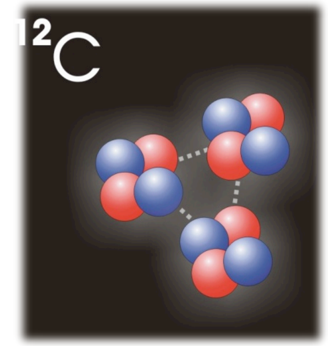

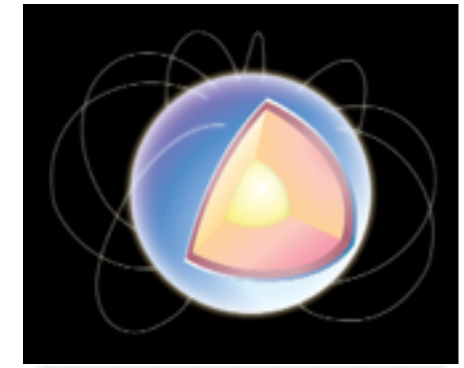

Electroweak response functions of Carbon-12

Alessandro Lovato

In collaboration with:
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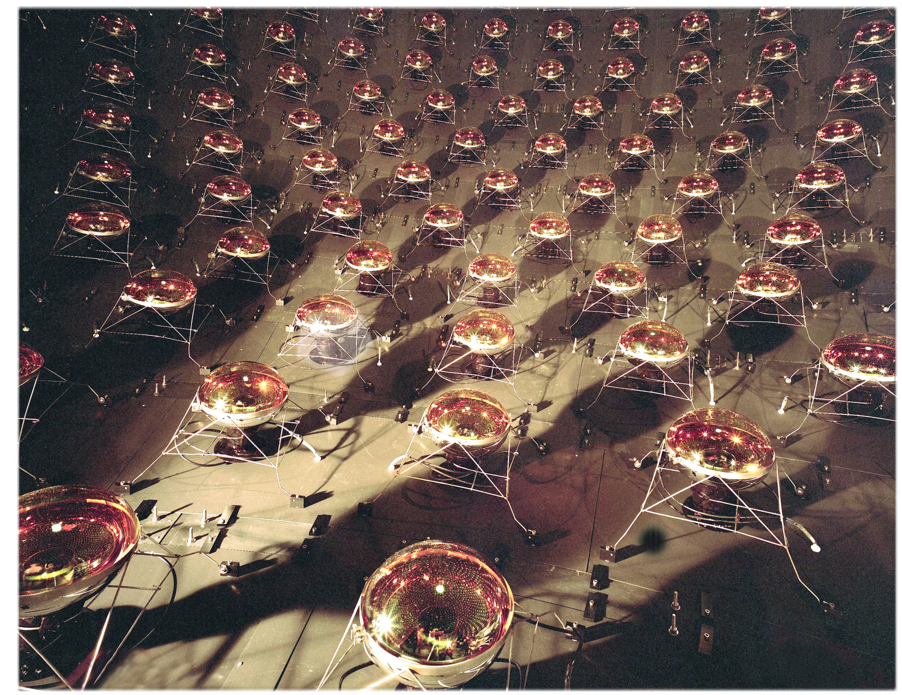
Argonne NATIONAL LABORATORY

Introduction

- The electroweak response is a fundamental ingredient to describe the neutrino - ^{12}C scattering

  Excess, at relatively low energy, of measured cross section relative to theoretical calculations.



- We have first computed the sum rules for the electromagnetic response of ^{12}C. We want to predict the results of Jefferson lab experiment.

  A model unable to describe electron-nucleus scattering is unlikely to describe neutrino-nucleus scattering.

Electromagnetic response

The electromagnetic <u>inclusive</u> cross section of the process

$$e +^{12}\mathrm{C} \to e' + X$$

where the target final state is <u>undetected</u>, can be written as

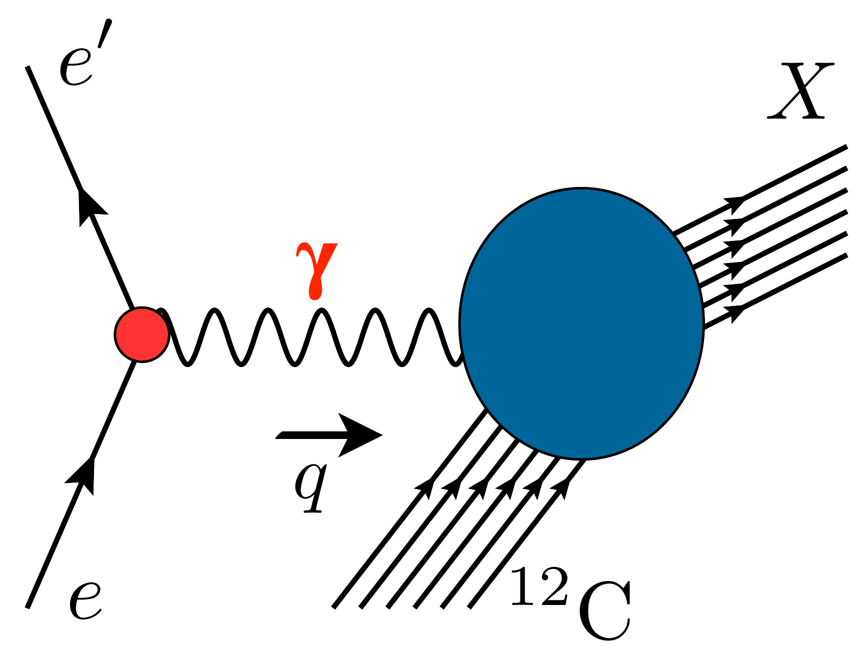
$$\frac{d^2\sigma}{d\Omega_{e'} dE_{e'}} = -\frac{\alpha^2}{q^4}\frac{E_{e'}}{E_e} L_{\mu\nu}^{EM} W_{EM}^{\mu\nu} \ ,$$

The leptonic tensor is fully specified by the measured electron kinematic variables

$$L_{\mu\nu}^{EM} = 2[k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}(kk')]$$

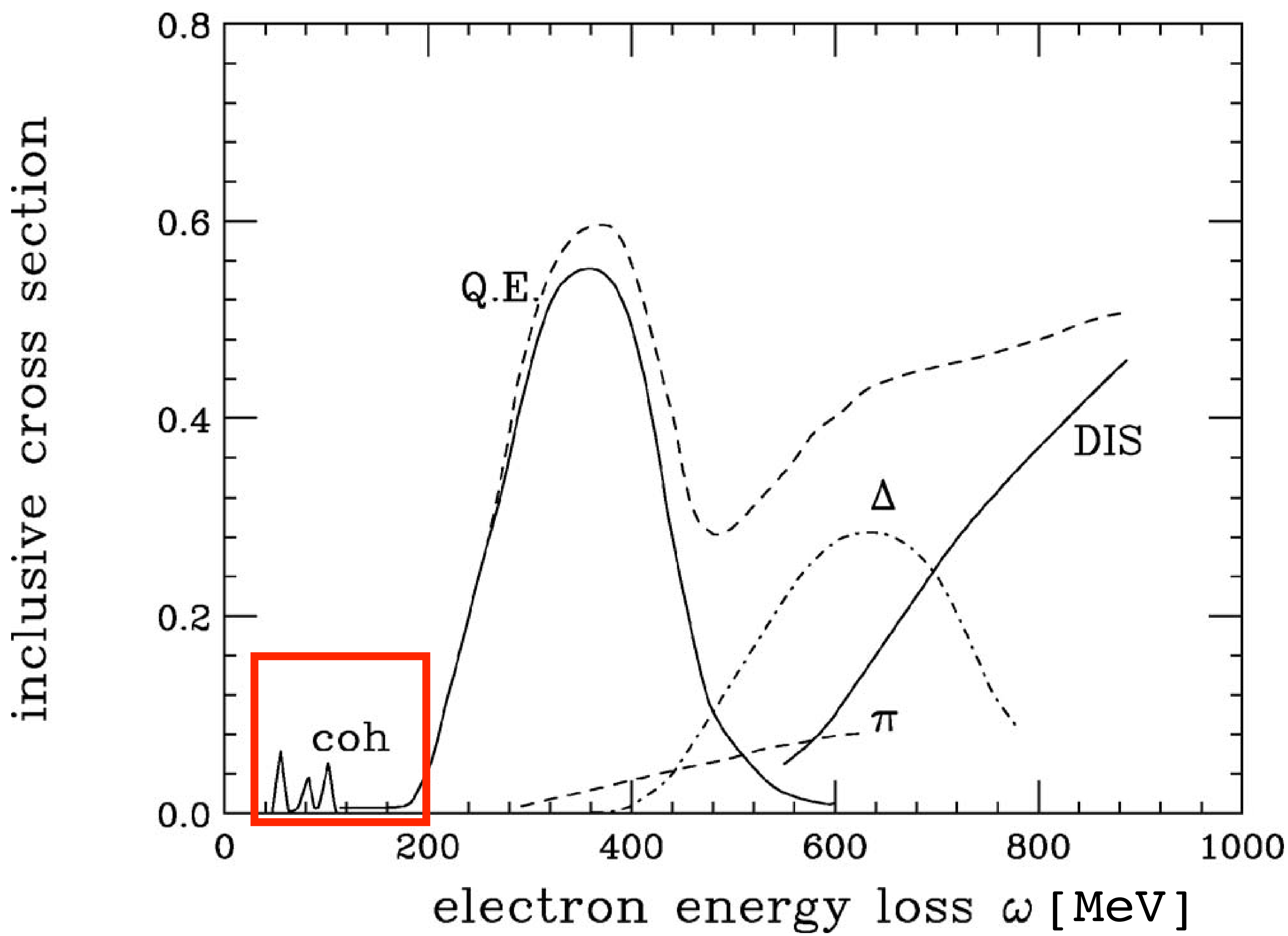
The Hadronic tensor contains all the information on target structure.

$$W_{EM}^{\mu\nu} = \sum_X \langle\Psi_0|J_{EM}^{\mu\,\dagger}|\Psi_X\rangle\langle\Psi_X|J_{EM}^{\nu}|\Psi_0\rangle\delta^{(4)}(p_0 + q - p_X)$$



Electromagnetic response

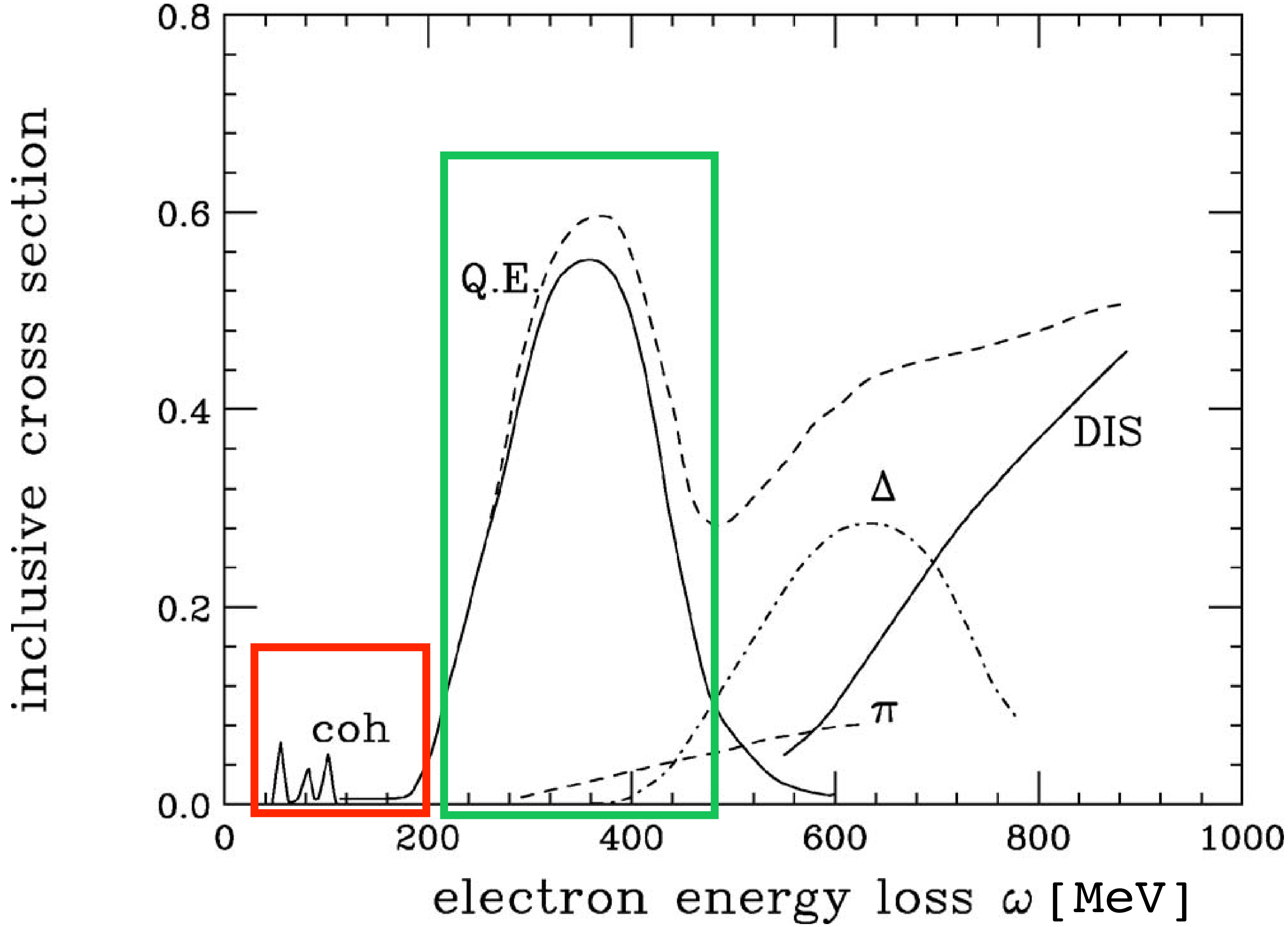
<u>Schematic</u> representation of the inclusive cross section as a function of the energy loss.



- Elastic scattering and inelastic excitation of discrete nuclear states

Electromagnetic response

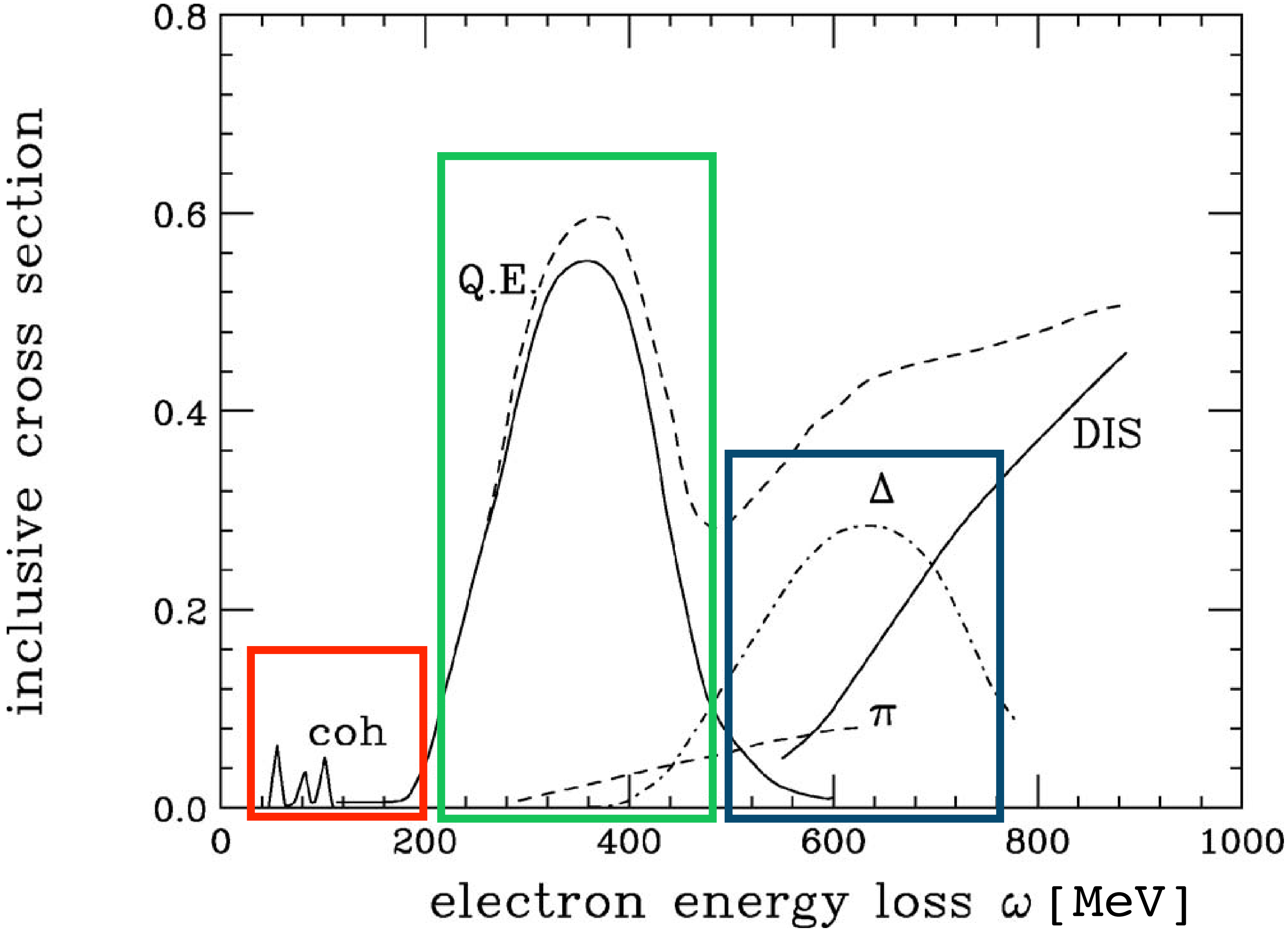
Schematic representation of the inclusive cross section as a function of the energy loss.



- Elastic scattering and inelastic excitation of discrete nuclear states.
- Broad peak due to quasi-elastic electron-nucleon scattering.

Electromagnetic response

<u>Schematic</u> representation of the inclusive cross section as a function of the energy loss.



- Elastic scattering and inelastic excitation of discrete nuclear states.
- Broad peak due to quasi-elastic electron-nucleon scattering.
- Excitation of the nucleon to distinct resonances (like the Δ) and pion production.

Electromagnetic response

- At moderate momentum transfer, the hadronic tensor (and the cross section) can be written in terms of the longitudinal and transverse response functions

Longitudinal $$R_L(q,\omega) = \sum_X \langle\Psi_0|\rho^\dagger|\Psi_X\rangle\langle\Psi_X|\rho|\Psi_0\rangle\delta(E_0+\omega-E_X)$$

Transverse $$R_T(q,\omega) = \sum_X \langle\Psi_0|\vec{j}_T^\dagger|\Psi_X\rangle\langle\Psi_X|\vec{j}_T|\Psi_0\rangle\delta(E_0+\omega-E_X)$$

- An expansion of the current operator in powers of $|\mathbf{q}|/m$ has been performed.

- Realistic models for the electromagnetic charge and current operators include one- and two-body terms, the latter assumed to be due to exchanges of effective pseudo-scalar and vector mesons.

$$\rho = \rho_{1b} + \rho_{2b} \qquad \vec{j} = \vec{j}_{1b} + \vec{j}_{2b}$$

Electromagnetic sum rules

- The direct calculation of the response requires the knowledge of all the transition amplitudes: $\langle \Psi_X | \rho | \Psi_0 \rangle$ and $\langle \Psi_X | \vec{j} | \Psi_0 \rangle$.

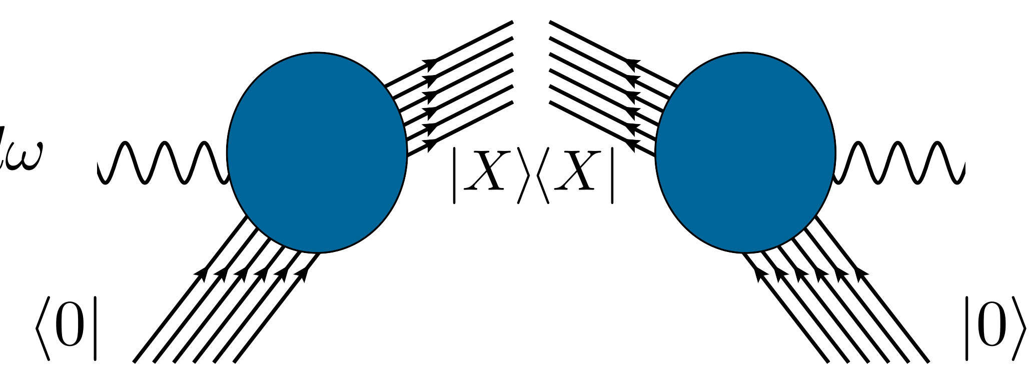
- The <u>sum rules</u> provide an useful tool for studying integral properties of the electron-nucleus scattering.

$$S_\alpha(q) = C_\alpha \int_{\omega_{\rm th}^+}^{\infty} {\rm d}\omega \, \frac{R_\alpha(q,\omega)}{G_E^{p\,2}(Q^2)}$$

→ Proton electric form factor

- Using the completeness relation, they can be expressed as <u>ground-state expectation values of the charge and current operators</u>.

$$S_\alpha(q) = \sum_X \int d\omega$$



Longitudinal and transverse sum rules.

Longitudinal sum rule

$$S_L(\mathbf{q}) = C_L \Big[\frac{1}{G_E^{p2}(Q^2_{qe})} \langle 0|\rho^\dagger(\mathbf{q})\rho(\mathbf{q})|0\rangle - \frac{1}{G_E^{p2}(Q^2_{el})} |\langle 0;\mathbf{q}|\rho(\mathbf{q})|0\rangle|^2 \Big] \quad ; \quad C_L = \frac{1}{Z}$$

The elastic contribution, proportional to the longitudinal form factor has been removed.

$$F_L(\mathbf{q}) = C_L \langle 0;\mathbf{q}|\rho(\mathbf{q})|0\rangle$$

Transverse sum rule

$$S_T(\mathbf{q}) = \frac{C_T}{G_E^p(Q^2_{qe})} \langle 0|\vec{j}_T^{\,\dagger}(\mathbf{q})\vec{j}_T(\mathbf{q})|0\rangle \quad ; \quad C_T = \frac{2}{(Z\,\mu_p^2 + N\,\mu_n^2)} \frac{m^2}{q^2}$$

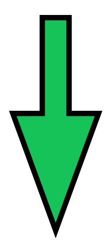
- C_L and C_T have been introduced in order for $S_\alpha(q \to \infty) \to 1$ in the limit where nuclear charge and current operators originate from the charge and spin magnetization of individual protons and neutrons and relativistic corrections are ignored.

Comparison with experiment

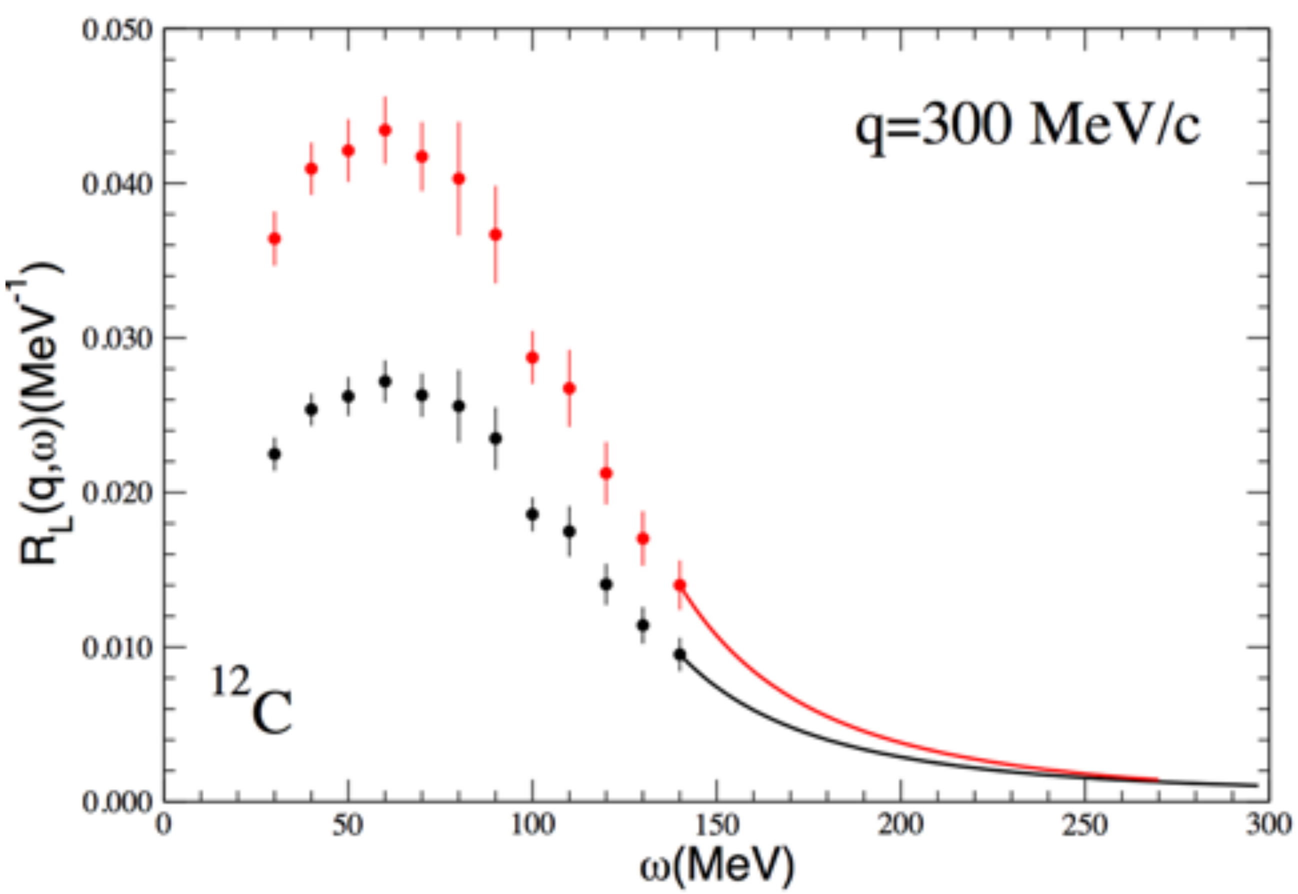
Direct comparison between the calculated and experimentally extracted sum rules cannot be made unambiguously for two reasons

- The experimental determination of S_α requires measuring the associated R_α in the whole energy-transfer region, from threshold up to ∞.

Inclusive electron scattering experiments only allow access to the region where $\omega < q$



Extrapolation needed



- Inadequacy of the dynamical framework to account for explicit pion production mechanisms.

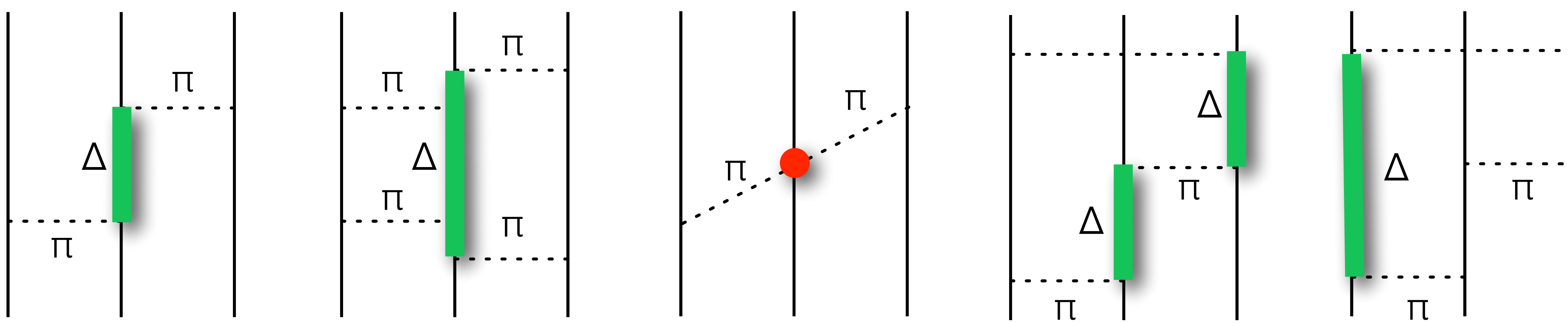
Ab-initio few-nucleon calculation

- The density and current operators have to be consistent with the realistic nucleon-nucleon (NN) interaction.

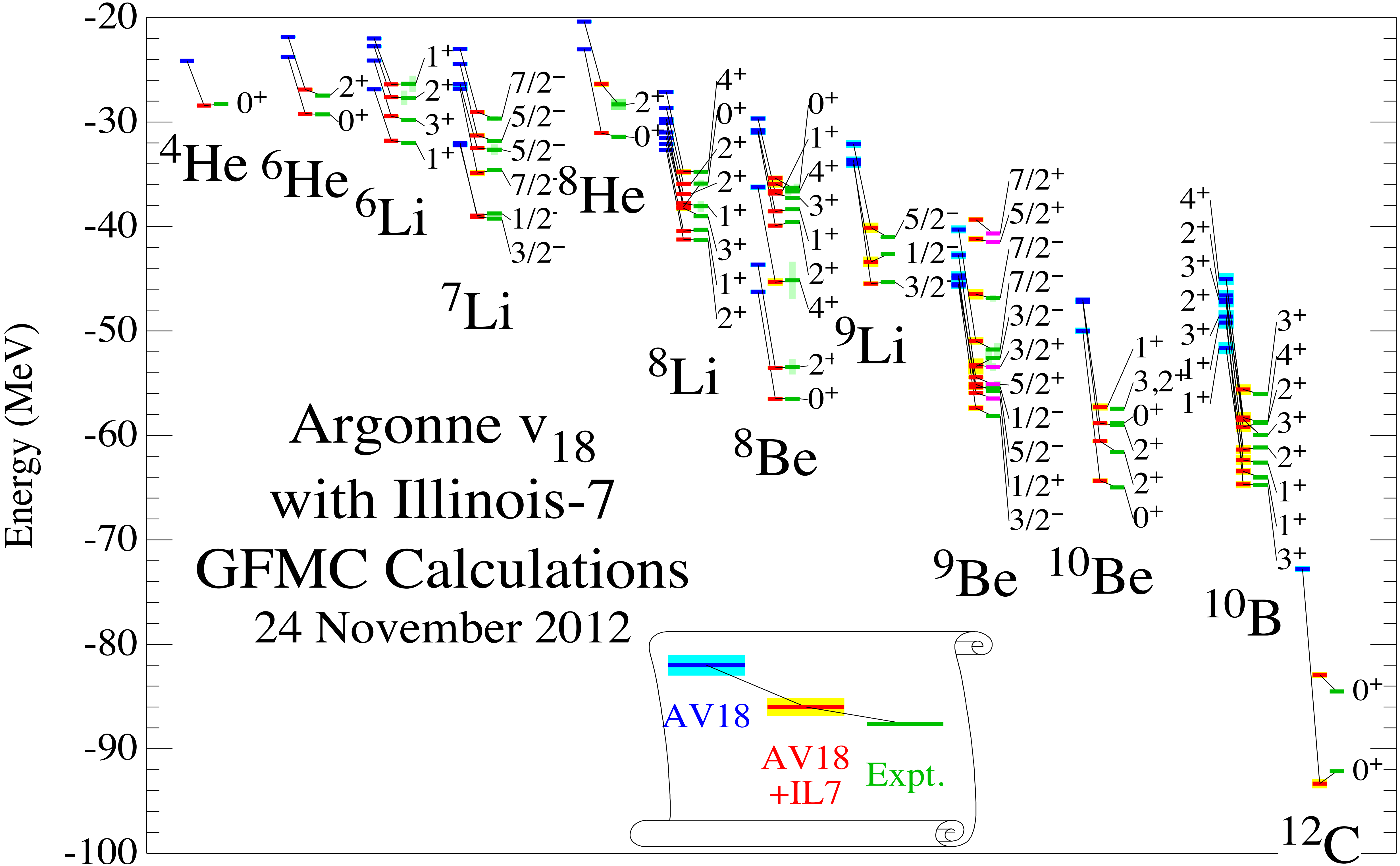
Argonne v18 : $$v_{18}(r_{12}) = \sum_{p=1}^{18} v^p(r_{12})\hat{O}^p_{12}$$

is controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database.

- To compute the sum rules and the longitudinal form factor, the ground state wave function of ^{12}C needs to be precisely known. The accurate **Illinois 7** three body potential has been included in the hamiltonian



Ab-initio few-nucleon calculation



Green's Function Monte Carlo

GFMC algorithms use projection techniques to enhance the ground-state component of a starting trial wave function

$$\Psi_0(x_1 \ldots x_A) = \lim_{\tau\to\infty} e^{-(\hat{H}-E_0)\tau}\Psi_T(x_1 \ldots x_A)$$

The trial wave-function can be expanded on the complete set of eigenstates of the the hamiltonian

$$\Psi_T(x_1 \ldots x_A) = \sum_n c_n \Psi_n(x_1 \ldots x_A)$$

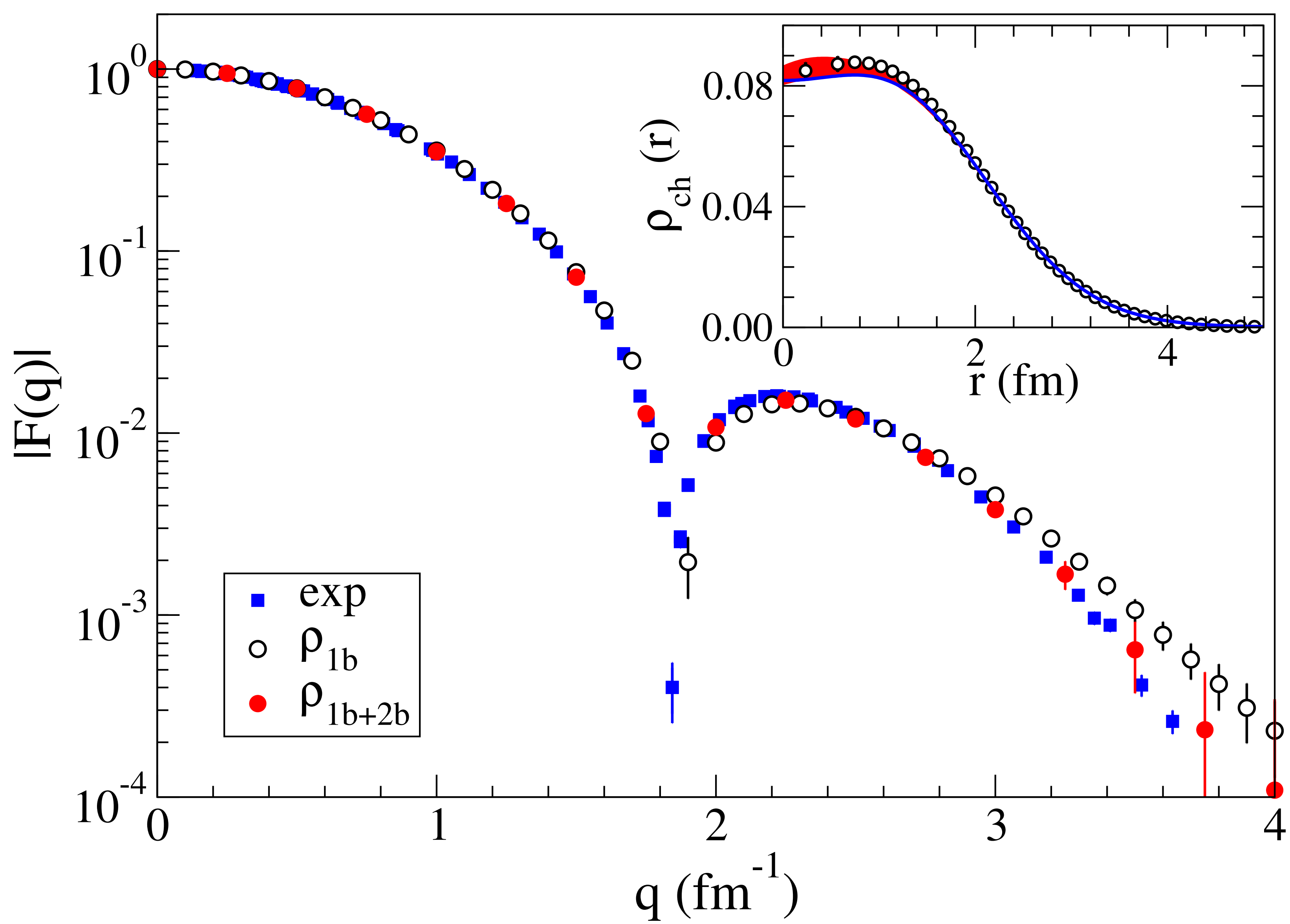
The evolution in imaginary time projects out the ground state from a trial wave function, provided that it is not orthogonal to the ground state

$$\lim_{\tau\to\infty} e^{-(\hat{H}-E_0)\tau}\Psi_T(x_1 \ldots x_A) = \sum_n c_n e^{-(E_n-E_0)\tau}\Psi_n(x_1 \ldots x_A)$$

Results - Longitudinal form factor

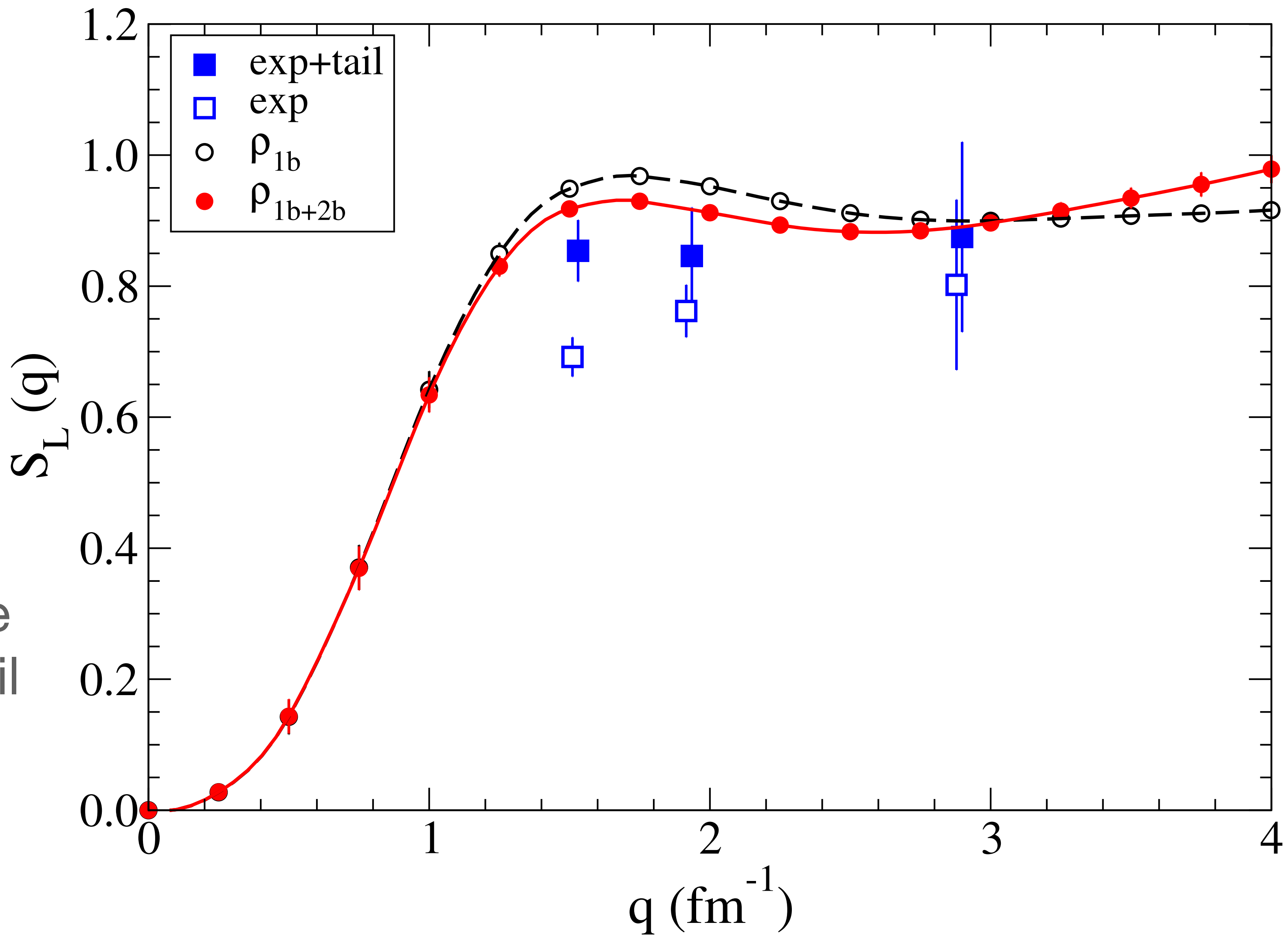
- Experimental data are well reproduced by theory over the whole range of momentum transfers;

- Two-body terms become appreciable only for $q > 2\text{fm}^{-1}$, where they interfere destructively with the one-body contributions bringing theory into closer agreement with experiment.



Results - Longitudinal sum rule

- S_L vanishes quadratically at small momentum transfer.
- The one-body sum rule in the large q limit differs from unity because of relativistic correction and convection term.
- Satisfactory agreement with the experimental values, including tail contributions.
- <u>No significant quenching of longitudinal strength is observed.</u>

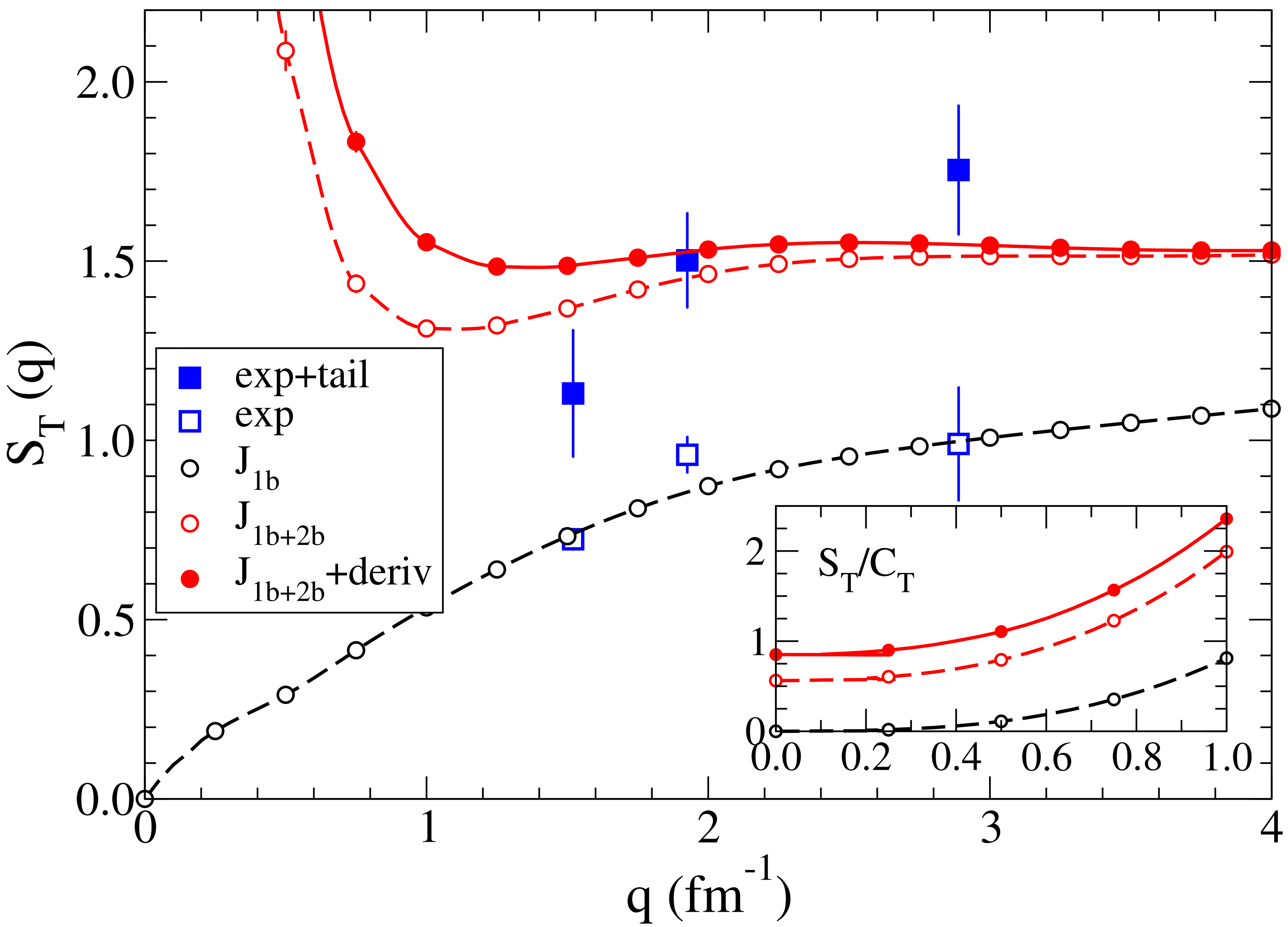


Results - Transverse sum rule

- Large two-body contribution, most likely from the quasi-elastic region, needed for a better agreement with experimental data.

- Divergent behavior at small q due to the normalization factor C_T.

- Comparison with experimental data made difficult by the Δ peak.



Neutral-current response

The neutral current inclusive cross section of the process

$$\nu_\ell + A \to \nu_{\ell'} + X$$

where the target final state is undetected, can be written in the Born approximation as

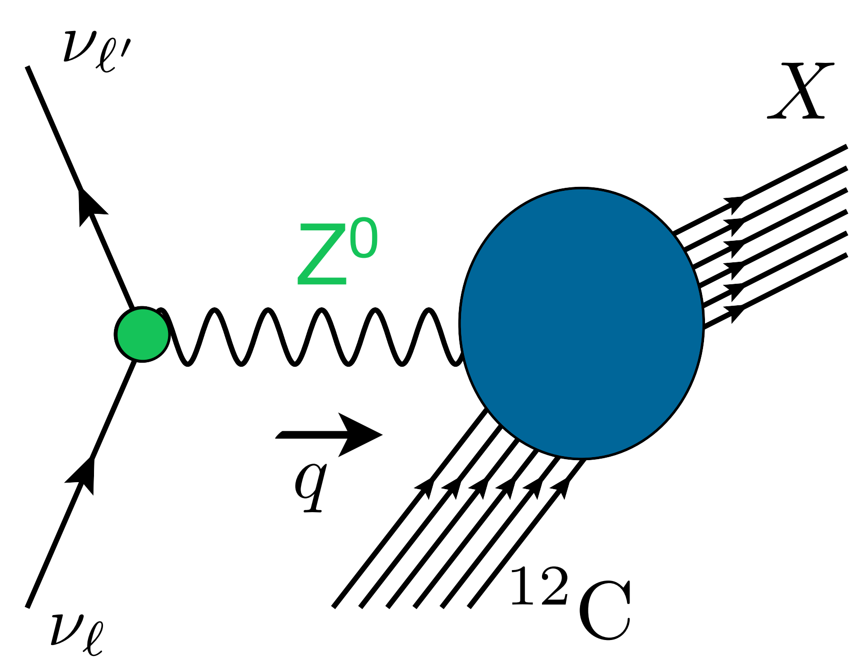
$$\frac{d^2\sigma}{d\Omega_{\nu'} dE_{\nu'}} = \frac{G_F^2}{4\pi^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} L_{\mu\nu}^{NC} W_{NC}^{\mu\nu}$$

The leptonic tensor is fully specified by the measured neutrino kinematic variables

$$L_{\mu\nu}^{NC} = 8 \left[k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu}(k \cdot k') - i\,\varepsilon_{\mu\nu\alpha\beta} k'^\beta k^\alpha \right]$$

The Hadronic tensor contains all the information on target structure.

$$W_{NC}^{\mu\nu} = \sum_X \langle \Psi_0 | J_{NC}^{\mu\,\dagger} | \Psi_X \rangle \langle \Psi_X | J_{NC}^{\nu} | \Psi_0 \rangle \delta^{(4)}(p_0 + q - p_X)$$



Neutral-current response

The neutral current operator can be written as

$$J^{\mu}_{NC} = -2\sin^2\theta_W J^{\mu}_{\gamma,S} + (1 - 2\sin^2\theta_W) J^{\mu}_{\gamma,z} + J^{\mu\,5}_{z}$$

- Weinberg angle $\sin^2\theta_W = 0.2312$
- Isoscalar and isovector terms of the electromagnetic current.

$$J^{\mu}_{EM} = J^{\mu}_{\gamma,S} + J^{\mu}_{\gamma,z}$$

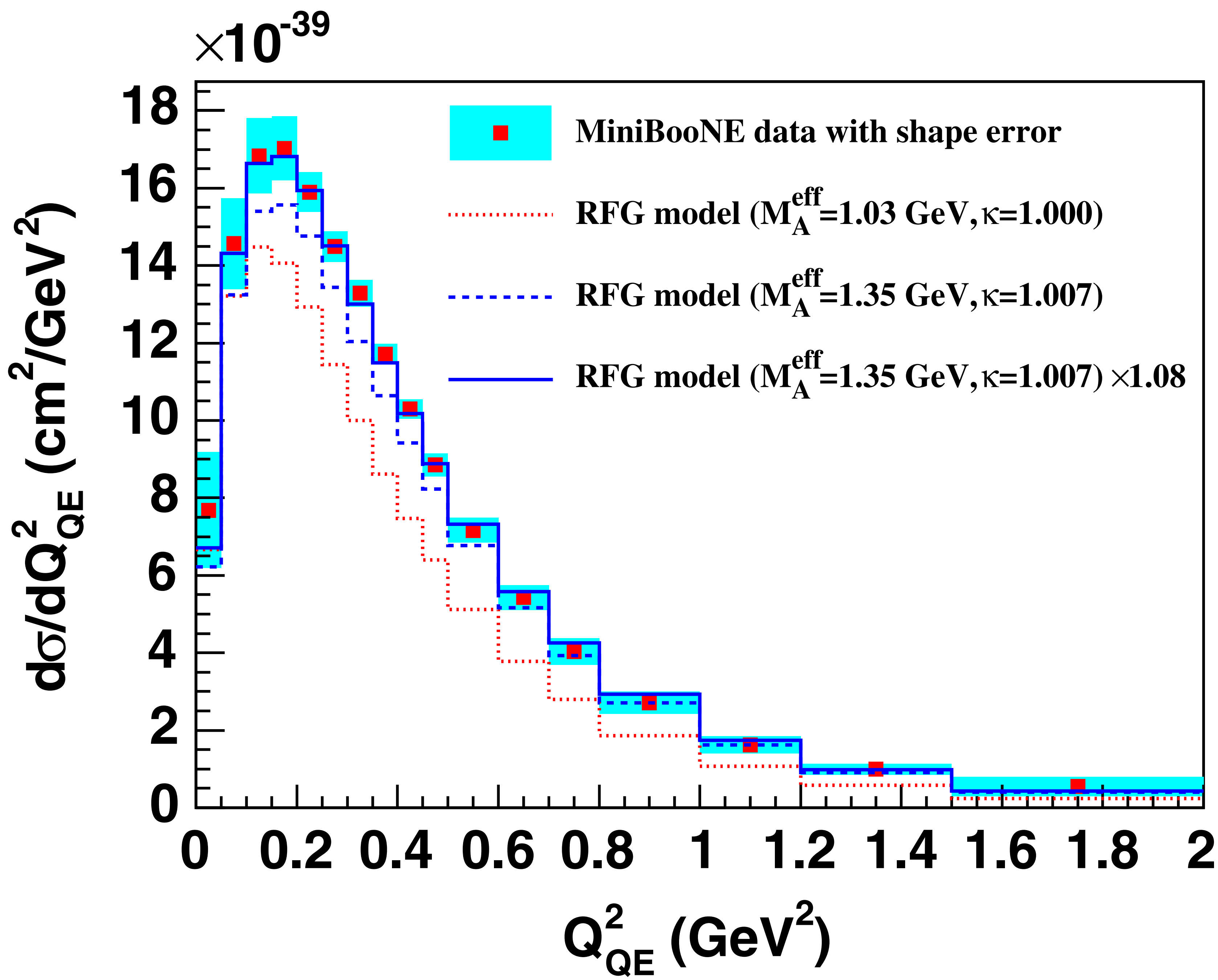
- Isovector term of the axial current, the one-body contributions of which are proportional to the axial form factor, often written in the simple dipole form

$$J^{\mu\,5}_{z} \propto G_A(Q^2) = \frac{g_A}{(1 + Q^2/\Lambda_A^2)^2}$$

The value of the axial mass obtained on neutrino-deuteron and neutrino-proton scattering data is $\Lambda_A \sim 1.03\,\mathrm{GeV}$.

Neutral-current response

Relativistic Fermi gas calculations requires an increase of the nucleon axial mass to reproduce the data.



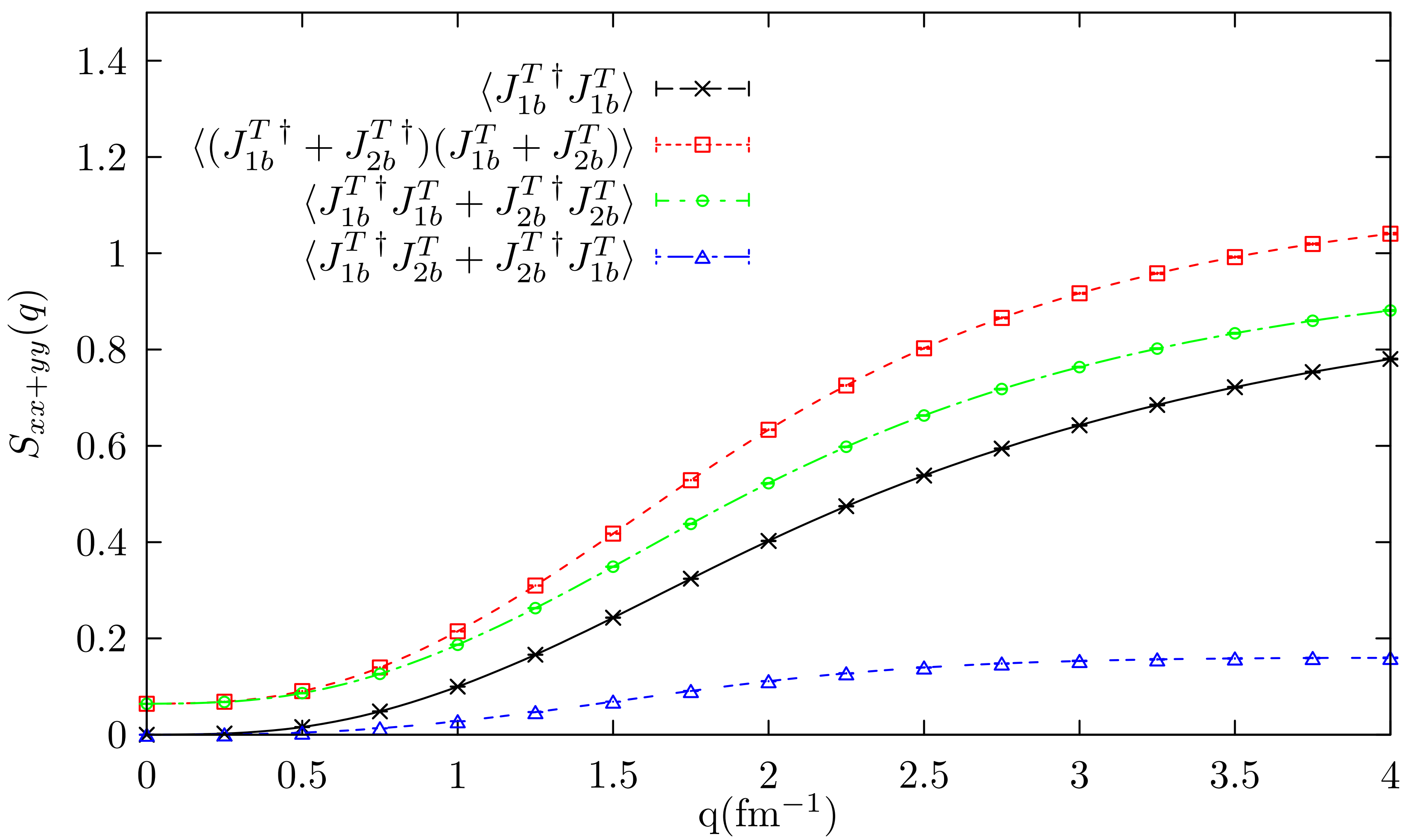
- Two-body currents?
- Correlations?
- Both?

Correlations for NC sum rules

Oversimplified models of nuclear dynamics (like RFG model) do not account for correlations induced by the nuclear interaction.

Their importance can be estimated by the interference term, which vanishes in any independent particle model.

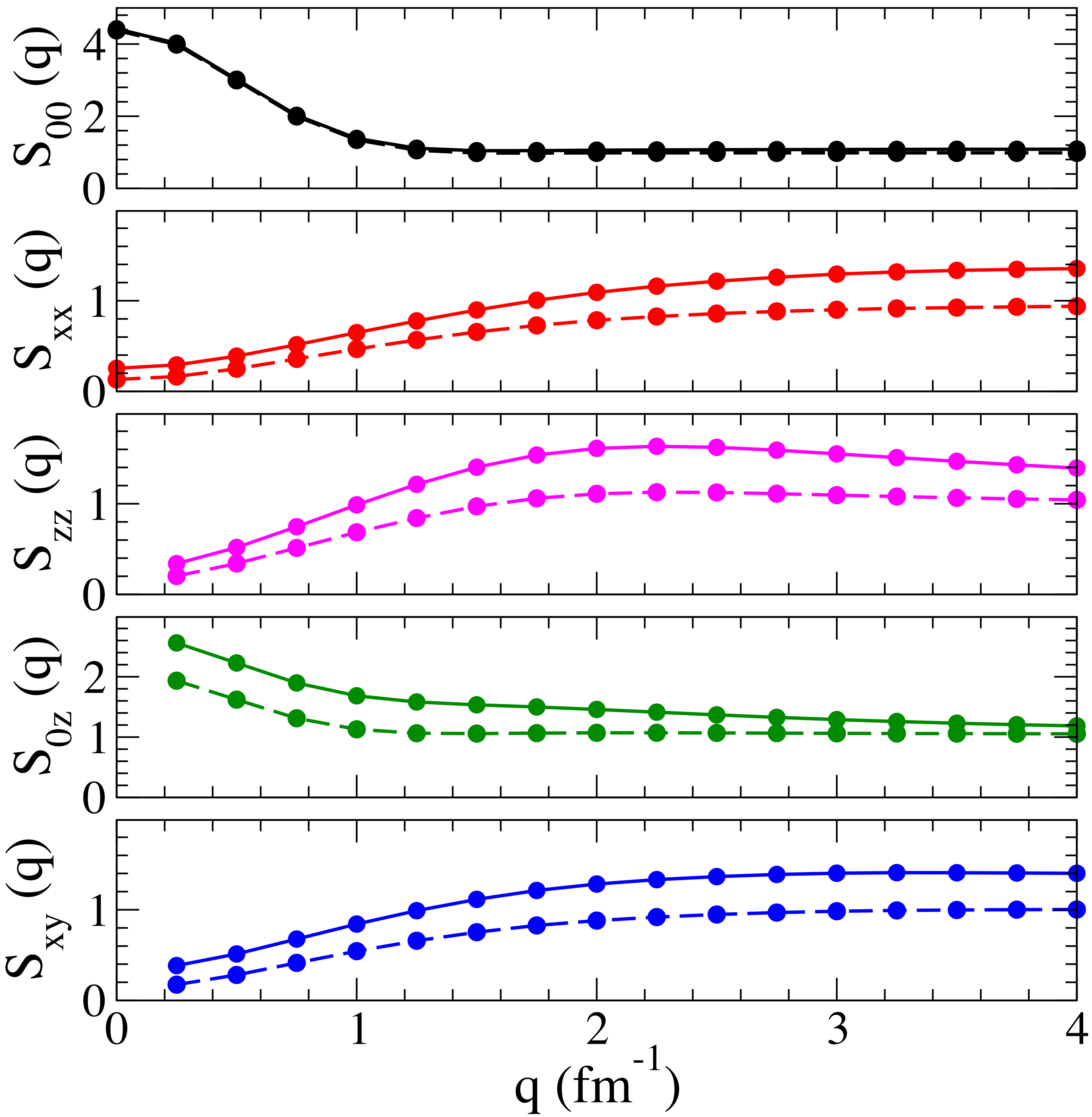
$$\langle (O_{1b}^\dagger + O_{2b}^\dagger)(O_{1b} + O_{2b})\rangle = \langle O_{1b}^\dagger O_{1b}\rangle + \langle O_{2b}^\dagger O_{2b}\rangle + \langle O_{1b}^\dagger O_{2b}\rangle + \langle O_{2b}^\dagger O_{1b}\rangle$$



Neutral-current response of ^{12}C

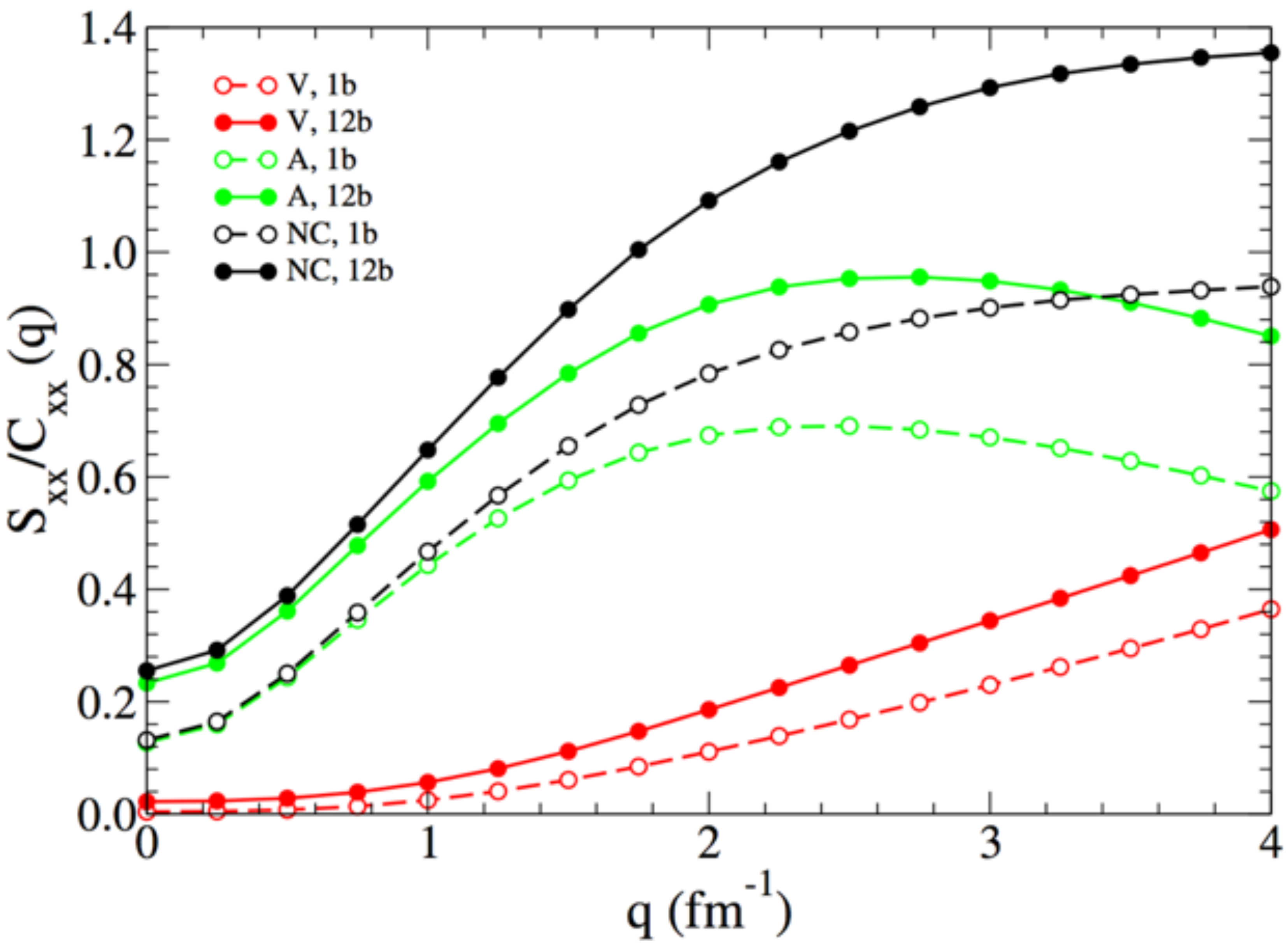
- Except for, the $S_{00}(q)$ case, the normalized sum rules of the response functions of ^{12}C exhibit a sizable enhancement due to two-body terms.

- A direct calculation of the response functions is needed to determine how this excess strength is distributed in energy transfer.



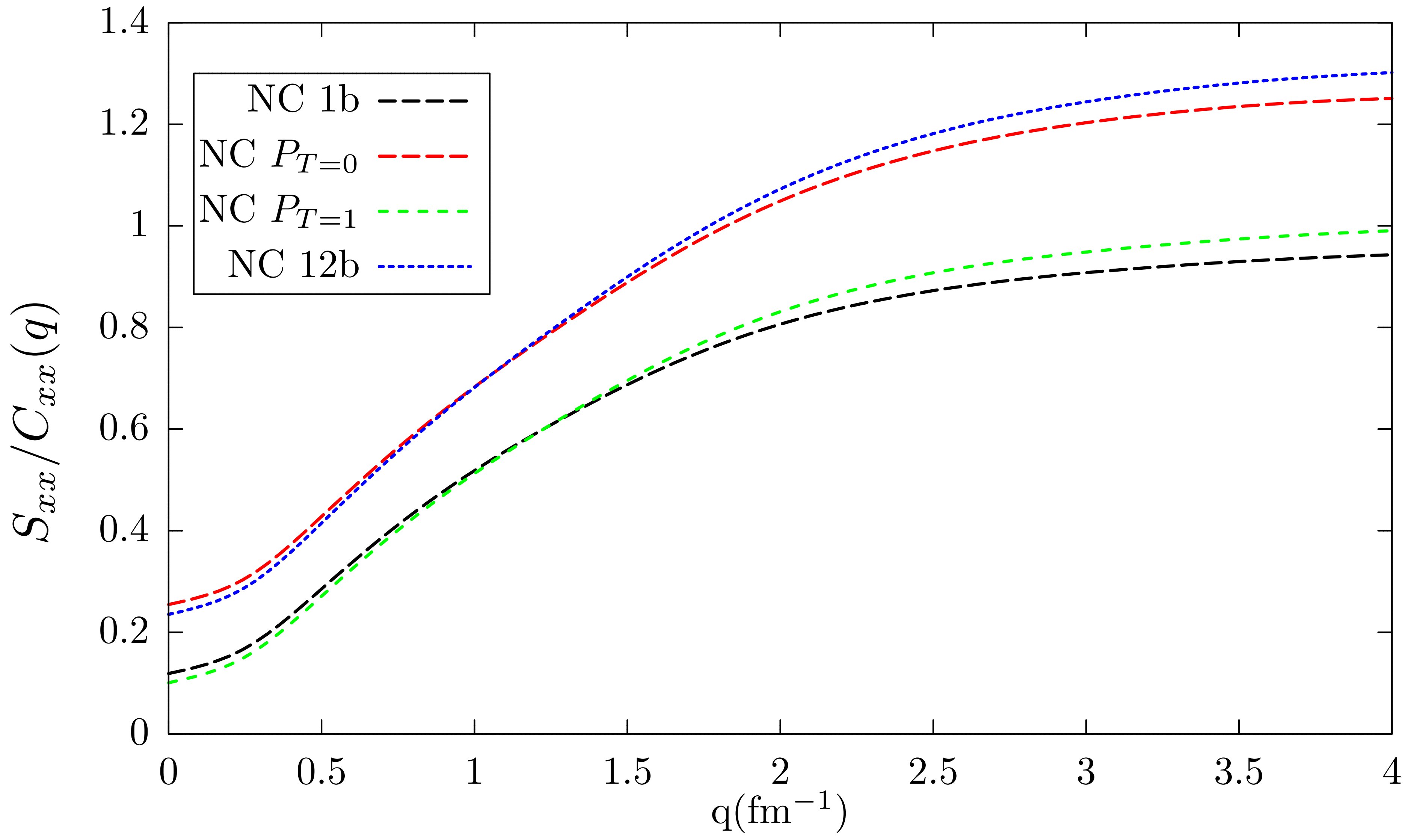
Neutral-current response of ^{12}C

The axial vector component of the $S_{xx}(q)$ sum rule has large two-body contributions, of the order of 30% relative to the one-body ones.

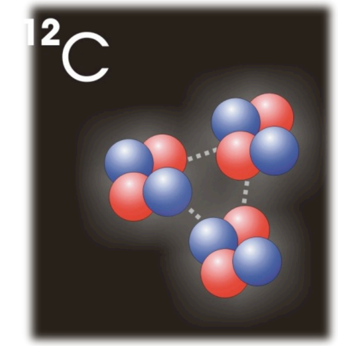


Neutral-current response of ^{12}C

The tensor force plays a larger role in np pairs than in nn and pp pairs. The enhancement in the weak response due to two-nucleon currents is dominated by T=0 pairs : tensor correlations!



Conclusions



Electromagnetic

- Very good description of the longitudinal form factor; two body terms bring theory into closer agreement with experiment.
- As for the longitudinal sum rule, we find satisfactory agreement with the experimental values, including tail contributions.
- In the transverse sum rule the large contribution of the two-body terms is needed for a better agreement with experimental data.
- Sizable interplay between correlations and two-body currents.

Neutral current

- Large two-body contributions from both the axial and vector sum rules provide a sizable enhancement of the neutral-current sum rule.
- All the processes but the ρ-meson exchanges in the two-body axial current contribute to the enhancement of the corresponding transverse sum-rule.

Current developments

- Euclidean electromagnetic and neutral-current response calculation

$$E_\alpha(\mathbf{q},\tau) = \int_{\omega_{th}}^{\infty} e^{-(\omega - E_0)\tau} R_\alpha(q,\omega)\, d\omega$$

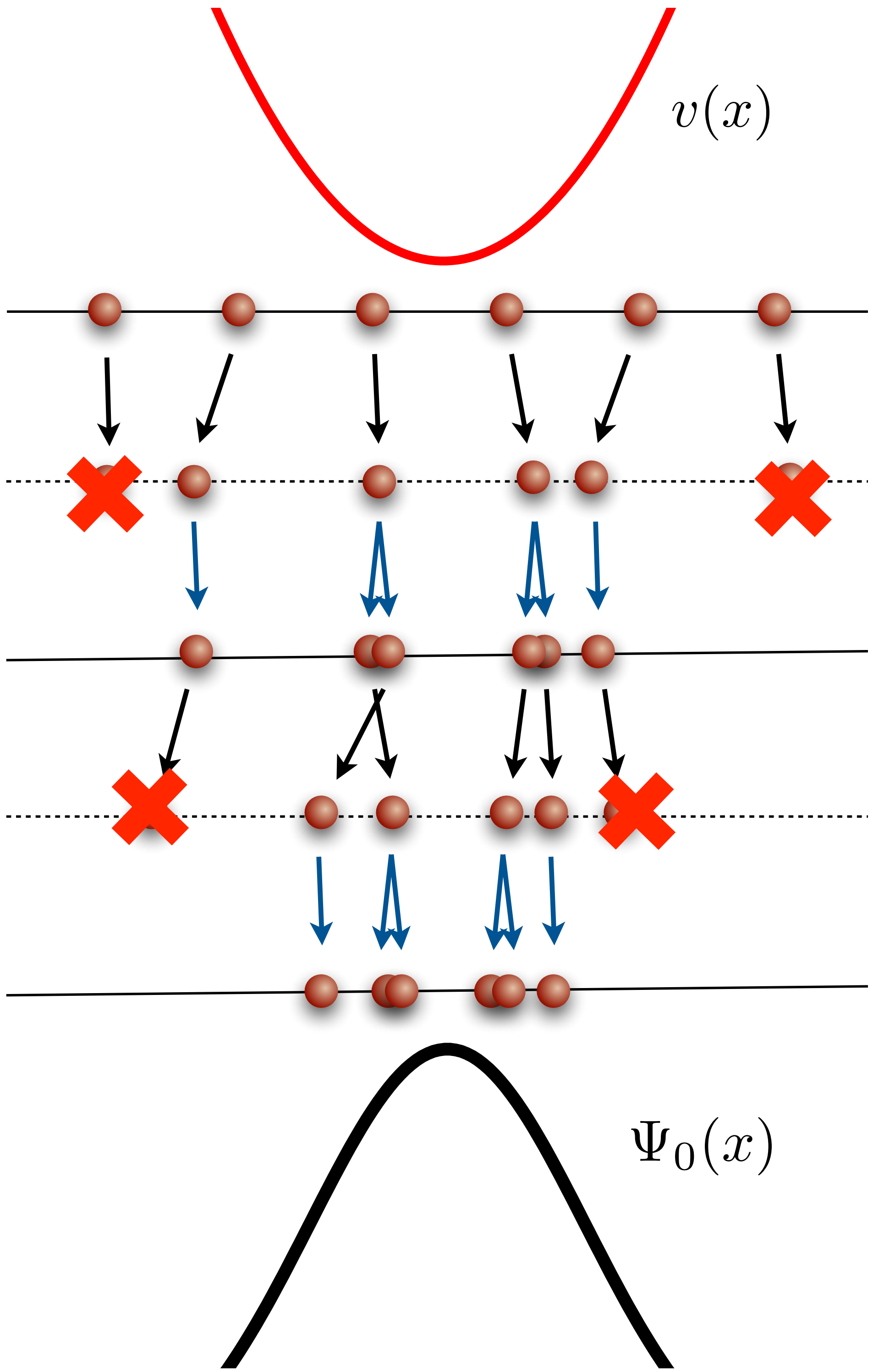
will enable us to make a more direct comparison with data. Its implementation in quantum Monte Carlo algorithms consists in the evaluation of

$$M(\tau) = \frac{\langle 0|O_\alpha^\dagger e^{-(H-E_0)\tau} O_\alpha|0\rangle}{\langle 0|e^{-(H-E_0)\tau}|0\rangle}$$

- Interplay with spectral function calculations, able to encompass relativistic effects within the impulse approximation scheme.

The two-nucleon spectral function of uniform and isospin symmetric nuclear matter at equilibrium density has been calculated within the CBF formalism; possible calculation using the improved AFDMC method we recently developed.

Diffusion Quantum Monte Carlo



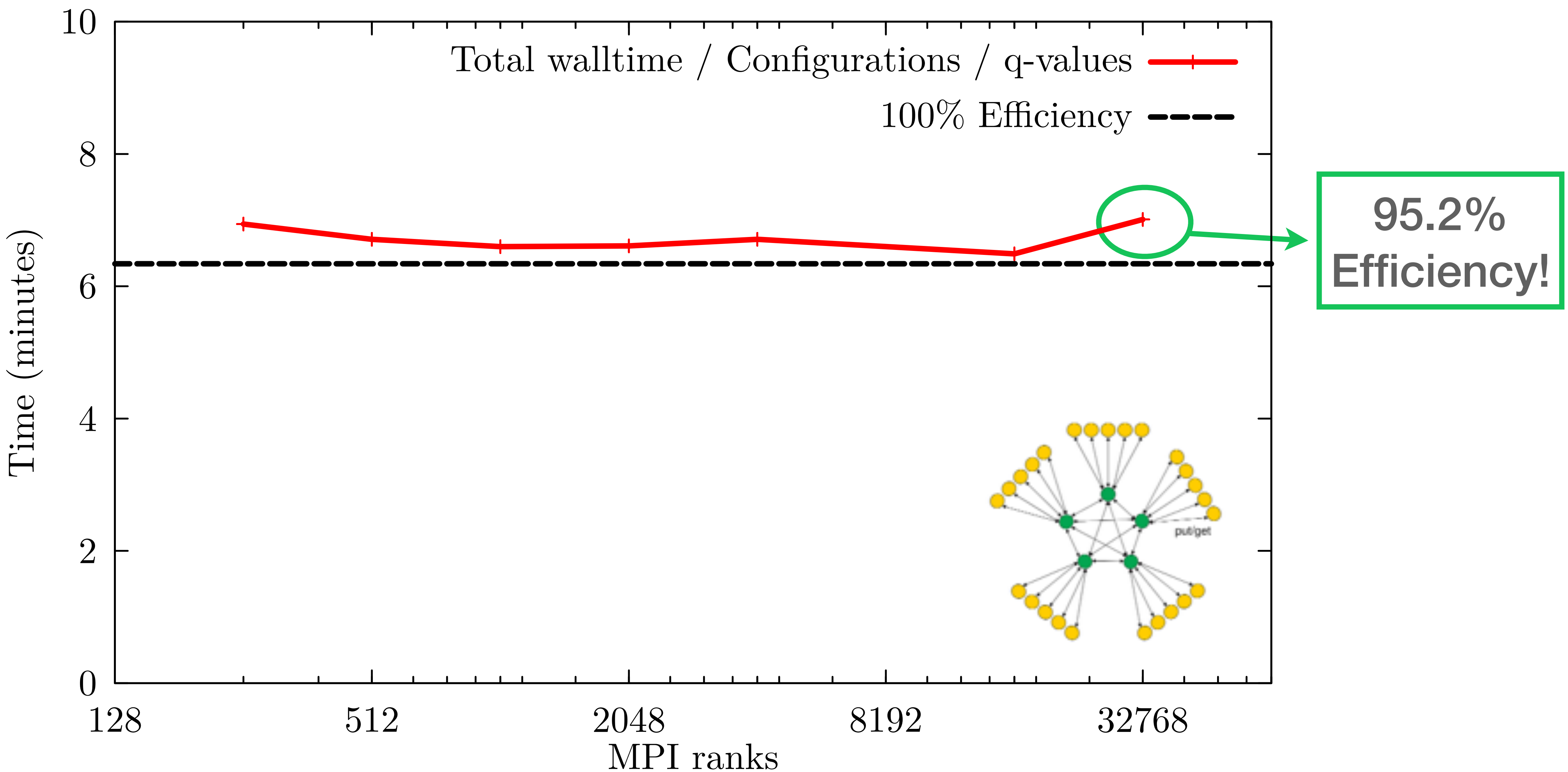
- A set of walkers is sampled from the trial wave function
- Gaussian drift for the kinetic energy
- Branching and killing of the walkers induced by the potential

The ground-state expectation values of observables that commute with $\hat{H}$ can be estimated by

$$\langle \hat{O} \rangle = \frac{\sum_{\{x\}} \langle x | \hat{O} | \Psi_T \rangle}{\sum_{\{x\}} \langle x | \Psi_T \rangle}$$

Parallelization: ADLB library performance

- Very good scaling of the calculation: total time per configuration per q-value very close to the ideal case.



Neutral-current response of ^{4}He

The most important terms of the two-body axial current are those associated with the π- and ρ-meson exchanges, the axial ρπ transition mechanism, and a Δ excitation term.