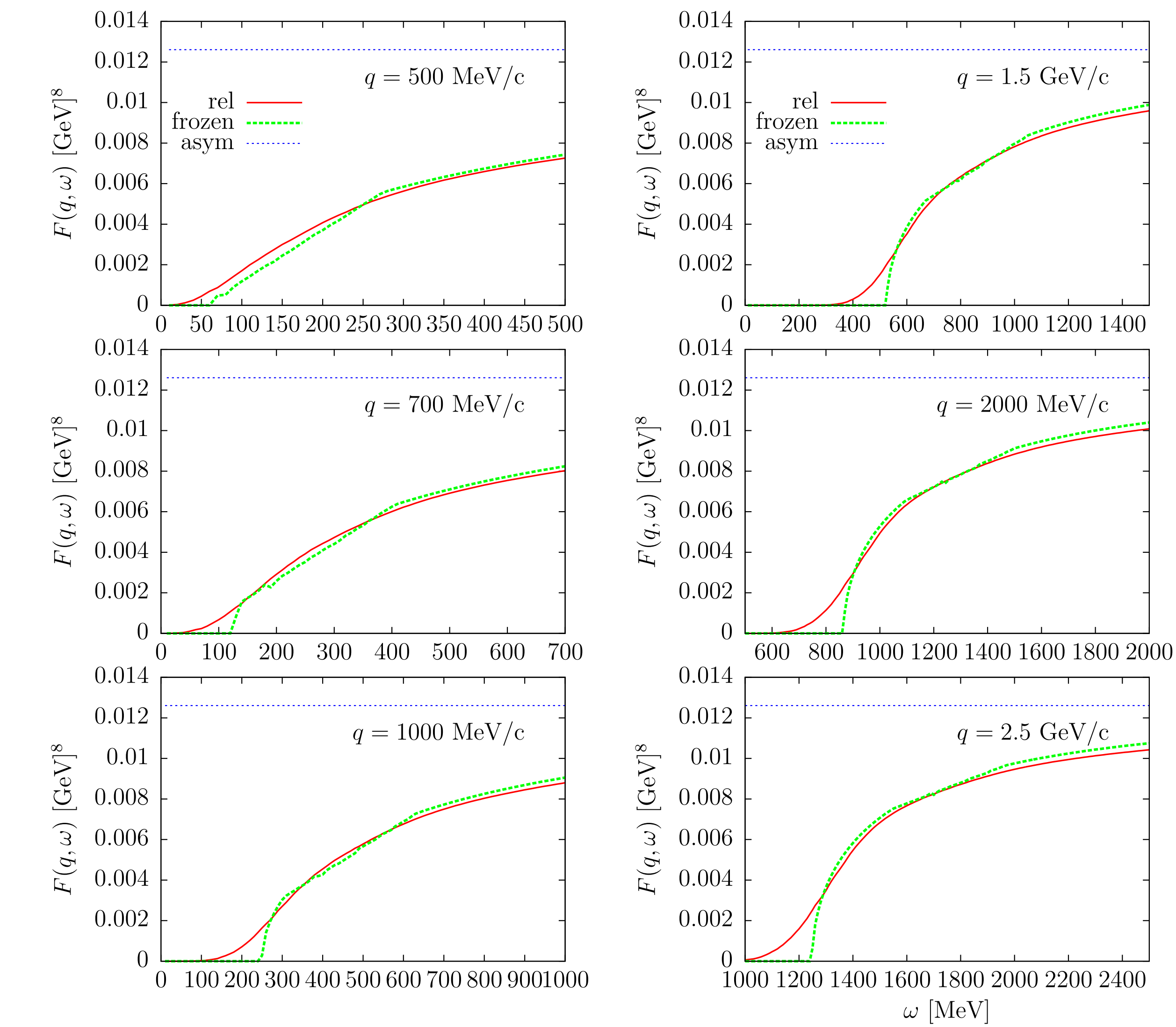
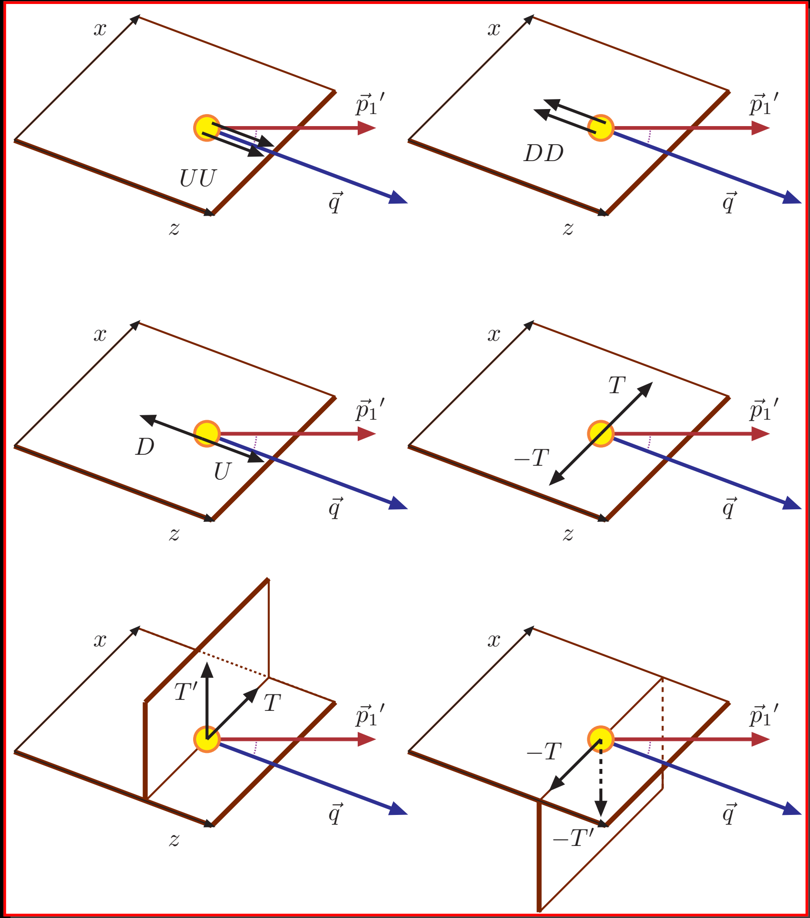
Frozen nucleon approximation

- The frozen approximation is good for moderate to high momentum transfer except for very low energy tail



Average momentum approximation

- The hole momenta $\mathbf{h}_1, \mathbf{h}_2$ are set to a constant inside the integral
- $F(q,\omega)$ is the average over all the $\mathbf{h}_1, \mathbf{h}_2$ configurations.
- Pairs of configurations with opposite total momentum $\mathbf{p} = \mathbf{h}_1 + \mathbf{h}_2$ average to the frozen nucleon approximation.
- Check with parallel, anti-parallel and perpendicular configurations



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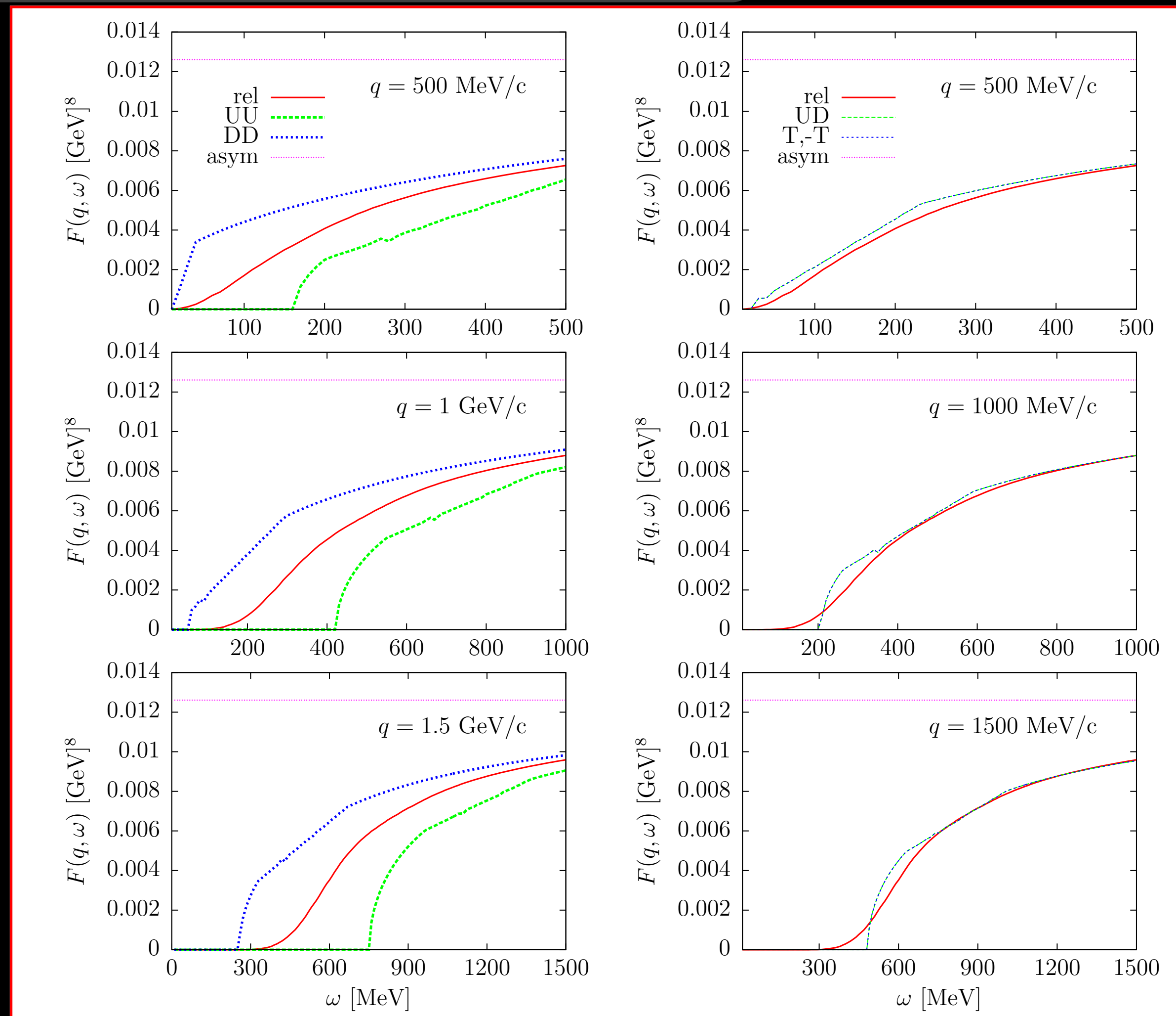
Parallel and anti-parallel configurations

Why the forzen approximation works?

- UU vs. DD average to the frozen approximation
- UD and $T, -T$ are pairs with high relative momentum, like correlated nucleons. They contribute the same as the frozen approximation ecause the total momentum is zero.



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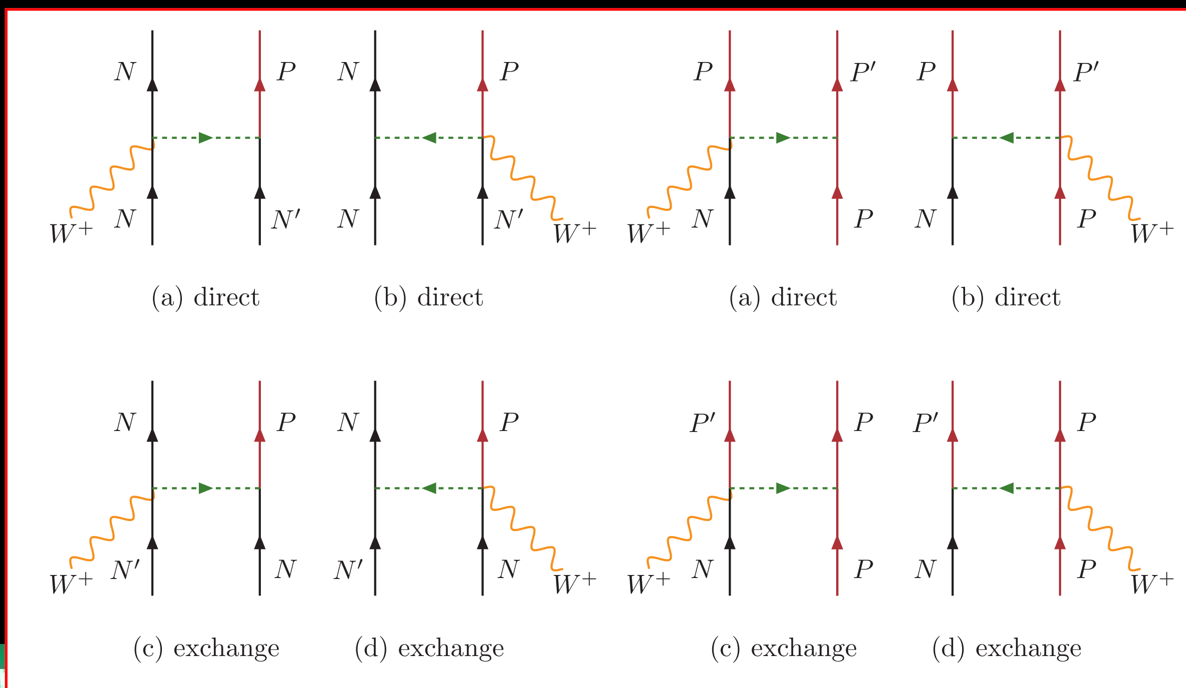
Application to 2p-2h MEC in neutrino reactions

Weak CC Seagull operator

$$j_s^\mu(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2) = [\tau_0 \otimes \tau_{+1} - \tau_{+1} \otimes \tau_0] J^\mu(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2)\,,$$

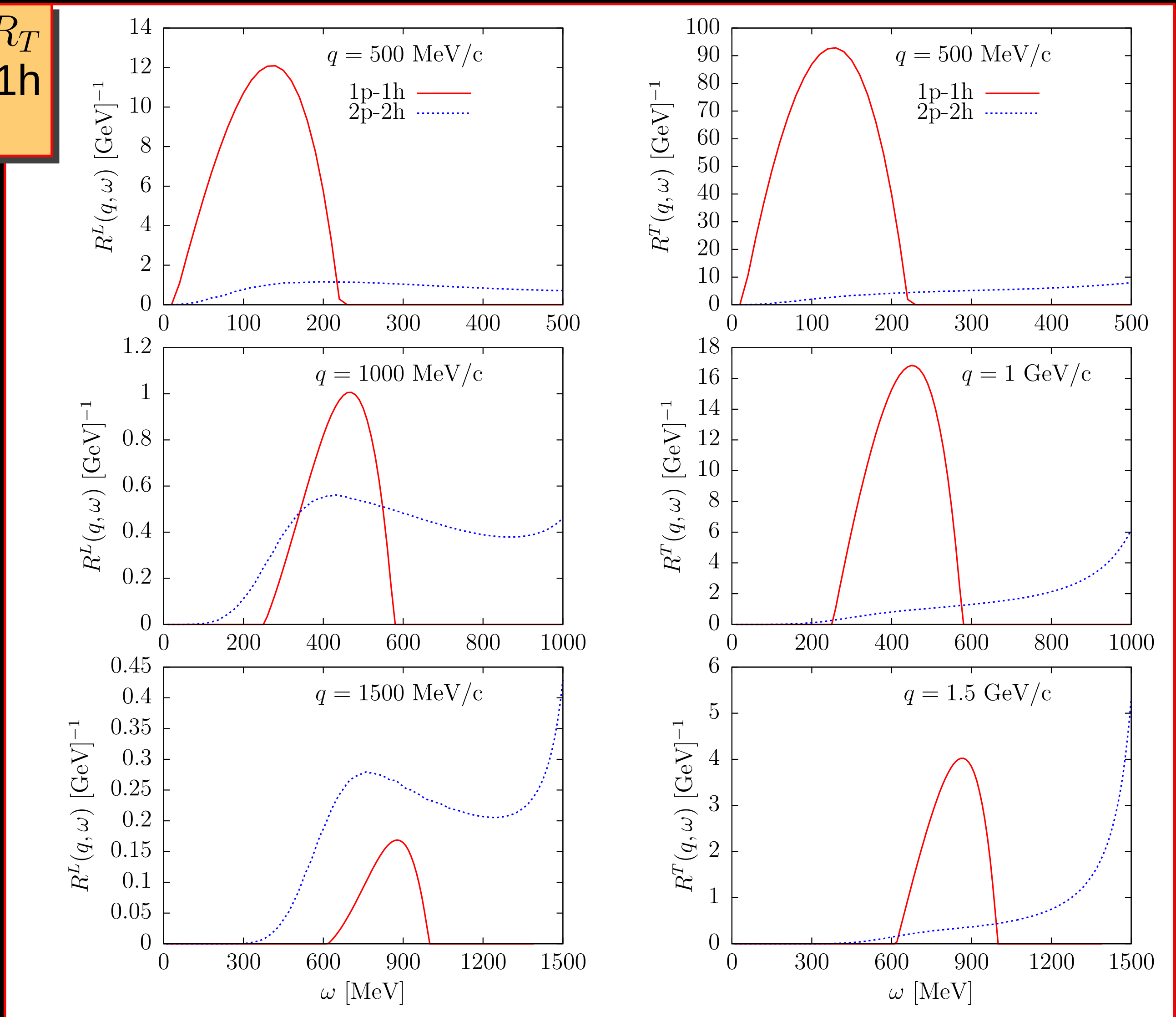
with vector and axial currents

$$J^\mu(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2) = \frac{f}{m_\pi}\frac{1}{\sqrt{2}f_\pi}\overline{u}(\mathbf{p}'_1)\gamma_5\, \not{K}_1 u(\mathbf{h}_1)\frac{\overline{u}(\mathbf{p}'_2)\left[g_A F_1^V(Q^2)\gamma_5\gamma^\mu + F_\rho(K_2^2)\gamma^\mu\right] u(\mathbf{h}_2)}{K_1^2 - m_\pi^2} - (1 \leftrightarrow 2)$$



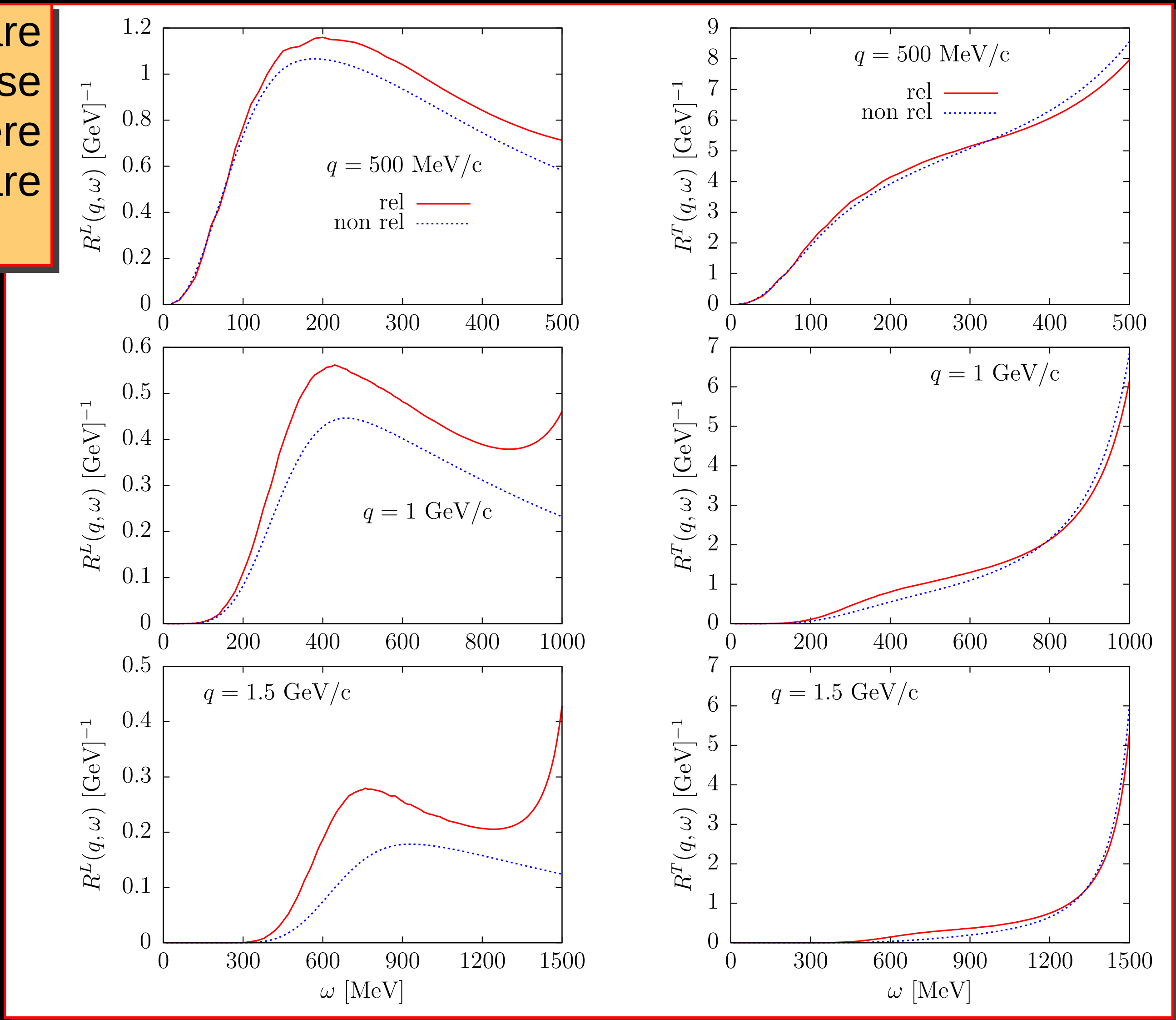
CC response functions

Fully relativistic R_L and R_T compared to the OB 1p-1h RFG



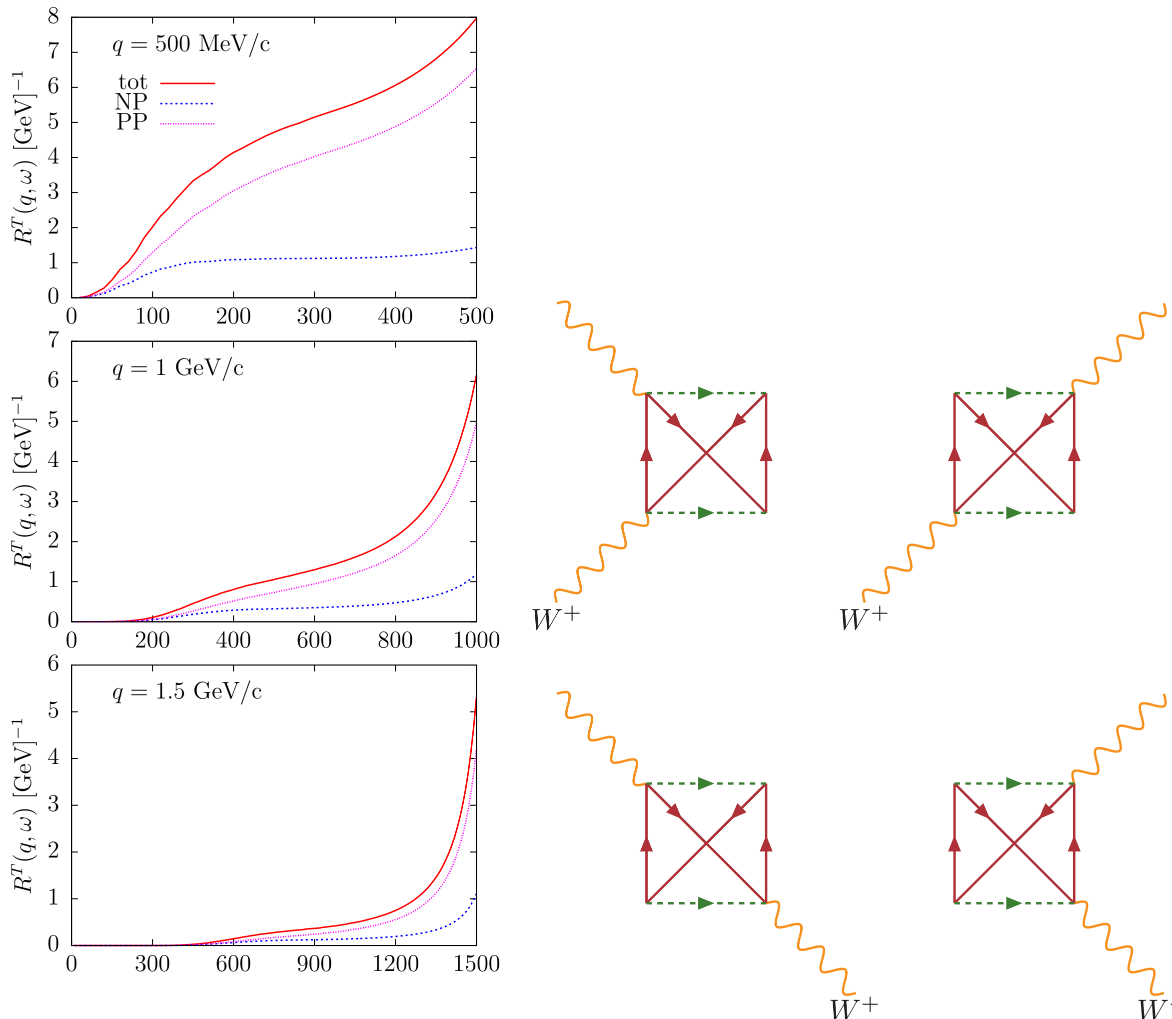
Relativistic effects

Relativistic effects are small in R_T because $F_1^V(Q^2)$ is small where relativistic effects are large.



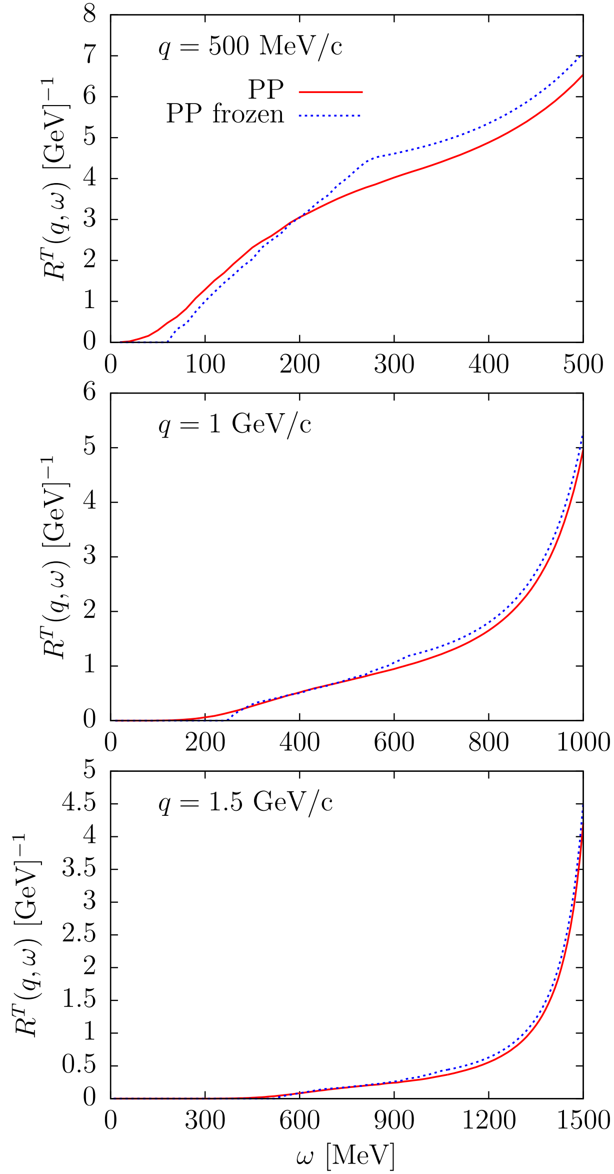
Isospin channels

- PP emission dominates over PN
- The difference is due to the interference direct-exchange (D-X)
- D-X interference diagrams are NOT negligible



Frozen nucleon approximation

- The frozen approximation is good for the seagull current and for the (D-X) interference diagrams.



Summary

- Optimization of the 7D integral in 2p-2h response functions
- Phase space function $F(q, \omega)$
- Angular distribution in the frozen approximation has divergencies for some angles
- Found the allowed angular regions and integrate analitycally around the divergencies
- CPU time reduced by 100
- Relativistic results converge to the non relativistic ones
- Test for electron and neutrino reactions with the contact operator.
- Frozen approximation (1D) very close to the exact (7D) results.
- We are working in the implementation of a complete set of MEC operators, including the axial part.

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THANK YOU