Watson's theorem, Goldberger–Treiman relation and the $\pi N\Delta$ axial $C_5^A(0)$ coupling constant

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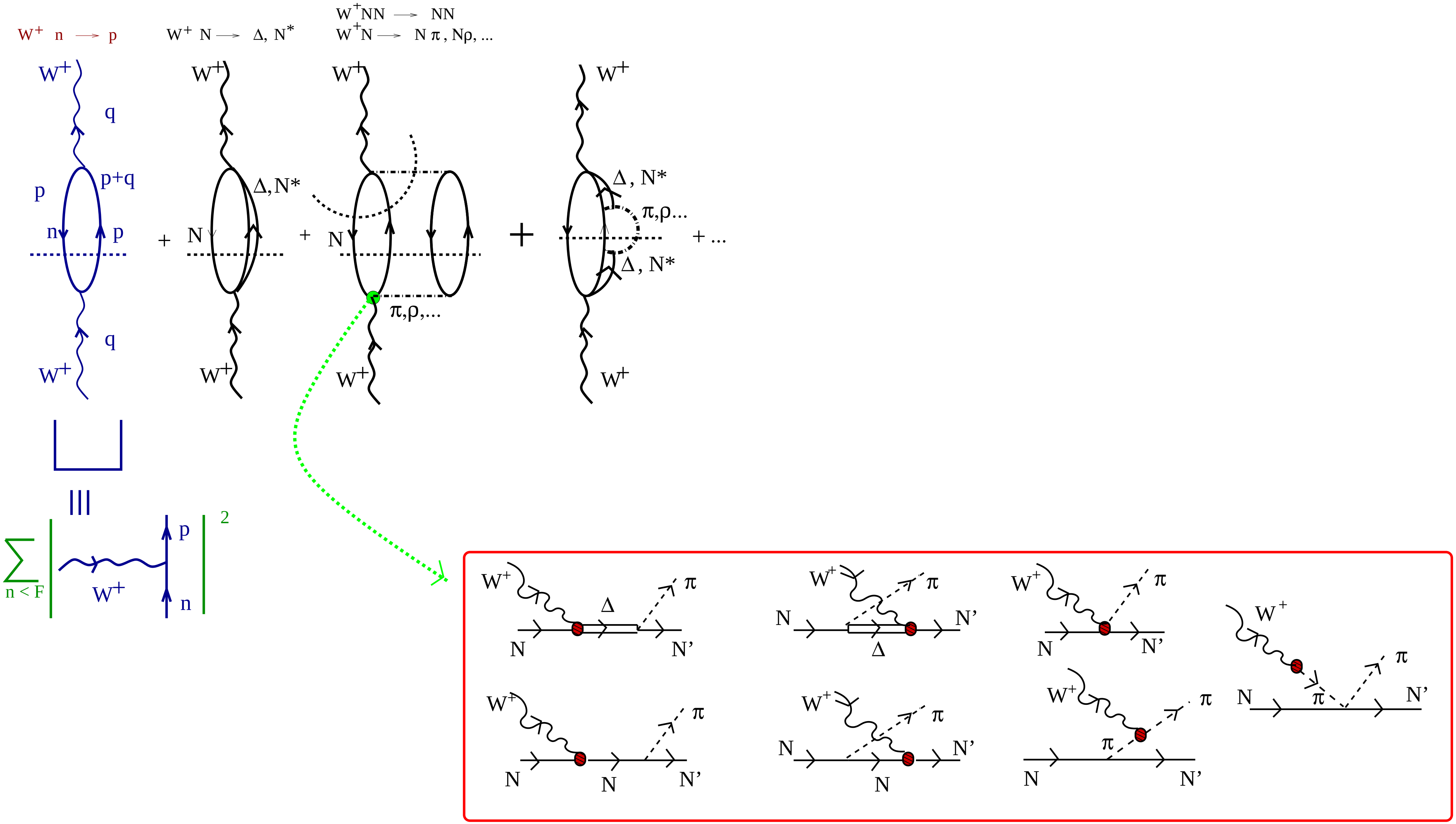
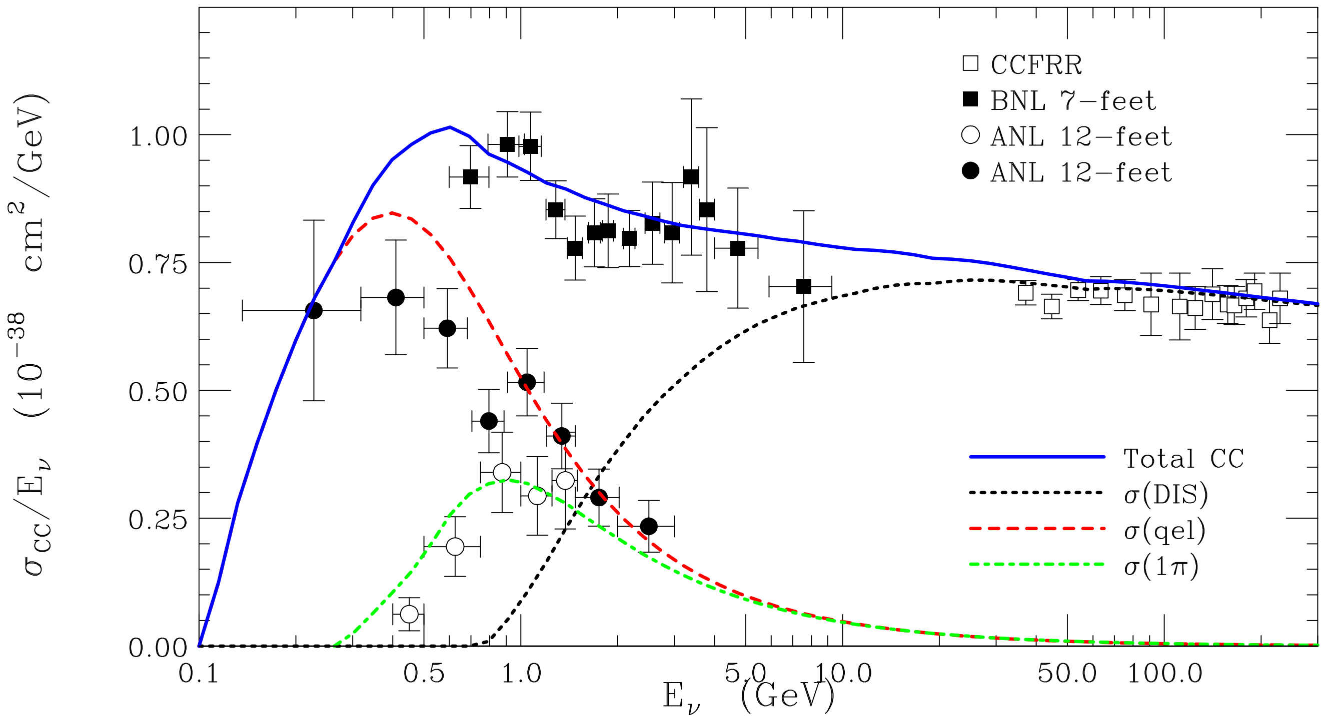
- PRD76 (2007) 033005, PLB647 (2007) 452: Chiral symmetry background terms and $C_5^A(0)$ fitted to ANL
- PRD81 (2010) 085046: Deuteron effects and $C_5^A(0)$ fitted to ANL & BNL, **and work in progress...**

Motivation :

1. Explore some aspects of hadron dynamics/structure, no accessible with electrons/photons, by using a **weak probe**
2. First step to study **neutrino–nucleus inclusive scattering** above the QE peak


W+ n → p
W+ N → Δ, N*
W+NN → NN
W+N → N π , Nρ, ...
W+
q
p
p+q
n
p
Δ,N*
N
π,ρ,...
Δ, N*
π,ρ...
+ ...
n < F
2
Δ
π
N
N'
W+



Theoretical knowledge of the one pion cross section is important to carry out a precise data analysis... Furthermore...

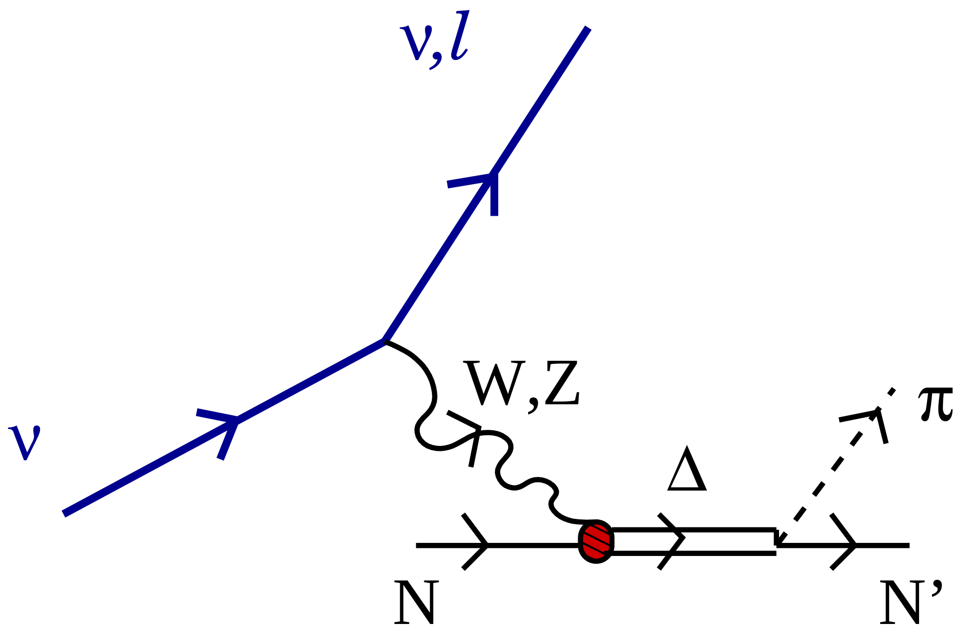
Pion production → identify *incorrectly* one-Čerenkov-ring events, which are assumed to be **CC QE** $\nu_\alpha A \to l_\alpha A'$.

- **Appearance probability** $P(\nu_\mu \to \nu_e)$**:** CC QE $\nu_e A \to e A'$ signal, which is used to identify ν_e, could be confused with that from 1π NC $\nu_\mu A \to \nu_\mu A \pi^0$ process

- **Survival probability** $P(\nu_\mu \to \nu_\mu)$**:** CC QE $\nu_\mu A \to \mu A'$ signal, which is used to identify ν_μ, could be confused with that from CC/NC 1π $\nu_{\tau,\mu} A \to (\nu_{\tau,\mu}$ ó $\tau, \mu) A' \pi$ signal, **if only one particle radiates Čerenkov light.**

  For instance, $(\nu_\mu, \mu\pi)$ **Incorrect E_ν re-construction** $\to L/E$ analysis?

Theoretical Model $\nu_l N \to l N' \pi$, $\nu_l N \to \nu_l N' \pi$ **(C.H. Llewellyn Smith, 1972)**: weak excitation of the $\Delta(1232)$ resonance and its subsequent decay into $N\pi$,



$$\langle \Delta^+; p_\Delta = p + q \, | j^\mu_{cc+}(0) | n; p \rangle = \bar{u}_\alpha(\vec{p}_\Delta) \Gamma^{\alpha\mu}(p,q) \, u(\vec{p}) \cos\theta_C,$$

$$\Gamma^{\alpha\mu} = \left[\frac{\mathbf{C_3^A}}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) + \frac{\mathbf{C_4^A}}{M^2} (g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p^\mu_\Delta) + \mathbf{C_5^A} g^{\alpha\mu} + \frac{\mathbf{C_6^A}}{M^2} q^\mu q^\alpha \right]$$

$$+ \left[\frac{\mathbf{C_3^V}}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) + \frac{\mathbf{C_4^V}}{M^2} (g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p^\mu_\Delta) + \frac{\mathbf{C_5^V}}{M^2} (g^{\alpha\mu} q \cdot p - q^\alpha p^\mu) \right.$$

$$\left. + \mathbf{C_6^V} g^{\mu\alpha} \right] \gamma_5,$$

$\mathbf{C^A_{3,4,5,6}}$ axial FF′s, $\mathbf{C^V_{3,4,5,6}}$ vector FF′s, furthermore

$$\mathcal{L}_{\pi N \Delta} = \frac{f^*}{m_\pi} \bar{\Psi}_\mu \vec{T}^\dagger (\partial^\mu \vec{\phi}) \Psi + \text{h.c.}, \qquad f^* = 2.14$$

$$\mathbf{G^{\mu\nu}(p_\Delta)} = \frac{\not{p}_\Delta + M_\Delta}{p^2_\Delta - M^2_\Delta + i M_\Delta \Gamma_\Delta} \left[-\mathbf{g^{\mu\nu}} + \frac{\mathbf{1}}{\mathbf{3}} \gamma^\mu \gamma^\nu + \frac{\mathbf{2}}{\mathbf{3}} \frac{\mathbf{p^\mu_\Delta p^\nu_\Delta}}{\mathbf{M^2_\Delta}} - \frac{\mathbf{1}}{\mathbf{3}} \frac{\mathbf{p^\mu_\Delta} \gamma^\nu - \mathbf{p^\nu_\Delta} \gamma^\mu}{\mathbf{M_\Delta}} \right]$$

$eN \to e'\Delta \to e'N'\pi \Rightarrow C^V_{3,4,5,6}$ **FF's.** **CVC** $\Rightarrow C^V_6 = 0$ and $(M_V = 0.84$ GeV)

$$\frac{\mathbf{C^V_3(q^2)}}{2.13} = \frac{\mathbf{C^V_4(q^2)}}{-1.51} = \frac{1-\frac{q^2}{0.776M_V^2}}{1-\frac{q^2}{4M_V^2}}\frac{\mathbf{C^V_5(q^2)}}{0.48} = \frac{1}{(1-q^2/M_V^2)^2}\times\frac{1}{1-\frac{q^2}{4M_V^2}}$$

$C^A_{3,4,5,6}$ **Axial FF's :** Δ^{++} $(\nu_\mu p \to \mu^- p\pi^+)$ **data taken in the ANL and BNL bubble chambers (filled in with deuterium)**

Dominant form factor: $\mathbf{C^A_5(q^2)}$. $\mathbf{C^A_3(q^2)}$ and $\mathbf{C^A_4(q^2)}$ contributions are small and we have taken as (**Adler's model 1968**)

$$\mathbf{C^A_4(q^2)} = -\frac{\mathbf{C^A_5(q^2)}}{4}, \quad \mathbf{C^A_3(q^2)} = 0$$

PCAC ($\partial_\mu A^\mu \propto m_\pi^2$) and Goldberger–Treiman

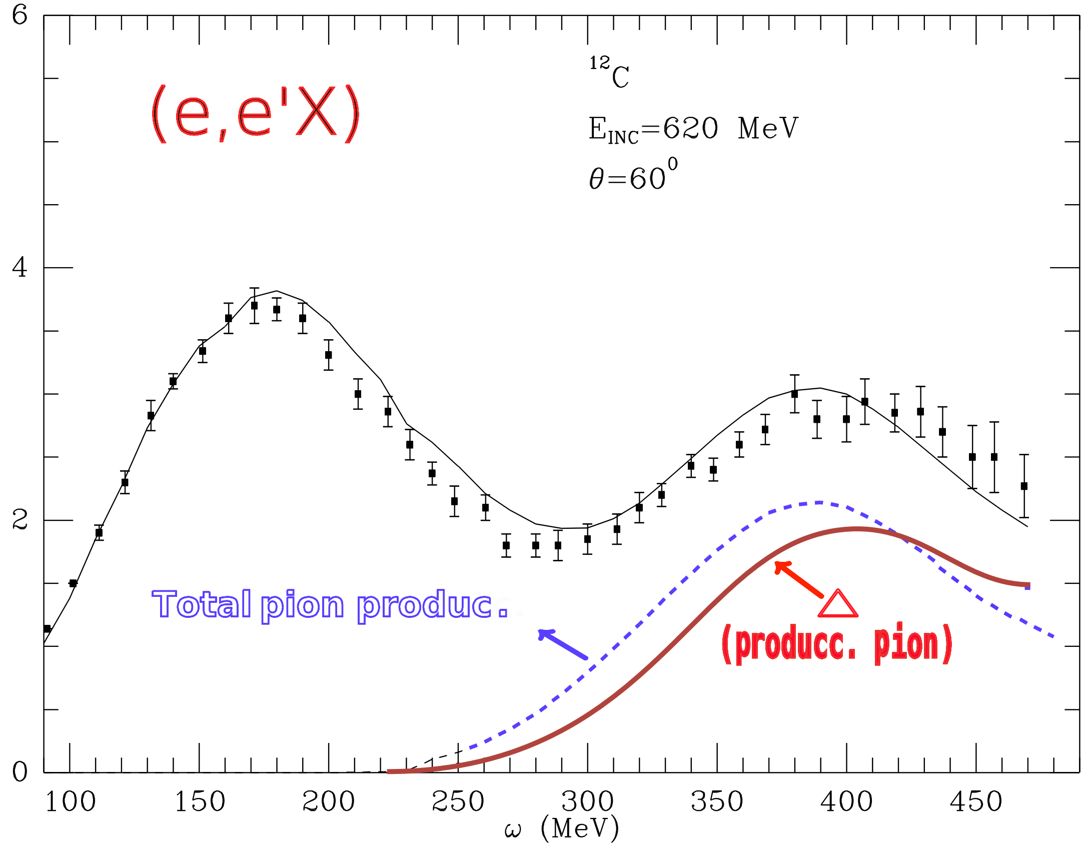
$$C_5^A(0) \sim \sqrt{\frac{2}{3}}\frac{f_\pi}{m_\pi} f^* = 1.2$$

$$\mathbf{C_5^A(q^2)} = \frac{\mathbf{1.2}}{(1-q^2/\mathbf{M_{A\Delta}^2})^2} \times \frac{1}{1-\frac{q^2}{3\mathbf{M_{A\Delta}^2}}}, \quad \underbrace{\mathbf{C_6^A(q^2)} = \mathbf{C_5^A(q^2)}\frac{M^2}{m_\pi^2 - q^2}}_{\text{PCAC}}$$

$M_{A\Delta}$ **fitted to the q^2 dependence of the $\nu_\mu p \to \mu^- p \pi^+$** cross section (neutrino energy averaged) **with ($M(\pi N) < 1.4$ GeV) measured at ANL and BNL.** It varies in the range 0.95 GeV (ANL) – 1.28 GeV (BNL).

E. Paschos, J-Y. Yu and M. Sakuda (PRD69, 014013 (2004)),

$$\mathbf{M_{A\Delta} \sim 1.05\ GeV}$$

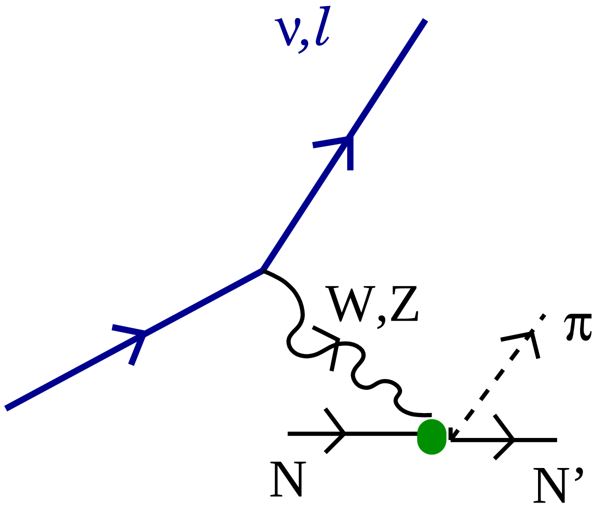


... but **only the Δ pole** contribution turns out to be **an <u>insufficient</u>** model, even at the Δ peak, and **specially close to pion threshold**. Close to pion threshold, the pion from the $(\nu_\mu, \mu\pi)$ **reaction will not radiate Čerenkov light** and thus it would be necessary an improved theoretical model to carry out a proper L/E **oscillation analysis.**

Such model for the $\nu_l N \to l N'\pi$, $\nu_l N \to \nu_l N'\pi$ should include **<u>non</u> <u>resonant</u>** <u>terms</u> ⇒ **Realization of the axial and vector currents, which couple to the W, Z^0 bosons, for a system of pions and nucleons.**

Non-linear $\sigma-$Model: EFT involving pions and nucleons which implements spontaneous chiral symmetry breaking.

If $\Psi_q = \begin{pmatrix} \Psi_u \\ \Psi_d \end{pmatrix}$, the CC and NC, which induce $W(Z^0)N \to N'\pi$



$$j^\mu_{cc\pm} = \cos\theta_C \bar\Psi_q \gamma^\mu (1-\gamma_5)\left(-\frac{\boxed{\tau^1_{\pm 1}}}{\sqrt{2}}\right)\Psi_q$$

$$j^\mu_{nc} = \bar\Psi_q \gamma^\mu (1 - 2\sin^2\theta_W - \gamma_5)\boxed{\tau^1_0}\Psi_q$$

$$- \boxed{4\sin^2\theta_W \mathbf{s}^\mu_{\mathrm{em,IS}} - \bar{\mathbf{\Psi}}_\mathbf{s}\gamma^\mu(\mathbf{1}-\gamma_\mathbf{5})\mathbf{\Psi}_\mathbf{s}}$$

$$s^\mu_{em} = \underbrace{\frac{1}{6}\bar\Psi_q\gamma^\mu\Psi_q - \frac{1}{3}\bar\Psi_s\gamma^\mu\Psi_s}_{\mathbf{s}^\mu_{\mathrm{em,IS}}} + \frac{1}{\sqrt{2}}\bar\Psi_q\gamma^\mu\frac{\boxed{\tau^1_0}}{\sqrt{2}}\Psi_q$$

$$\langle N'\pi|\,\mathbf{j}^\mu_{cc+}(\mathbf{0}), \mathbf{j}^\mu_{cc-}(\mathbf{0}), \mathbf{j}^\mu_{nc}(\mathbf{0})|N\rangle = ? \Leftarrow$$ **QCD and its pattern of SχSB**

If $\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$, $U = \frac{f_\pi}{\sqrt{2}} e^{\,\mathrm{i}\,\vec{\tau}\cdot\boxed{\vec{\phi}}/f_\pi} = \frac{f_\pi}{\sqrt{2}}\xi^2$, with $f_\pi \sim 93$ MeV,

$$\begin{aligned}\mathcal{L}_{N\pi} &= \bar{\Psi}\mathrm{i}\gamma^\mu\left[\partial_\mu + \mathcal{V}_\mu\right]\Psi - M\bar{\Psi}\Psi + g_A\bar{\Psi}\gamma^\mu\gamma_5\mathcal{A}_\mu\Psi \\ &+ \frac{1}{2}\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right]\boxed{+m_\pi^2\frac{f_\pi}{2\sqrt{2}}\mathrm{Tr}(U + U^\dagger - \sqrt{2}f_\pi)}\end{aligned}$$

$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right) \qquad \mathcal{A}_\mu = \frac{\mathrm{i}}{2}\left(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi\right)$$

Isospin rotat. $\xi \to \mathbf{T_V}\,\xi\,\mathbf{T_V^\dagger}$, $\mathbf{\Psi} \to \mathbf{T_V}\,\mathbf{\Psi}$, $\quad T_V = e^{-\mathrm{i}\,\frac{\vec{\tau}\cdot\vec{\theta}_V}{2}}$

Axial rotat. $\xi \to \mathbf{T_A^\dagger}\,\xi\,\mathbf{T_\Lambda^\dagger} = \mathbf{T_\Lambda}\,\xi\,\mathbf{T_A^\dagger}$, $\mathbf{\Psi} \to \mathbf{T_\Lambda}\,\mathbf{\Psi}$, $\quad T_{\Lambda,A} = e^{-\mathrm{i}\,\frac{\vec{\tau}\cdot\vec{\theta}_{\Lambda,A}}{2}}$

Isospin rotat. $\Rightarrow \delta\mathcal{L}_{\mathbf{N}\pi} = \mathbf{0}$, **Axial rotat.** $\Rightarrow \delta\mathcal{L}_{\mathbf{N}\pi}\boxed{\propto m_\pi^2 \neq 0}$

Up to order $\mathcal{O}(1/f_\pi^4)$, $\mathcal{L}_{N\pi}$ reads,

$$\mathcal{L}_{\mathbf{N}\pi} = \bar{\Psi}[i\partial\!\!\!/ - M]\Psi + \frac{1}{2}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \frac{1}{2}m_\pi^2\vec{\phi}^2 \quad (\textbf{kinetic}) +$$

$$\frac{\mathbf{g_A}}{\mathbf{f}_\pi}\bar{\mathbf{\Psi}}\gamma^\mu\gamma_5\frac{\vec{\tau}}{\mathbf{2}}(\partial_\mu\vec{\phi})\mathbf{\Psi} - \frac{\mathbf{1}}{\mathbf{4f_\pi^2}}\bar{\mathbf{\Psi}}\gamma_\mu\vec{\tau}\left(\vec{\phi}\times\partial^\mu\vec{\phi}\right)\mathbf{\Psi} - \frac{\mathbf{g_A}}{\mathbf{6f_\pi^3}}\bar{\mathbf{\Psi}}\gamma^\mu\gamma_5\left[\vec{\phi}^{\mathbf{2}}\frac{\vec{\tau}}{\mathbf{2}}\partial_\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})\frac{\vec{\tau}}{\mathbf{2}}\vec{\phi}\right]\mathbf{\Psi}$$

$$-\frac{\mathbf{1}}{\mathbf{6f_\pi^2}}\left(\vec{\phi}^{\mathbf{2}}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})(\vec{\phi}\partial^\mu\vec{\phi})\right) + \frac{\mathbf{m_\pi^2}}{\mathbf{24f_\pi^2}}(\vec{\phi}^{\mathbf{2}})^{\mathbf{2}} + \mathcal{O}(1/f_\pi^4)$$

Contact interactions $NN\pi$, $\underbrace{NN\pi\pi}_{\text{WT}}$, $NN\pi\pi\pi$ and $\pi\pi\pi\pi$.

<u>Parameters: f_π and g_A</u> . Noether's currents

$$j^\mu = \frac{\partial\mathcal{L}_{N\pi}}{\partial(\partial_\mu\varphi_a)}\delta\varphi_a, \quad a = 1, 2, \cdots$$

up to order $\mathcal{O}(1/f_\pi^3)$ ...

$$\vec{\mathbf{V}}^\mu = \vec{\phi}\times\partial^\mu\vec{\phi} + \frac{g_A}{2f_\pi}\bar{\Psi}\gamma^\mu\gamma_5(\vec{\phi}\times\vec{\tau})\Psi + \bar{\Psi}\gamma^\mu\frac{\vec{\tau}}{2}\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi$$

$$- \frac{\vec{\phi}^{\,2}}{3f_\pi^2}(\vec{\phi}\times\partial^\mu\vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3}), \quad \partial_\mu\vec{\mathbf{V}}^\mu = \mathbf{0}$$

$$\vec{\mathbf{A}}^\mu = f_\pi\partial^\mu\vec{\phi} + \frac{1}{2f_\pi}\bar{\Psi}\gamma^\mu(\vec{\phi}\times\vec{\tau})\Psi + g_A\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{2}{3f_\pi}\left[\vec{\phi}(\vec{\phi}\cdot\partial^\mu\vec{\phi}) - \vec{\phi}^{\,2}\partial^\mu\vec{\phi}\right]$$

$$- \frac{g_A}{4f_\pi^2}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3}), \quad \underbrace{\partial_\mu\vec{A}^\mu \propto m_\pi^2 \ldots}_{\text{PCAC}}$$

+ isospin relations ⇒ evaluate CC $\langle N'\pi|\, j^\mu_{cc+}(0), j^\mu_{cc-}(0)|N\rangle$

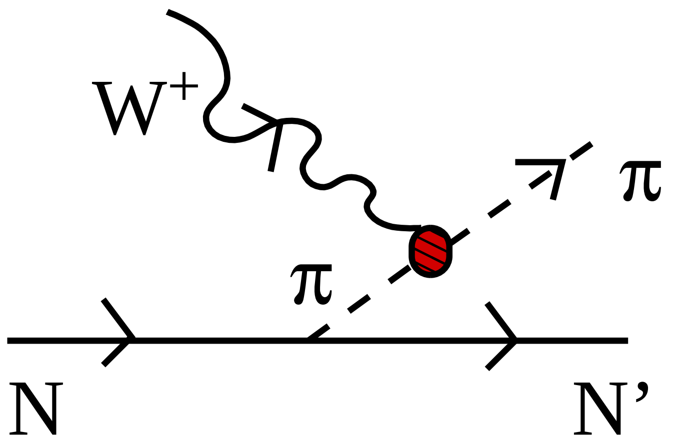
$$\langle p\pi^0|j^\mu_{cc+}(0)|n\rangle = -\frac{1}{\sqrt{2}}\left[\langle \mathbf{p}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{p}\rangle - \langle \mathbf{n}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{n}\rangle\right]$$

$$\langle p\pi^-|j^\mu_{cc-}(0)|p\rangle = \langle \mathbf{n}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{n}\rangle$$

$$\langle n\pi^-|j^\mu_{cc-}(0)|n\rangle = \langle \mathbf{p}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{p}\rangle$$

$$\langle n\pi^0|j^\mu_{cc-}(0)|p\rangle = -\langle p\pi^0|j^\mu_{cc+}(0)|n\rangle = \frac{1}{\sqrt{2}}\left[\langle \mathbf{p}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{p}\rangle - \langle \mathbf{n}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{n}\rangle\right]$$

$$\vec{\mathbf{V}}^\mu = \vec{\phi}\times\partial^\mu\vec{\phi} + \frac{g_A}{2f_\pi}\bar\Psi\gamma^\mu\gamma_5(\vec{\phi}\times\vec{\tau})\Psi + \bar\Psi\gamma^\mu\frac{\vec{\tau}}{2}\Psi - \frac{1}{4f_\pi^2}\bar\Psi\gamma^\mu\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi$$
$$- \frac{\vec{\phi}^{\,2}}{3f_\pi^2}(\vec{\phi}\times\partial^\mu\vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3})$$
$$\vec{A}^\mu = f_\pi\partial^\mu\vec{\phi} + \frac{1}{2f_\pi}\bar\Psi\gamma^\mu(\vec{\phi}\times\vec{\tau})\Psi + g_A\bar\Psi\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{2}{3f_\pi}\left[\vec{\phi}(\vec{\phi}\cdot\partial^\mu\vec{\phi}) - \vec{\phi}^{\,2}\partial^\mu\vec{\phi}\right]$$
$$- \frac{g_A}{4f_\pi^2}\bar\Psi\gamma^\mu\gamma_5\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3})$$

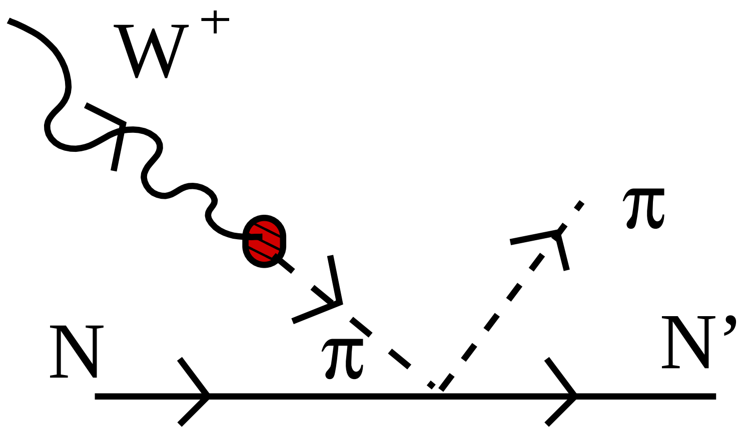


$$j^\mu_{cc+}\Big|_{PF} = \mp\,\mathrm{i}\,\mathbf{F_{PF}(q^2)}\frac{\sqrt{2}Mg_A}{f_\pi}\cos\theta_C\frac{(2k_\pi - q)^\mu}{(k_\pi - q)^2 - m_\pi^2}\bar u(\vec{p}\,')\gamma_5 u(\vec{p})$$

$$(\; - \Rightarrow W^+p\to p\pi^+,\; + \Rightarrow W^+n\to n\pi^+\;)$$

$$\vec{V}^\mu = \vec{\phi}\times\partial^\mu\vec{\phi} + \frac{g_A}{2f_\pi}\bar{\Psi}\gamma^\mu\gamma_5(\vec{\phi}\times\vec{\tau})\Psi + \bar{\Psi}\gamma^\mu\frac{\vec{\tau}}{2}\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi$$
$$- \frac{\vec{\phi}^{\,2}}{3f_\pi^2}(\vec{\phi}\times\partial^\mu\vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3})$$

$$\vec{\mathbf{A}}^\mu = \mathbf{f}_\pi\partial^\mu\vec{\phi} + \frac{1}{2f_\pi}\bar{\Psi}\gamma^\mu(\vec{\phi}\times\vec{\tau})\Psi + g_A\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{2}{3f_\pi}\left[\vec{\phi}(\vec{\phi}\cdot\partial^\mu\vec{\phi}) - \vec{\phi}^{\,2}\partial^\mu\vec{\phi}\right]$$
$$- \frac{g_A}{4f_\pi^2}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3})$$



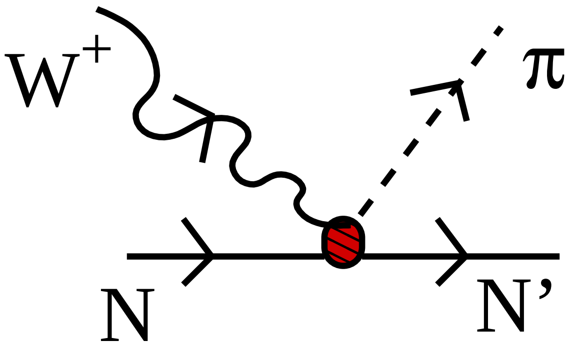
$$j^\mu_{cc+}\Big|_{PP} = \mp\,\mathrm{i}\,\mathbf{F}_\rho\left((\mathbf{q}-\mathbf{k}_\pi)^2\right)\frac{\cos\theta_C}{\sqrt{2}f_\pi}\frac{q^\mu}{q^2-m_\pi^2}\bar{u}(\vec{p}\,')\not{q}\,u(\vec{p})$$

$$(\,-\Rightarrow W^+p\to p\pi^+,\ +\Rightarrow W^+n\to n\pi^+\,)$$

$$\mathbf{F}_\rho(\mathbf{t}) = \frac{\mathbf{1}}{\mathbf{1}-\mathbf{t}/\mathbf{m}_\rho^2}$$

$$\vec{\mathbf{V}}^\mu = \vec{\phi}\times\partial^\mu\vec{\phi} + \frac{\mathbf{g_A}}{\mathbf{2f_\pi}}\bar{\mathbf{\Psi}}\gamma^\mu\gamma_5(\vec{\phi}\times\vec{\tau})\mathbf{\Psi} + \bar{\Psi}\gamma^\mu\frac{\vec{\tau}}{2}\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi$$
$$- \frac{\vec{\phi}^{\,2}}{3f_\pi^2}(\vec{\phi}\times\partial^\mu\vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3})$$

$$\vec{\mathbf{A}}^\mu = f_\pi\partial^\mu\vec{\phi} + \frac{\mathbf{1}}{\mathbf{2f_\pi}}\bar{\mathbf{\Psi}}\gamma^\mu(\vec{\phi}\times\vec{\tau})\mathbf{\Psi} + g_A\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{2}{3f_\pi}\left[\vec{\phi}(\vec{\phi}\cdot\partial^\mu\vec{\phi}) - \vec{\phi}^{\,2}\partial^\mu\vec{\phi}\right]$$
$$- \frac{g_A}{4f_\pi^2}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3})$$



$$j^\mu_{cc+}\Big|_{CT} = \mp\,\mathrm{i}\,\frac{\cos\theta_C}{\sqrt{2}f_\pi}\bar{u}(\vec{p}\,')\gamma^\mu\left(g_A\mathbf{F^V_{CT}}(\mathbf{q^2})\gamma_5 - \mathbf{F}_\rho\left((\mathbf{q}-\mathbf{k}_\pi)^\mathbf{2}\right)\right)u(\vec{p})$$

$$(\, - \Rightarrow W^+p\to p\pi^+,\; + \Rightarrow W^+n\to n\pi^+\,)$$

$$
\begin{aligned}
\vec{\mathbf{V}}^{\mu} &= \vec{\phi} \times \partial^{\mu} \vec{\phi} + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^{\mu} \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi + \bar{\boldsymbol{\Psi}} \gamma^{\mu} \frac{\vec{\tau}}{\mathbf{2}} \boldsymbol{\Psi} - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma^{\mu} \left[\vec{\tau} \vec{\phi}^{\,2} - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi \\
&- \frac{\vec{\phi}^{\,2}}{3f_\pi^2} (\vec{\phi} \times \partial^{\mu} \vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3}) \\
\vec{\mathbf{A}}^{\mu} &= f_\pi \partial^{\mu} \vec{\phi} + \frac{1}{2f_\pi} \bar{\Psi} \gamma^{\mu} (\vec{\phi} \times \vec{\tau}) \Psi + \mathbf{g_A} \bar{\boldsymbol{\Psi}} \gamma^{\mu} \gamma_5 \frac{\vec{\tau}}{\mathbf{2}} \boldsymbol{\Psi} + \frac{2}{3f_\pi} \left[\vec{\phi} (\vec{\phi} \cdot \partial^{\mu} \vec{\phi}) - \vec{\phi}^{\,2} \partial^{\mu} \vec{\phi} \right] \\
&- \frac{g_A}{4f_\pi^2} \bar{\Psi} \gamma^{\mu} \gamma_5 \left[\vec{\tau} \vec{\phi}^{\,2} - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi + \mathcal{O}(\frac{1}{f_\pi^3})
\end{aligned}
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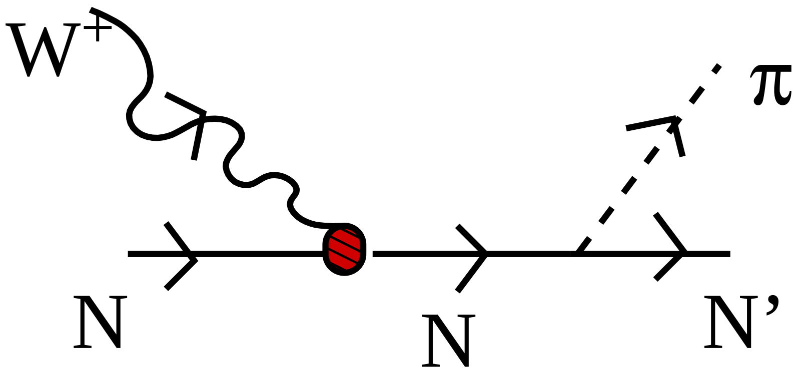
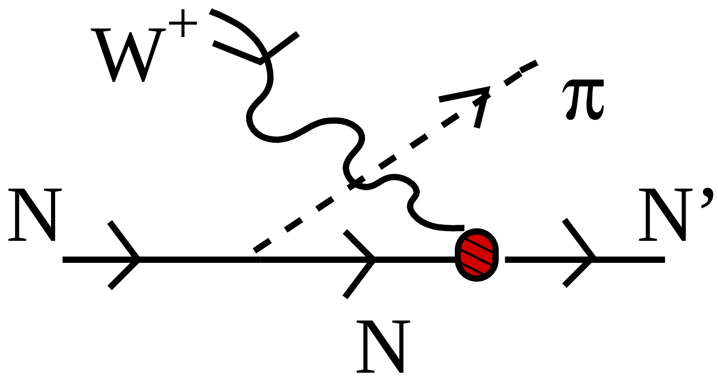




... improve the WNN transition vertex

$$\langle p; \vec{p}\,' = \vec{p}+\vec{q}\,|\mathbf{j}^{\alpha}_{\mathbf{cc+}}(\mathbf{0})|n; \vec{p}\rangle = \cos\theta_C\, \bar{u}(\vec{p}\,')(\mathbf{V}^{\alpha}_{\mathbf{N}}(\mathbf{q}) - \mathbf{A}^{\alpha}_{\mathbf{N}}(\mathbf{q}))u(\vec{p})$$

$$\mathbf{V}^{\alpha}_{\mathbf{N}}(\mathbf{q}) = 2 \times \left(\mathbf{F}^{\mathbf{V}}_{\mathbf{1}}(\mathbf{q^2})\gamma^{\alpha} + \mathrm{i}\mu_{\mathbf{V}} \frac{\mathbf{F}^{\mathbf{V}}_{\mathbf{2}}(\mathbf{q^2})}{2M} \sigma^{\alpha\nu} q_{\nu} \right)$$

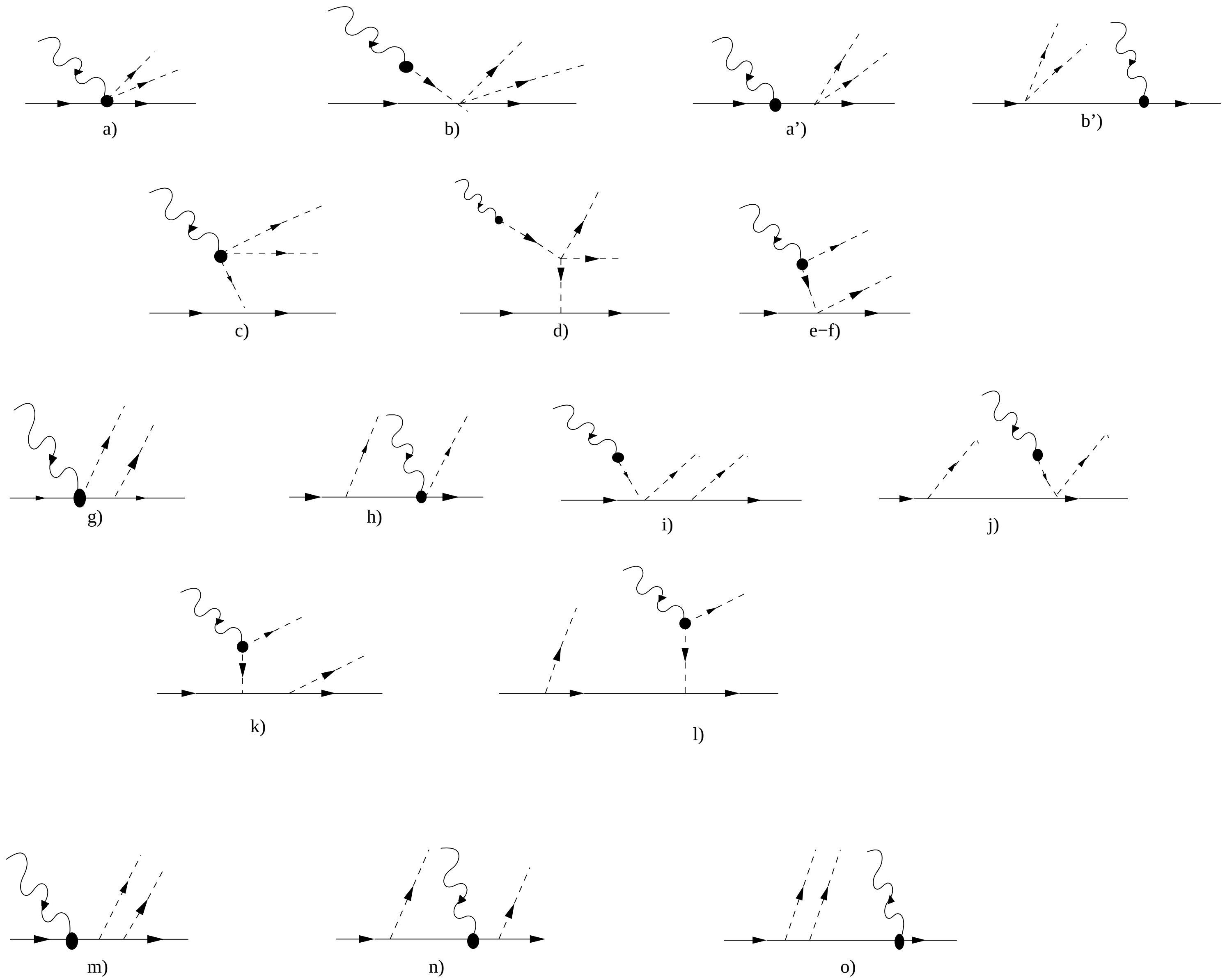
$$\mathbf{A}^{\alpha}_{\mathbf{N}}(\mathbf{q}) = \underbrace{\frac{g_A}{(1 - q^2/M_A^2)^2}}_{\mathbf{G_A}(\mathbf{q^2})} \times \left(\gamma^{\alpha}\gamma_5 + \underbrace{\frac{\not{q}}{m_{\pi}^2 - q^2}}_{\text{PCAC}} q^{\alpha}\gamma_5 \right), \quad \begin{cases} g_A = 1.26 \\ M_A = 1.05 \text{ GeV} \end{cases}$$

$$\mathbf{F}^{\mathbf{V}}_{\mathbf{1}}(\mathbf{q^2}) = \frac{1}{2}\left(\mathbf{F}^{\mathbf{p}}_{\mathbf{1}}(\mathbf{q^2}) - \mathbf{F}^{\mathbf{n}}_{\mathbf{1}}(\mathbf{q^2})\right), \qquad \mu_{\mathbf{V}}\mathbf{F}^{\mathbf{V}}_{\mathbf{2}}(\mathbf{q^2}) = \frac{1}{2}\left(\mu_{\mathbf{p}}\mathbf{F}^{\mathbf{p}}_{\mathbf{2}}(\mathbf{q^2}) - \mu_{\mathbf{n}}\mathbf{F}^{\mathbf{n}}_{\mathbf{2}}(\mathbf{q^2})\right),$$

furthermore **CVC** $\Rightarrow$ $\boxed{F_{PF}(q^2) = F^{V}_{CT}(q^2) = 2F^{V}_{1}(q^2) = F^{p}_{1} - F^{n}_{1}}$

$$
\begin{aligned}
\vec{\mathbf{V}}^{\mu} &= \vec{\phi}\times\partial^{\mu}\vec{\phi} + \frac{g_A}{2f_\pi}\bar{\Psi}\gamma^{\mu}\gamma_5(\vec{\phi}\times\vec{\tau})\Psi + \bar{\Psi}\gamma^{\mu}\frac{\vec{\tau}}{2}\Psi - \frac{\mathbf{1}}{\mathbf{4f_\pi^2}}\bar{\mathbf{\Psi}}\gamma^{\mu}\left[\vec{\tau}\vec{\phi}^{\mathbf{2}} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\mathbf{\Psi} \\
&- \frac{\vec{\phi}^{\mathbf{2}}}{\mathbf{3f_\pi^2}}(\vec{\phi}\times\partial^{\mu}\vec{\phi}) + \mathcal{O}(\frac{\mathbf{1}}{\mathbf{f_\pi^3}}) \\
\vec{\mathbf{A}}^{\mu} &= f_\pi\partial^{\mu}\vec{\phi} + \frac{1}{2f_\pi}\bar{\Psi}\gamma^{\mu}(\vec{\phi}\times\vec{\tau})\Psi + g_A\bar{\Psi}\gamma^{\mu}\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{\mathbf{2}}{\mathbf{3f_\pi}}\left[\vec{\phi}(\vec{\phi}\cdot\partial^{\mu}\vec{\phi}) - \vec{\phi}^{\mathbf{2}}\partial^{\mu}\vec{\phi}\right] \\
&- \frac{\mathbf{g_A}}{\mathbf{4f_\pi^2}}\bar{\mathbf{\Psi}}\gamma^{\mu}\gamma_{\mathbf{5}}\left[\vec{\tau}\vec{\phi}^{\mathbf{2}} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\mathbf{\Psi} + \mathcal{O}(\frac{\mathbf{1}}{\mathbf{f_\pi^3}})
\end{aligned}
$$

$\nu_l N \to l N'\pi\pi$, $\nu_l N \to \nu_l N'\pi\pi$ **close to threshold.** $N^*(1440)$ **degrees of freedom (PRD77 (2008) 053009)**


a)
b)
a')
b')
c)
d)
e−f)
g)
h)
i)
j)
k)
l)
m)
n)
o)

Evaluation of NC $\langle N'\pi|\, j^\mu_{\rm nc}(0)|N\rangle$:

$$j^\mu_{\rm nc} = \bar\Psi_q\gamma^\mu(1-2\sin^2\theta_W-\gamma_5)\boxed{\tau^1_0}\Psi_q - \boxed{4\sin^2\theta_W \mathbf{s}^\mu_{\rm em,IS} - \bar{\mathbf{\Psi}}_{\rm s}\gamma^\mu(\mathbf{1}-\gamma_5)\mathbf{\Psi}_{\rm s}}$$

$$s^\mu_{\rm em} = \underbrace{\frac{1}{6}\bar\Psi_q\gamma^\mu\Psi_q - \frac{1}{3}\bar\Psi_s\gamma^\mu\Psi_s}_{\mathbf{s}^\mu_{\rm em,IS}} + \frac{1}{\sqrt2}\bar\Psi_q\,\gamma^\mu\frac{\boxed{\tau^1_0}}{\sqrt2}\,\Psi_q$$

- ME's j^μ_{cc+} $\Rightarrow$ ME's **isovector** (τ^1_0) $j^\mu_{\rm nc}$ **contribution**
- **Δ does not contribute to the isoscalar $j^\mu_{\rm nc}$ part**

$$\langle n\pi^+|s^\mu_{{\rm em},IS}|p\rangle = \langle p\pi^-|s^\mu_{{\rm em},IS}|n\rangle = \sqrt2\langle \mathbf{p}\pi^{\mathbf{0}}|\mathbf{s}^\mu_{\rm em,IS}|\mathbf{p}\rangle = -\sqrt2\langle n\pi^0|s^\mu_{{\rm em},IS}|n\rangle$$

$$\langle p\pi^0|s^\mu_{{\rm em},IS}|p\rangle = -\frac{\langle n\pi^0|s^\mu_{\rm em}(0)|n\rangle - \langle p\pi^0|s^\mu_{\rm em}(0)|p\rangle}{2}$$

$$\mathbf{s}^\mu_{\rm em} = \underbrace{\bar\Psi\gamma^\mu\left(\frac{1+\tau_z}{2}\right)\Psi}_{\rm PN,PNC} + \underbrace{\frac{\mathrm{i}g_A}{2f_\pi}\bar\Psi\gamma^\mu\gamma_5\left(\tau^1_{-1}\phi^\dagger+\tau^1_{+1}\phi\right)\Psi}_{\rm CT} + \mathrm{i}\underbrace{\left(\phi^\dagger\partial^\mu\phi-\phi\partial^\mu\phi^\dagger\right)}_{PF}+\cdots$$

CT, PF do not contribute $\Rightarrow$ PN and PNC $\Rightarrow$ ME's of $s^\mu_{{\rm em},IS}$

- ME's $\mathbf{j}^\mu_{\rm nc,str} = \bar{\mathbf{\Psi}}_{\mathbf{s}}\gamma^\mu(\mathbf{1}-\gamma_{\mathbf{5}})\mathbf{\Psi}_{\mathbf{s}}$: *nucleon strange content*

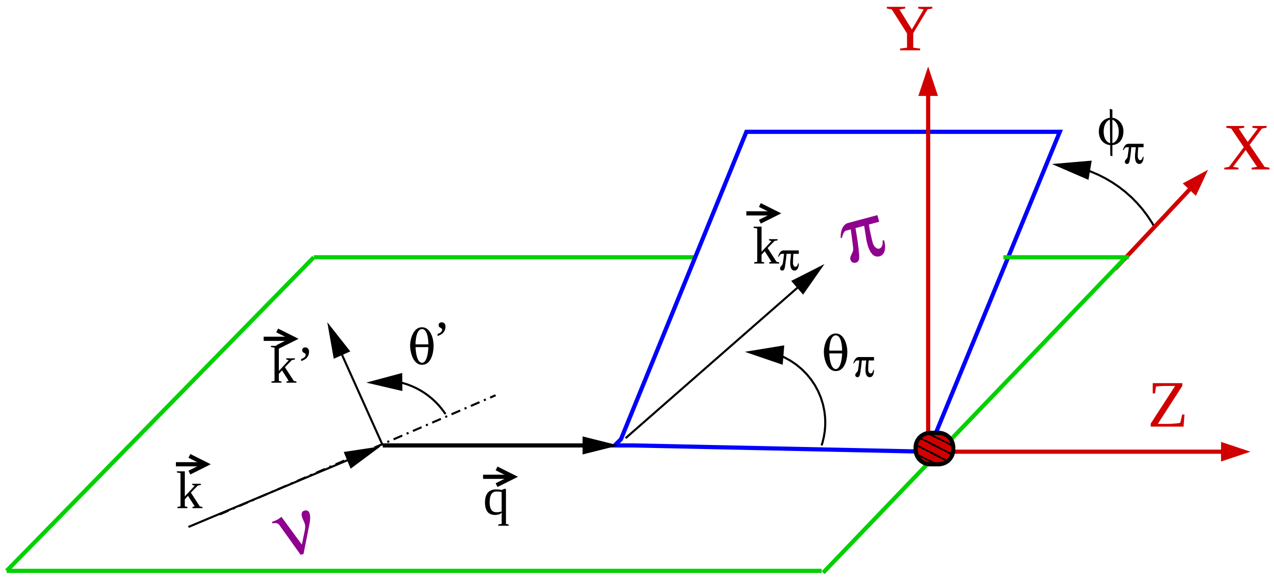
$$\langle p\pi^0|\mathbf{j}^\mu_{\rm nc,str}(\mathbf{0})|p\rangle = -\mathrm{i}\,\frac{g_A}{2f_\pi}\bar u(\vec p\,')\Big\{\not k_\pi\gamma_5\frac{\not p+\not q+M}{(p+q)^2-M^2+i\epsilon}\left[\mathbf{V}^\mu_{\mathbf{N,s}}(\mathbf{q})-\mathbf{A}^\mu_{\mathbf{N,s}}(\mathbf{q})\right]$$

$$+\left[\mathbf{V}^\mu_{\mathbf{N,s}}(\mathbf{q})-\mathbf{A}^\mu_{\mathbf{N,s}}(\mathbf{q})\right]\frac{\not p'-\not q+M}{(p'-q)^2-M^2+i\epsilon}\not k_\pi\gamma_5\Big\}u(\vec p) \quad \Leftarrow \mathbf{PN}+\mathbf{PNC}$$

$$\mathbf{V}^\mu_{\mathbf{N,s}}(\mathbf{q}) = \underbrace{F_1^s(q^2)}_{\approx 0}\gamma^\mu + \mathrm{i}\mu_s\frac{\overbrace{F_2^s(q^2)}^{\approx 0}}{2M}\sigma^{\mu\nu}q_\nu, \qquad \mathbf{A}^\mu_{\mathbf{N,s}}(\mathbf{q}) = \overbrace{G_A^s(q^2)}^{\boxed{g_S}\,(\mathbf{1}-\mathbf{q^2}/\mathbf{M_A^2})^{-2}}\gamma^\mu\gamma_5 + \underbrace{G_P^s}_{\rm don't\ contr.}q^\mu\gamma_5$$

Results :

$$\Rightarrow \mathrm{CC}: \nu_l(k) + N(p) \to l^-(k') + N(p') + \pi(k_\pi)$$



$$\frac{\mathbf{d^5}\sigma_{\nu_l \mathbf{l}}}{\mathbf{d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)}} = \frac{|\vec{k}'|}{|\vec{k}\ |}\frac{G^2}{4\pi^2}\int_0^{+\infty}\frac{d|\vec{k}_\pi||\vec{k}_\pi|^2}{E_\pi}\boxed{L^{(\nu)}_{\mu\sigma}\left(W^{\mu\sigma}_{\mathrm{CC}\pi}\right)^{(\nu)}}$$

$$(W^{\mu\sigma}_{\rm CC\pi})^{(\nu)} = \frac{1}{4M}\overline{\sum_{\rm spins}}\int \frac{d^3p'}{(2\pi)^3}\frac{1}{2E'_N}\delta^4(p'+k_\pi-q-p)\langle \mathbf{N}'\pi|\mathbf{j}^\mu_{\rm cc+}(\mathbf{0})|\mathbf{N}\rangle\langle \mathbf{N}'\pi|\mathbf{j}^\sigma_{\rm cc+}(\mathbf{0})|\mathbf{N}\rangle^*$$

$$\mathbf{L}^{(\nu)}_{\mu\sigma} = (\mathbf{L}^{(\nu)}_{\mathbf{s}})_{\mu\sigma} + {\rm i}(\mathbf{L}^{(\nu)}_{\mathbf{a}})_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma}k\cdot k' + {\rm i}\epsilon_{\mu\sigma\alpha\beta}k'^\alpha k^\beta$$

$$\Rightarrow\ {\rm CC}:\ \bar\nu_l(k) + N(p) \to l^+(k') + N(p') + \pi(k_\pi)$$

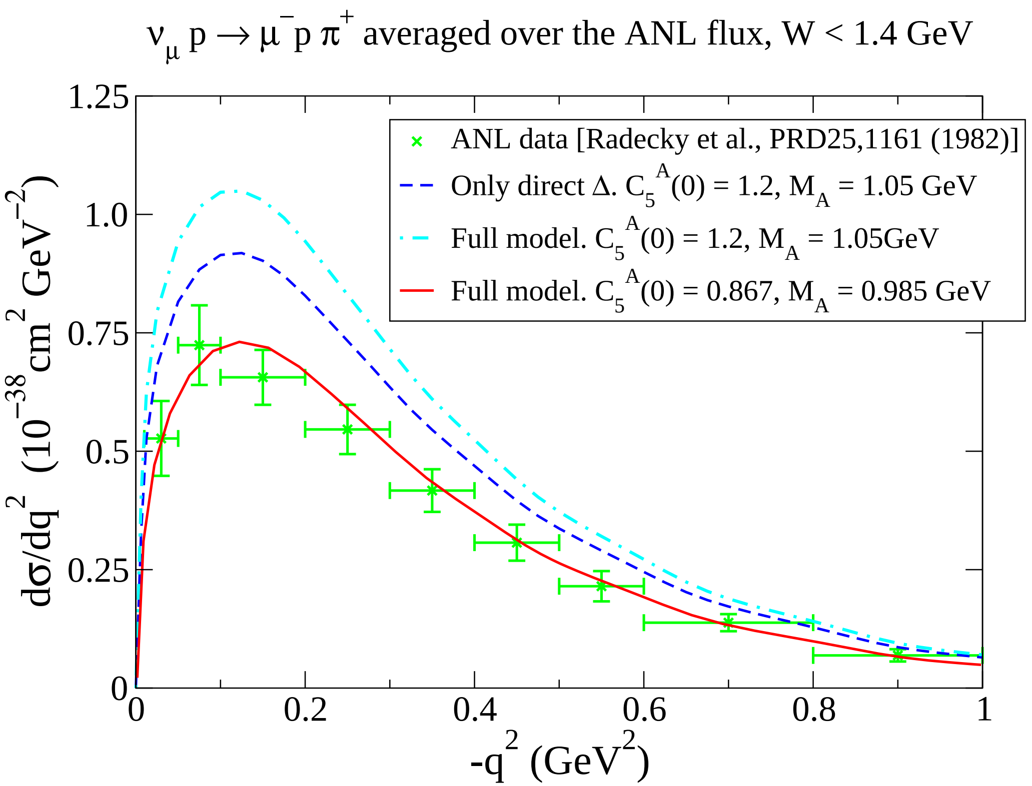
$$\mathbf{L}^{(\bar\nu)}_{\mu\sigma} = \mathbf{L}^{(\nu)}_{\sigma\mu},\quad \mathbf{j}^\sigma_{\rm cc+} \leftrightarrow \mathbf{j}^\sigma_{\rm cc-}$$

$$\Rightarrow\ {\rm NC}:\ \nu(k) + N(p) \to \nu(k') + N(p') + \pi(k_\pi)$$

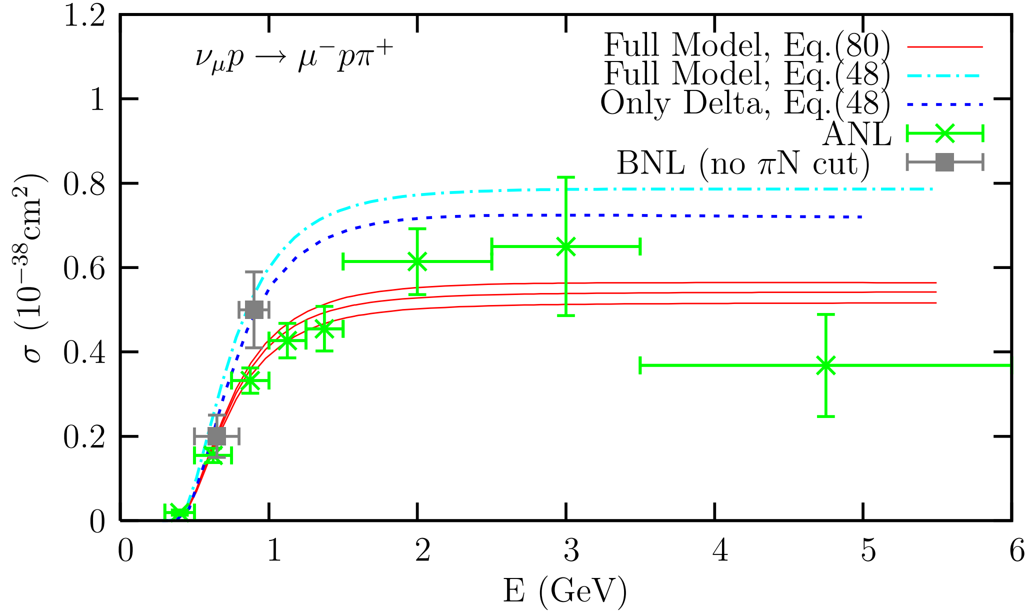
$$\mathbf{j}^\sigma_{\rm cc+} \leftrightarrow \frac{\mathbf{1}}{\mathbf{2}}\mathbf{j}^\sigma_{\rm nc},\quad (\mathbf{W}^{\mu\sigma}_{\rm NC\pi})^{(\nu)} = (\mathbf{W}^{\mu\sigma}_{\rm NC\pi})^{(\bar\nu)}$$

Note $\underbrace{(E',\theta')}_{\text{outgoing lepton}} \leftrightarrow q^2, \underbrace{W^2 = (p+q)^2}_{\pi\text{N inv. mass}}$

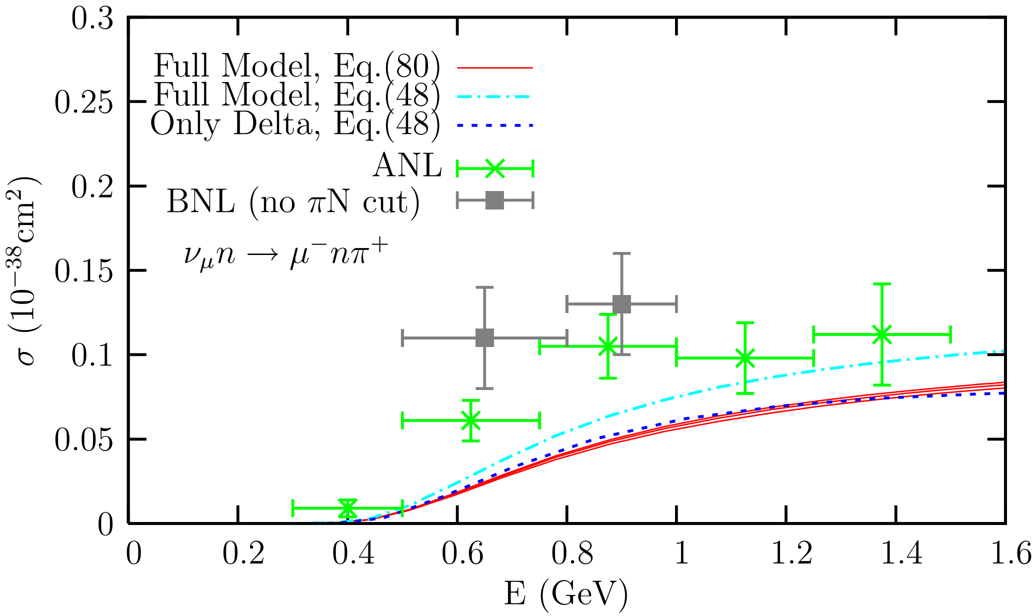
$$\int_{M+m_\pi}^{\boxed{1.4\,\mathrm{GeV}}} dW \frac{d\,\overline{\sigma}_{\nu_\mu \mu^-}}{dq^2 dW}, \qquad \nu_\mu \mathbf{p} \to \mu^- \mathbf{p} \pi^+$$



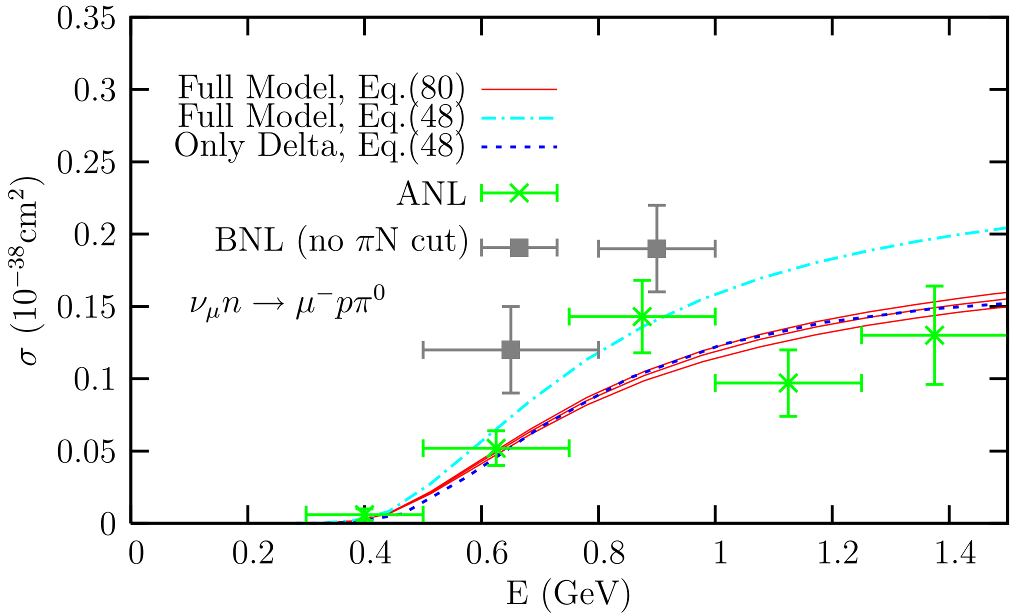
Fit to ANL : $C_5^A(0) = 0.867 \pm 0.075, \quad M_{A\Delta} = 0.985 \pm 0.082\,\mathrm{GeV}$



$\nu_\mu p \to \mu^- p \pi^+$
Full Model, Eq.(80)
Full Model, Eq.(48)
Only Delta, Eq.(48)
ANL
BNL (no πN cut)
σ (10^{-38}cm^2)
E (GeV)



Full Model, Eq.(80)
Full Model, Eq.(48)
Only Delta, Eq.(48)
ANL
BNL (no πN cut)
$\nu_\mu n \to \mu^- n \pi^+$
σ (10^{-38}cm^2)
E (GeV)



Full Model, Eq.(80)
Full Model, Eq.(48)
Only Delta, Eq.(48)
ANL
BNL (no πN cut)
$\nu_\mu n \to \mu^- p \pi^0$
σ (10^{-38}cm^2)
E (GeV)

How to reconcile ANL & BNL data and still have $C_5^A(0) \sim 1.2$

K.M. Graczyk et al. [Phys. Rev. D 80, 093001 (2009)]

- ANL and BNL data were measured in deuterium
  - Deuteron effects were estimated by L. Alvarez-Ruso et al [Phys. Rev. C 59, 3386 (1999)] to reduce the cross section by 5-10%.
- Large uncertainties in the neutrino flux normalization, 10% for BNL data and 20% for ANL data.

K.M. Graczyk et al. made a combined fit to both ANL&BNL data, assuming that only the Δ mechanism contributed, including deuteron effects, and treating flux uncertainties as systematic errors. They found

$$C_5^A(0) = 1.19 \pm 0.08, \qquad M_{A\Delta} = 0.94 \pm 0.03\,\text{GeV}$$

for a pure dipole parameterization for $C_5^A(q^2)$. Good agreement with the off-diagonal GTR is found! **No background terms included !**

Background terms included

PRD 81 085046 (2010): We included **background terms in a combined fit to ANL & BNL data that took into account deuteron effects and flux normalization uncertainties.**

We used a simpler dipole parameterization for $C_5^A(q^2)$

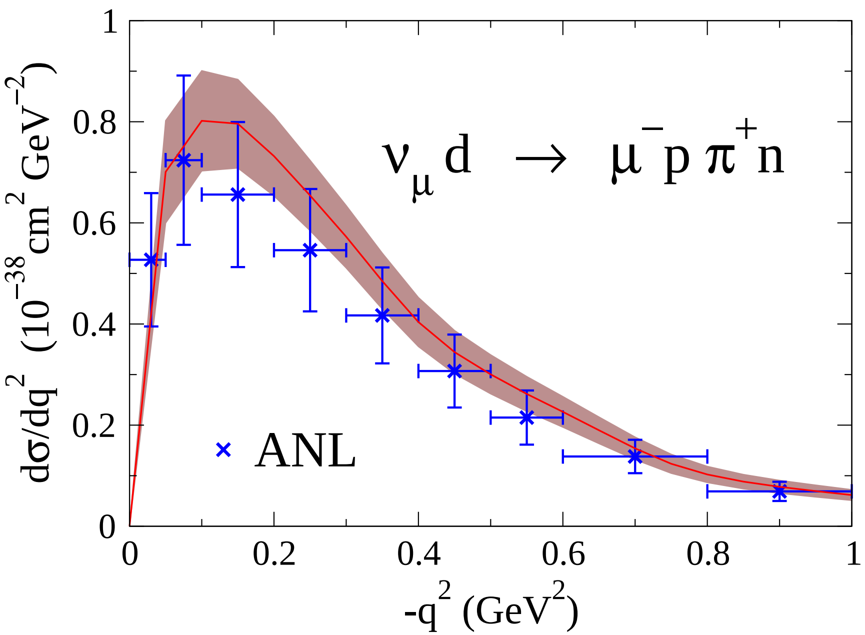
$$C_5^A(q^2) = \frac{C_5^A(0)}{\left(1 - q^2/M_{A\Delta}^2\right)^2}$$

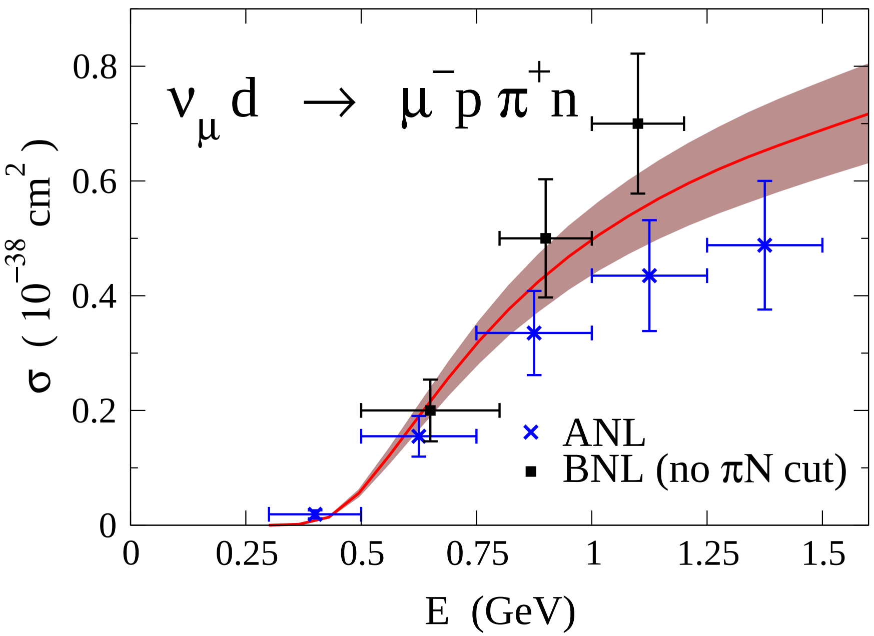
Using Adler's constraints we obtained

$$C_5^A(0) = 1.00 \pm 0.11, \qquad M_{A\Delta} = 0.93 \pm 0.07\,\mathrm{GeV}$$

$C_5^A(0)$ **compatible with its GTR value** (~ 1.2) **at the** 2σ **level**.

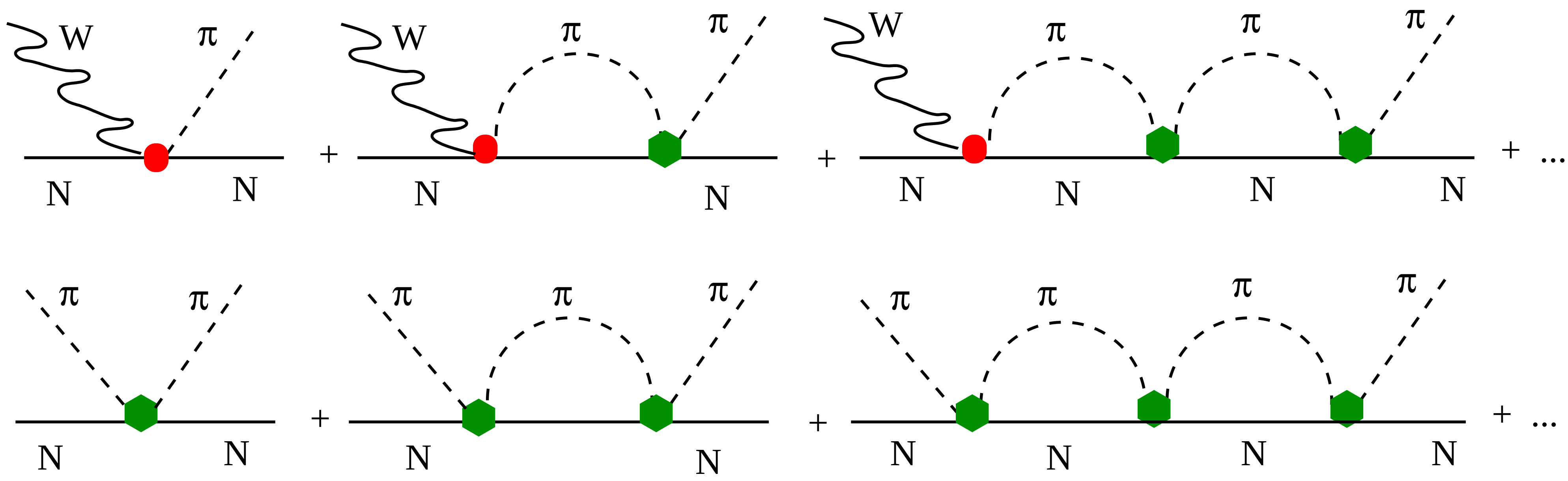
Comparison with ANL & BNL data





68% confidence level bands are shown. The total experimental errors shown contain flux uncertainties that are considered as systematic errors and have been added in quadratures to the statistical ones.

Watson's final-state-interaction theorem (unitarity and time–reversal invariance): The phase of an amplitude leading to a final state with two strongly interacting particles in a given partial wave is the same as the scattering phase of that pair, $\boxed{\delta}$. **[PRD 88 (1952) 1163]**



Optical theorem in partial waves

$$
\begin{aligned}
SS^\dagger = 1 \quad &\Leftrightarrow \quad i\left(T - T^\dagger\right) = T^\dagger T \\
a + b \quad &\rightarrow \quad 1 + 2 \\
i\left[\langle\lambda_1\lambda_2|T_J|\lambda_a\lambda_b\rangle - \langle\lambda_a\lambda_b|T_J|\lambda_1\lambda_2\rangle^*\right] \quad &\sim \quad \sum_{\lambda_1'\lambda_2'} \langle\lambda_1\lambda_2|T_J^\dagger|\lambda_1'\lambda_2'\rangle\langle\lambda_1'\lambda_2'|T_J|\lambda_a\lambda_b\rangle
\end{aligned}
$$

Optical theorem in partial waves

$$SS^\dagger = 1 \quad \Leftrightarrow \quad i\left(T - T^\dagger\right) = T^\dagger T$$

$$a + b \quad \rightarrow \quad 1 + 2$$

$$i[\langle\lambda_1\lambda_2|T_J|\lambda_a\lambda_b\rangle - \langle\lambda_a\lambda_b|T_J|\lambda_1\lambda_2\rangle^*] \quad \sim \quad \sum_{\lambda_1'\lambda_2'} \langle\lambda_1\lambda_2|T_J^\dagger|\lambda_1'\lambda_2'\rangle\langle\lambda_1'\lambda_2'|T_J|\lambda_a\lambda_b\rangle$$

Using CM helicity states $|p; JM\lambda_1\lambda_2\rangle$ and **invariance under time reversal,**

$$\underbrace{\langle\lambda_1\lambda_2|T_J|\lambda_a\lambda_b\rangle}_{\mathbf{a+b\to 1+2}} = \underbrace{\langle\lambda_a\lambda_b|T_J|\lambda_1\lambda_2\rangle}_{\mathbf{1+2\to a+b}}$$

$$\boxed{\mathbb{R} \ni \mathrm{Im}\langle\lambda_1\lambda_2|T_J|\lambda_a\lambda_b\rangle \sim \sum_{\lambda_1'\lambda_2'}\langle\lambda_1\lambda_2|T_J^\dagger|\lambda_1'\lambda_2'\rangle\langle\lambda_1'\lambda_2'|T_J|\lambda_a\lambda_b\rangle \in \mathbb{R}}$$

Considering **intermediate strong interacting πN states**, Watson's theorem for the **weak $WN \to N\pi$ process** implies,

$$\sum_{\lambda''_N} \underbrace{\langle \lambda'_N}_{N\pi} |T_J^\dagger(s)| \underbrace{\lambda''_N\rangle\langle\lambda''_N}_{N\pi} |T_J(s)| \underbrace{\lambda_N\lambda_W}_{NW}\rangle \in \mathbb{R}$$

In terms πN $|p; LSJM\rangle$ states

$$\sum_L \sqrt{\frac{2L+1}{2J+1}}(L\frac{1}{2}J|0\lambda'_N\lambda'_N)\underbrace{\langle L\frac{1}{2}J|T_J|L\frac{1}{2}J\rangle^*}_{\pi N\to\pi N}\underbrace{\langle L\frac{1}{2}J|T_J|\lambda_N\lambda_W\rangle}_{WN\to N\pi} \in \mathbb{R}$$

For $J = 3/2, T = 3/2$ and neglecting the $L = 2$ multipole,

$$\left\langle \mathbf{P_{33}} \left| \mathbf{T^{WN\to N\pi}_{J=\frac{3}{2},T=\frac{3}{2}}} \right| \mathbf{J = \frac{3}{2}, M = \lambda_N - \lambda_W, \lambda_N\lambda_W} \right\rangle \times \underbrace{e^{-i\delta_{P_{33}}(s)}}_{\mathbf{L_{2J2T}}\ \mathbf{N}\pi\ \text{phase shift}} \in \mathbb{R}$$

There is a total of 6 $[(\lambda_N = \pm\frac{1}{2}) \times (\lambda_W = 0, \pm 1)]$ amplitudes which should have the same phase ($\delta_{P_{33}}(s), s = (p_N + p_\pi)^2$).

Using CM three momentum helicity states $|p;\theta\phi\lambda_1\lambda_2\rangle$

$$|P_{33}M\rangle = \int d\Omega \sum_\lambda \sqrt{\frac{3}{4\pi}} \mathcal{D}^{\frac{3}{2}*}_{M\lambda}(\phi,\theta,-\phi)\left(1\frac{1}{2}\frac{3}{2}|0\lambda\lambda\right)|p;\theta\phi\lambda\rangle$$

$$|p;\theta=0\,\phi=0\lambda_N\lambda_W\rangle = \sum_J \sqrt{\frac{2J+1}{4\pi}}|p;JM=\lambda_N-\lambda_W,\lambda_N\lambda_W\rangle$$

$$\int \mathbf{d\Omega} \sum_\lambda \mathcal{D}^{\frac{3}{2}}_{\lambda_N-\lambda_W\lambda}(\phi,\theta,-\phi)\left(\mathbf{1}\frac{\mathbf{1}}{\mathbf{2}}\frac{\mathbf{3}}{\mathbf{2}}|\mathbf{0}\lambda\lambda\right)\underbrace{\left\langle p';\theta\phi\lambda\left|T^{WN\to N\pi}_{J=\frac{3}{2},T=\frac{3}{2}}\right|p;00\lambda_N\lambda_W\right\rangle}_{\text{related to } \bar{\mathbf{u}}(\mathbf{p}',\lambda)(\mathbf{O}_\mu\epsilon^\mu_{\lambda_\mathbf{W}})\mathbf{u}(\mathbf{p},\lambda_\mathbf{N})}\mathbf{e}^{-\mathbf{i}\delta_{\mathbf{P_{33}}}} \in \mathbb{R}$$

There is a total of 6 $[(\lambda_N = \pm\frac{1}{2})\times(\lambda_W = 0,\pm 1)]$ amplitudes which should have the same phase ($\delta_{P_{33}}(s), s=(p_N+p_\pi)^2$).

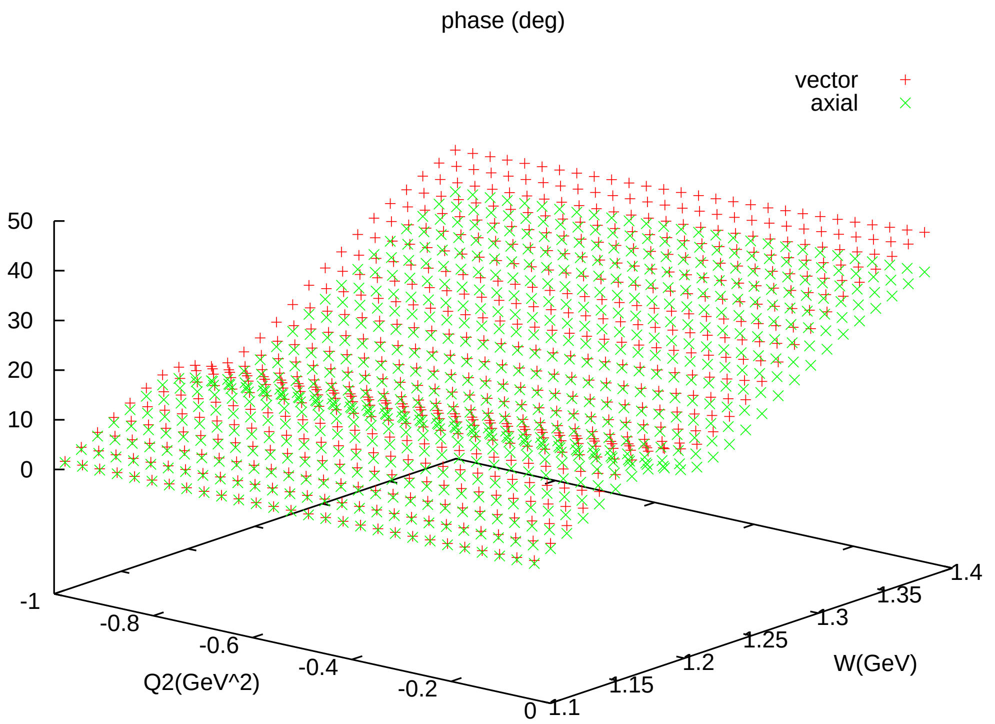
We force the correct phase for <u>two</u> different linear combinations of these amplitudes that correspond to the two multipoles where the Δ mechanism (vector and axial contributions) is dominant. For instance, in the case of the vector Δ contribution, this is the M_{1+} multipole. We denote the corresponding axial multipole as $\mathcal{A}_\Delta$.

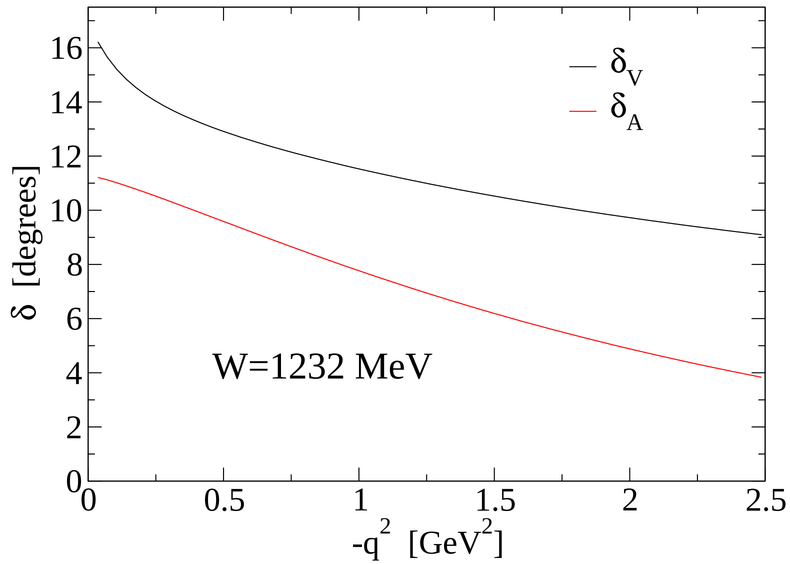
We follow a generalization of **M.G. Olsson's procedure [NPB 78 (1974) 55]** introducing two small phases $\phi_{\mathbf{V,A}}(\mathbf{s},\mathbf{q^2})$ which correct the vector and axial Δ contributions such that

$$\mathrm{Im}\left[\left(T_\Delta^{V,A}(s,q^2)\mathbf{e}^{\mathbf{i}\phi_{\mathbf{V,A}}(\mathbf{s},\mathbf{q^2})} + T_B^{V,A}(s,q^2)\right)^{M_{1+};\mathcal{A}_\Delta}\mathbf{e}^{-\mathbf{i}\delta_{\mathbf{P_{33}}}(\mathbf{s})}\right] = 0$$

$$\Gamma^{\alpha\mu} = \left[\frac{\mathbf{C_3^A}}{M} \left(g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu \right) + \frac{\mathbf{C_4^A}}{M^2} \left(g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p_\Delta^\mu \right) + \mathbf{C_5^A} g^{\alpha\mu} + \frac{\mathbf{C_6^A}}{M^2} q^\mu q^\alpha \right] \mathbf{e^{i\phi_A(s,q^2)}}$$

$$+ \left[\frac{\mathbf{C_3^V}}{M} \left(g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu \right) + \frac{\mathbf{C_4^V}}{M^2} \left(g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p_\Delta^\mu \right) + \frac{\mathbf{C_5^V}}{M^2} \left(g^{\alpha\mu} q \cdot p - q^\alpha p^\mu \right) \right] \mathbf{e^{i\phi_V(s,q^2)}} \gamma_5$$





We include **chiral background terms in a combined fit to ANL & BNL data that takes into account deuteron effects, flux normalization uncertainties and unitarity corrections (Watson's theorem)**

We use a simpler dipole parameterization for $C_5^A(q^2)$

$$C_5^A(q^2) = \frac{C_5^A(0)}{\left(1 - q^2/M_{A\Delta}^2\right)^2}$$

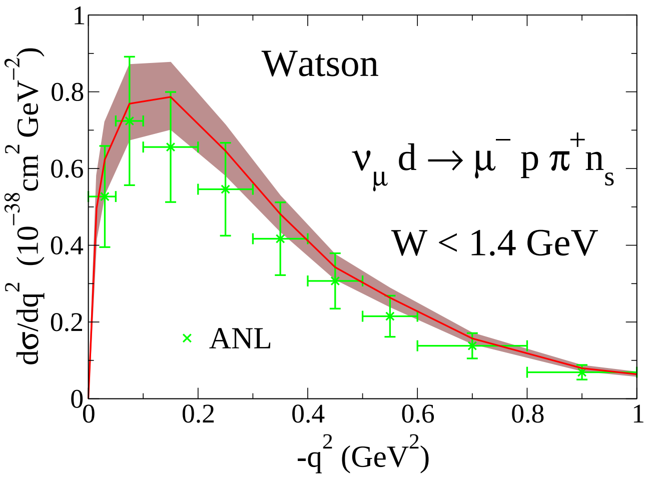
Using Adler's constraints we obtain **(preliminary results)**

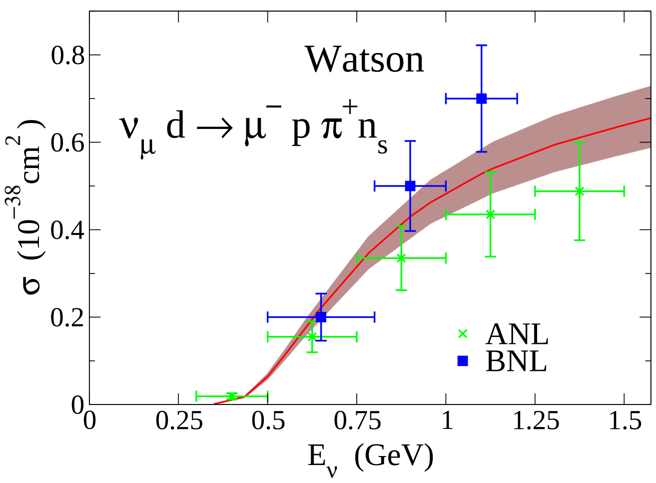
$$C_5^A(0) = 1.12 \pm 0.11, \quad M_{A\Delta} = 0.95 \pm 0.06\,\text{GeV}, \quad \text{(with unitarity corrections)}$$

$$C_5^A(0) = 1.00 \pm 0.11, \quad M_{A\Delta} = 0.93 \pm 0.07\,\text{GeV}, \quad \text{(without unitarity corrections)}$$

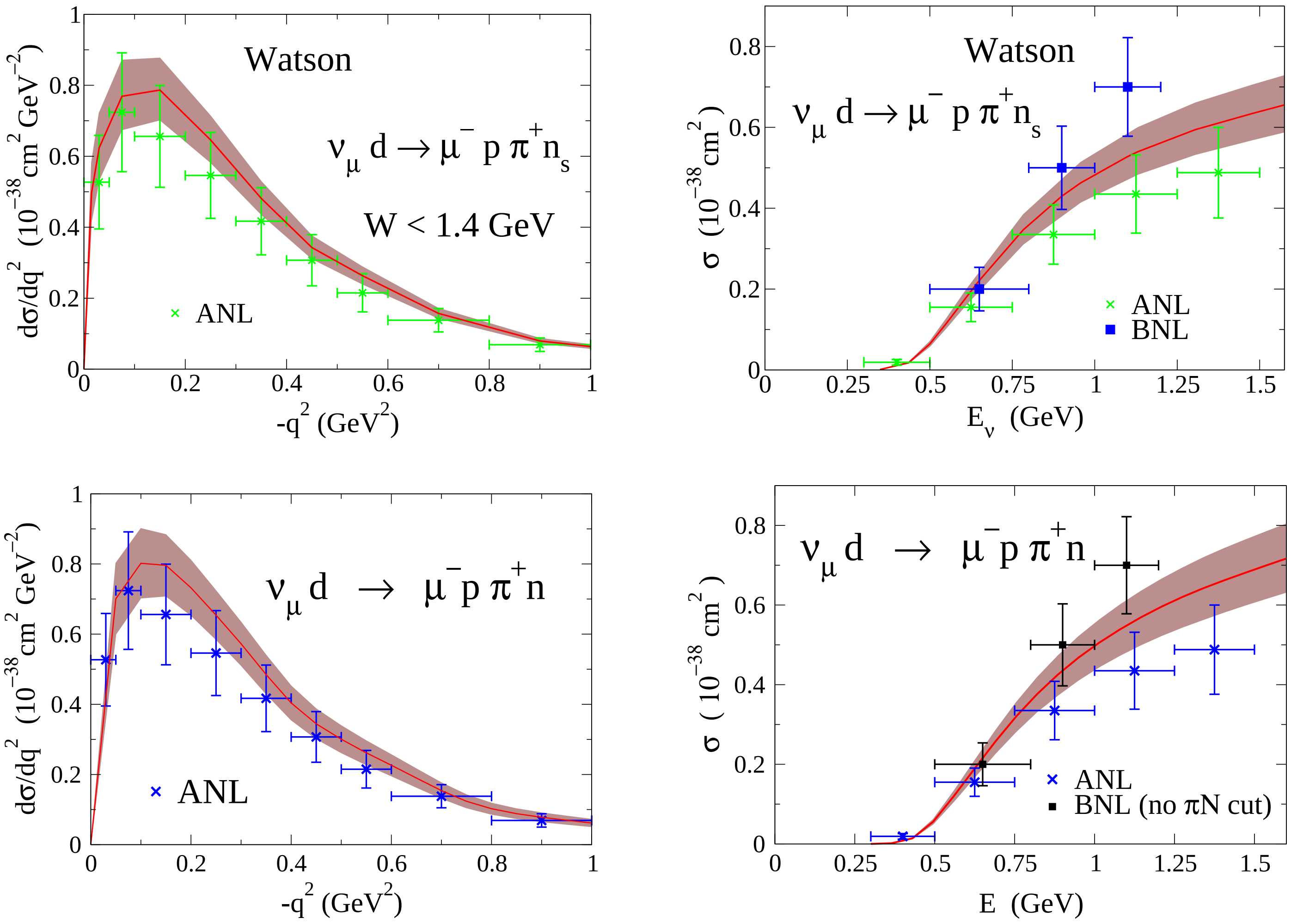
$C_5^A(0)$ **compatible with its GTR value** (~ 1.2) **at the** 1σ (2σ) **level.**

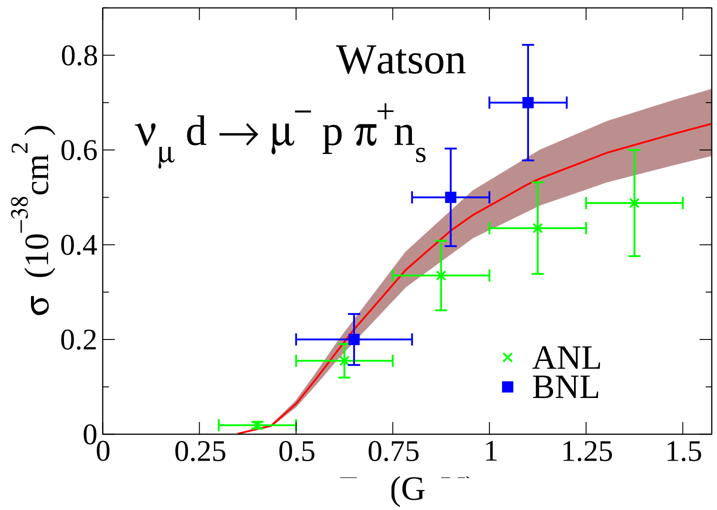
Comparison with ANL & BNL data

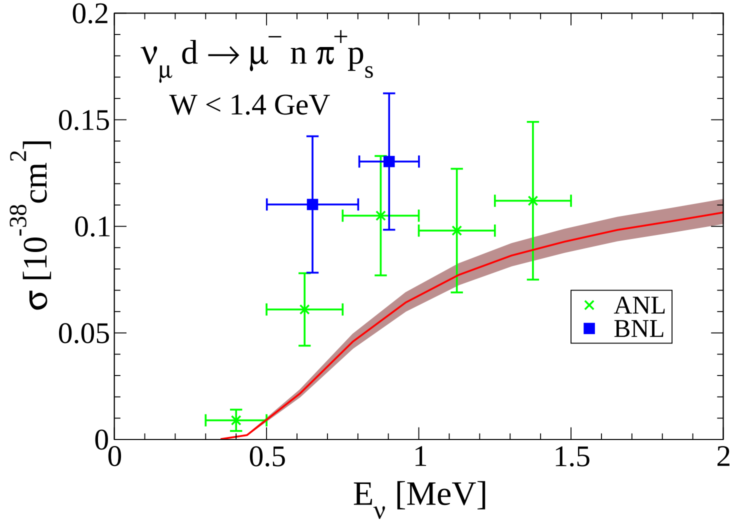


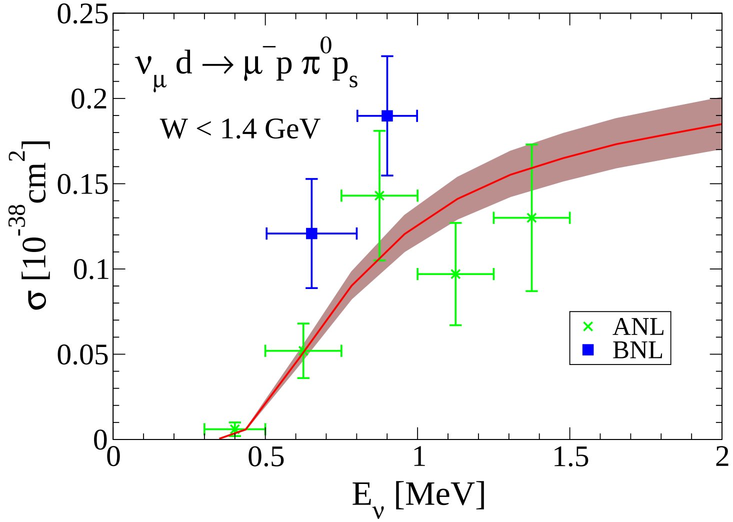


Comparison with ANL & BNL data

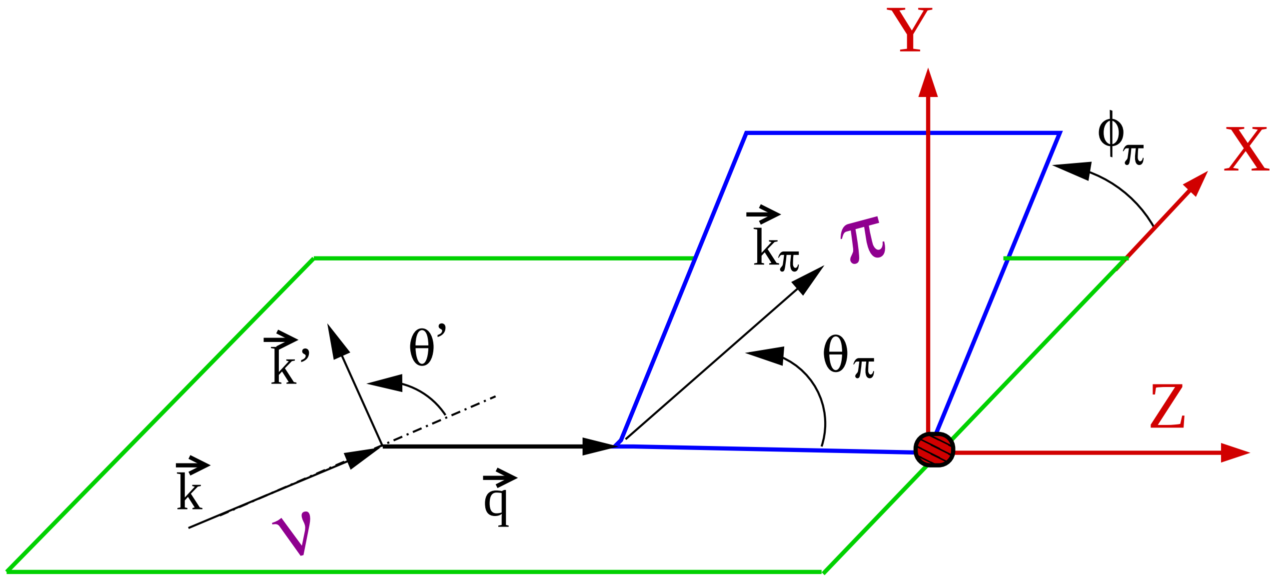



Watson
$\nu_\mu\, d \rightarrow \mu^- p\, \pi^+ n_s$
ANL
BNL
σ (10^{-38}cm^2)
0
0.2
0.4
0.6
0.8
0
0.25
0.5
0.75
1
1.25
1.5


$\nu_\mu\, d \rightarrow \mu^- n\, \pi^+ p_s$
W < 1.4 GeV
ANL
BNL
σ [10^{-38}cm^2]
E_ν [MeV]
0
0.05
0.1
0.15
0.2
0
0.5
1
1.5
2


$\nu_\mu\, d \rightarrow \mu^- p\, \pi^0 p_s$
W < 1.4 GeV
ANL
BNL
σ [10^{-38}cm^2]
E_ν [MeV]
0
0.05
0.1
0.15
0.2
0.25
0
0.5
1
1.5
2

Parity violation



$$\frac{d^5\sigma_{\nu_l l}}{d\Omega(\hat{k}')dE'd\Omega^*(\hat{k}_\pi)} = \frac{|\vec{k}'|}{|\vec{k}\,|}\frac{G^2}{4\pi^2}\Big\{ \overbrace{A^* + B^*\cos\phi^*_\pi + C^*\cos 2\phi^*_\pi}^{\text{Similar to } eN\to e'N\pi} + \underbrace{D^*\sin\phi^*_\pi + E^*\sin 2\phi^*_\pi}_{\text{parity violating}} \Big\}$$

$$L^{(\nu)}_{\mu\sigma} = (\mathbf{L}^{(\nu)}_{\mathbf{s}})_{\mu\sigma} + \mathrm{i}(\mathbf{L}^{(\nu)}_{\mathbf{a}})_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma} k\cdot k' + \mathrm{i}\epsilon_{\mu\sigma\alpha\beta} k'^\alpha k^\beta$$

By construction (similar for both CC and NC),

$$W^{\mu\sigma} = \mathbf{W}^{\mu\sigma}_{\mathbf{s}} + \mathrm{i}\mathbf{W}^{\mu\sigma}_{\mathbf{a}}, \quad W^{\mu\nu}_{s,a} = \left(W^{\mu\nu}_{s,a}\right)^{\mathrm{PC}} + \left(\mathbf{W}^{\mu\nu}_{\mathbf{s,a}}\right)^{\mathrm{PV}}$$

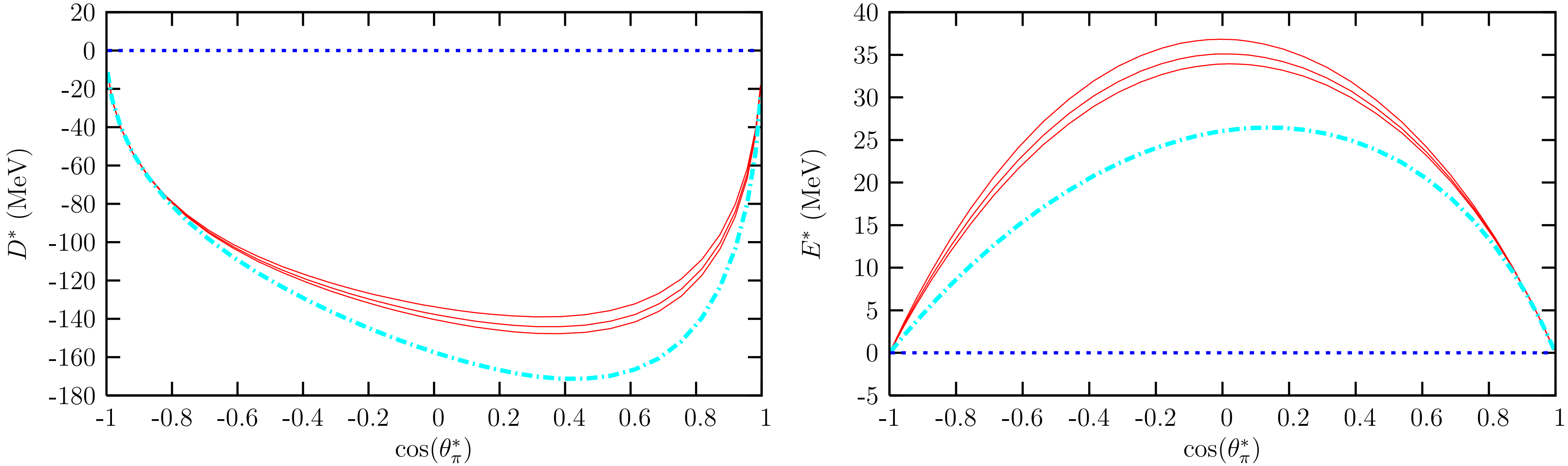
$$\begin{aligned}
(W^{\mu\nu}_s)^{\mathrm{PC}} &= W_1 g^{\mu\nu} + W_2 p^\mu p^\nu + W_3 q^\mu q^\nu + W_4 k^\mu_\pi k^\nu_\pi + \cdots \\
(W^{\mu\nu}_a)^{\mathrm{PC}} &= W_{14}\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta + W_{15}\epsilon^{\mu\nu\alpha\beta} p_\alpha k_{\pi\beta} + W_{16}\epsilon^{\mu\nu\alpha\beta} q_\alpha k_{\pi\beta} + \cdots \\
(\mathbf{W}^{\mu\nu}_{\mathbf{s}})^{\mathrm{PV}} &= \mathbf{W_8}\left(\mathbf{q}^\mu \epsilon^\nu_{.\alpha\beta\gamma}\mathbf{k}^\alpha_\pi \mathbf{p}^\beta \mathbf{q}^\gamma + \mathbf{q}^\nu \epsilon^\mu_{.\alpha\beta\gamma}\mathbf{k}^\alpha_\pi \mathbf{p}^\beta \mathbf{q}^\gamma\right) + \cdots \\
(\mathbf{W}^{\mu\nu}_{\mathbf{a}})^{\mathrm{PV}} &= \mathbf{W_{11}}(\mathbf{q}^\mu \mathbf{p}^\nu - \mathbf{q}^\nu \mathbf{p}^\mu) + \mathbf{W_{12}}(\mathbf{q}^\mu \mathbf{k}^\nu_\pi - \mathbf{q}^\nu \mathbf{k}^\mu_\pi) + \cdots
\end{aligned}$$

Under Parity

$$L^{(\nu)}_{\mu\nu} \to (L^{\nu\mu})^{(\nu)}, \quad (W_{\mu\nu})^{\mathrm{PC}} \to (W^{\nu\mu})^{\mathrm{PC}}, \quad (\mathbf{W}_{\mu\nu})^{\mathrm{PV}} \to -(\mathbf{W}^{\nu\mu})^{\mathrm{PV}}$$

- $d^5\sigma/d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)$ **is not inv. under parity,** since the pseudovector $\vec{k}\times\vec{k}'$ is used to define the Y axis.

- $d^3\sigma/d\Omega(\hat{k}')dE'$ **scalar,** except for the factor $|\vec{k}'|/|\vec{k}\,|$ $\Rightarrow$ parity violation **disappears** when performing the $\int d\Omega^*(\hat{k}_\pi)$



$\nu_\mu n \to \mu^- p\pi^0$ ($E = 1.5$ GeV, $W = M_\Delta$, $q^2 = -0.5$ GeV^2)

- **Non-resonant terms are needed to produce non-vanishing parity violating structure functions**

Conclusions: **We have derived a model** for CC and NC weak pion production off the nucleon

1. **In addition to the** Δ resonance, we include **non-resonant contributions** $\Leftarrow$ **QCD SχSB.**

2. **Non resonant contributions** are important $\Rightarrow$ **re-adjust of** $C_5^A(q^2)$. **GTR prediction** $C_5^A(0) \sim 1.2$.

   - Fit to ANL $\Rightarrow C_5^A(0) = 0.867 \pm 0.075$
   - Fit to ANL & BNL + normalization uncertainties + deuteron effects $\Rightarrow C_5^A(0) = 1.00 \pm 0.11$
   - Fit to ANL & BNL + normalization uncertainties + deuteron effects + unitarity corrections (Watson's theorem) $\Rightarrow$ $\mathbf{C_5^A(0) = 1.12 \pm 0.11}$

3. $\nu - \bar{\nu}$ **Asymmetries**, **distinguish ν_τ from $\bar{\nu}_\tau$?**
4. **Parity violation effects due to the interferences between the non resonant and Δ contributions.**
5. **Starting point** to study **inclusive and exclusive neutrino-nucleus scattering above the QE region.**
6. **Higher $W(N\pi)$, we include the $N^*(1520)$ resonance**

Back up material

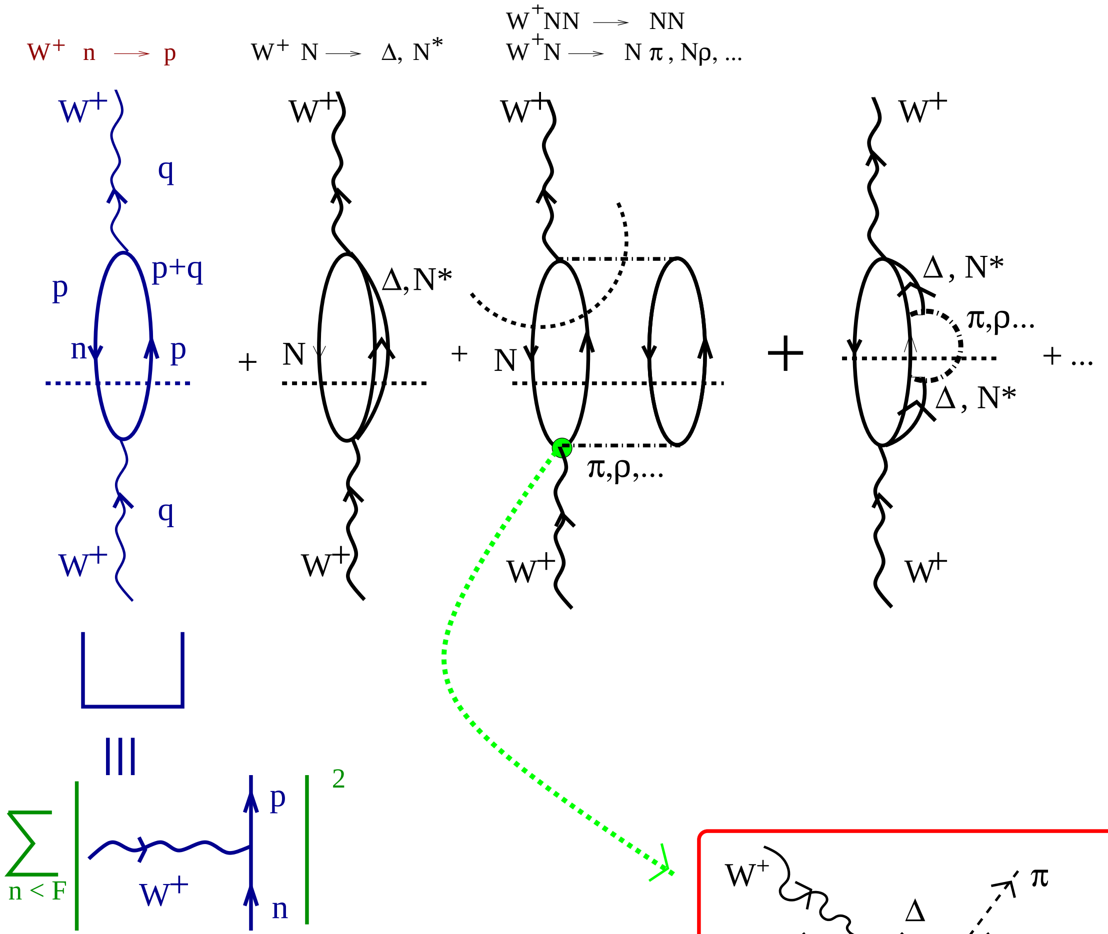
In some of the fits we relaxed Adler's constraints allowing

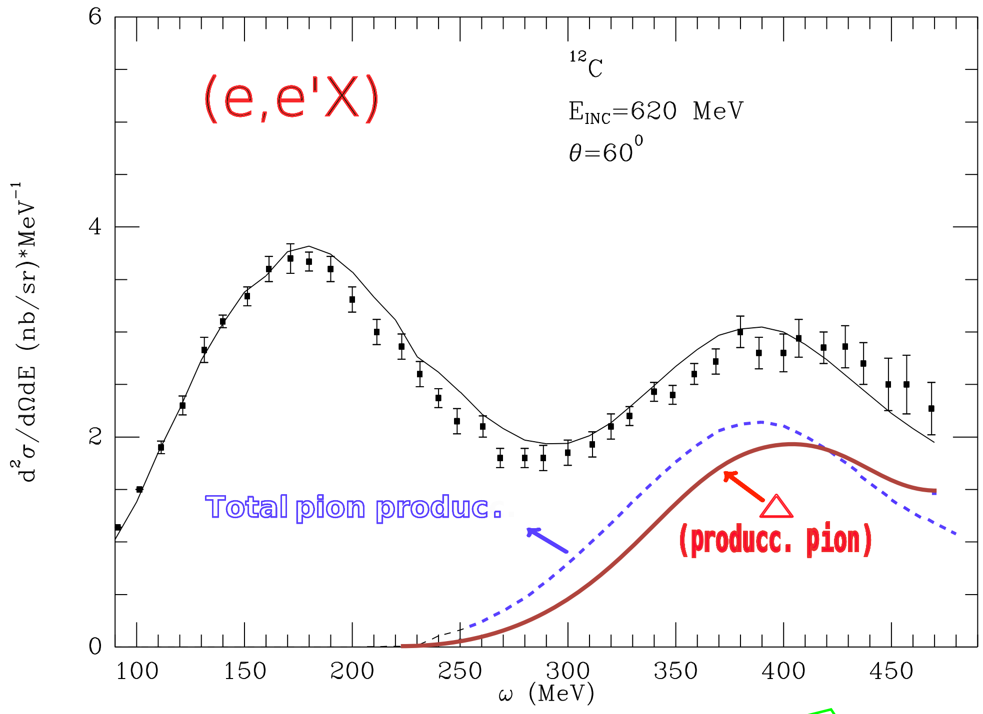
$$C^A_{3,4}(q^2) = C^A_{3,4}(0)\,\frac{C^A_5(q^2)}{C^A_5(0))}$$

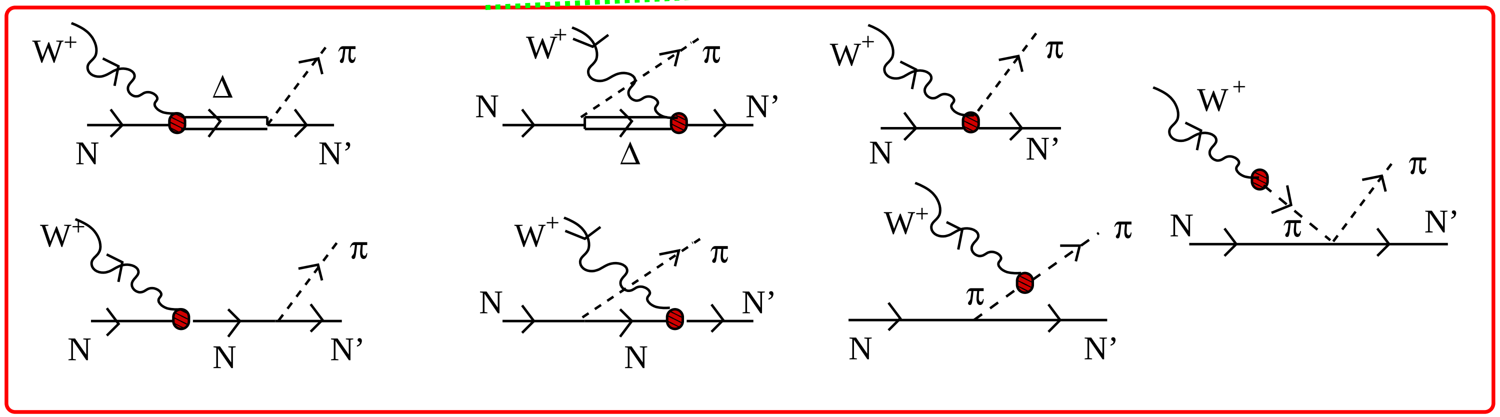
exploring the possibility of extracting some direct information on $C^A_{3,4}(0)$

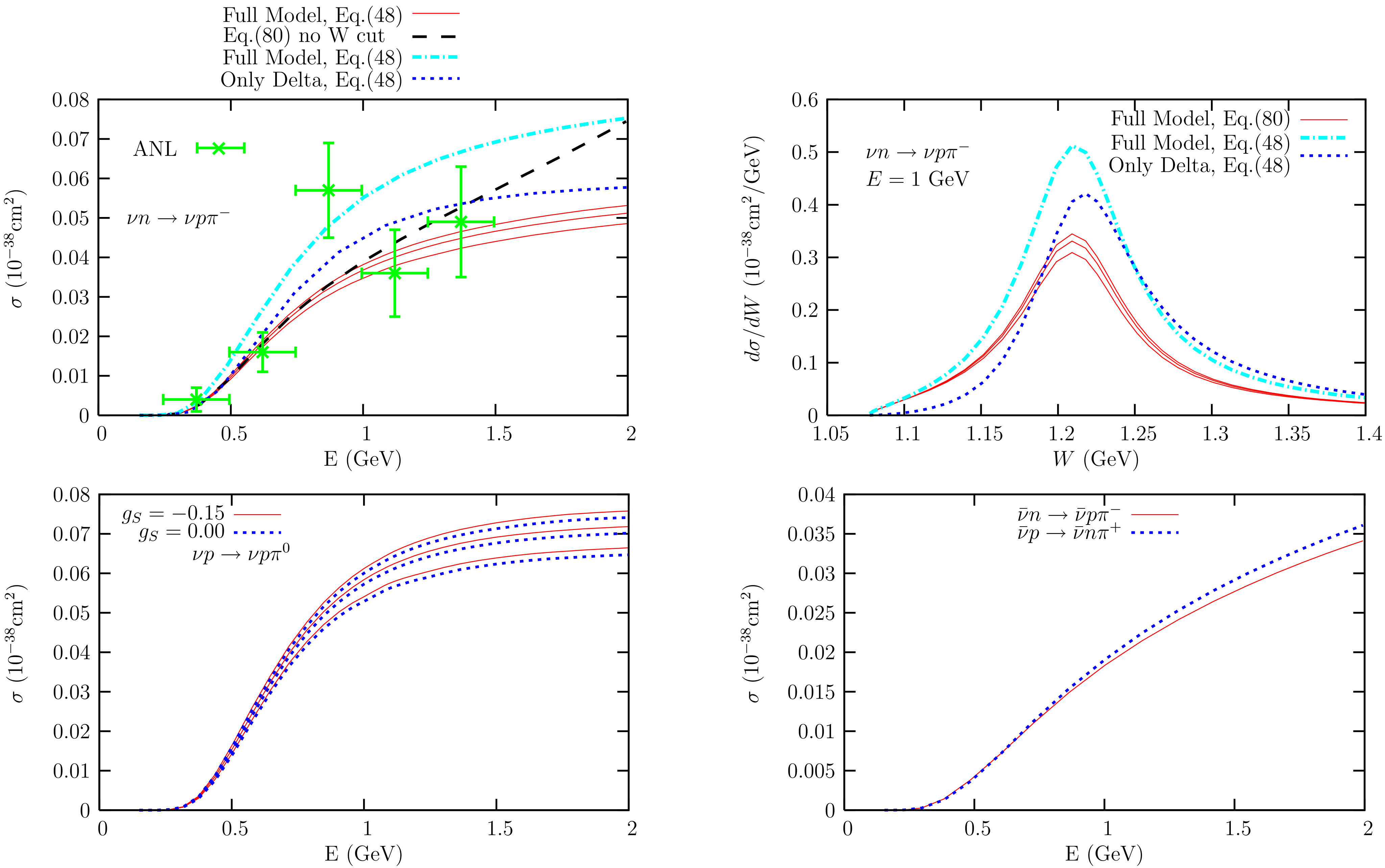
	$C^A_5(0)$	$M_{A\Delta}$/GeV	$C^A_3(0)$	$C^A_4(0)$	χ^2/dof
I* (only ΔP)	1.08 ± 0.10	0.92 ± 0.06	Ad	Ad	0.36
II*	0.95 ± 0.11	0.92 ± 0.08	Ad	Ad	0.49
III (only ΔP)	1.13 ± 0.10	0.93 ± 0.06	Ad	Ad	0.32
IV	1.00 ± 0.11	0.93 ± 0.07	Ad	Ad	0.42
V	1.08 ± 0.14	0.91 ± 0.10	-1.0 ± 1.4	Ad	0.40
VI	1.08 ± 0.14	0.86 ± 0.15	Ad	-1.0 ± 1.3	0.40
VII	1.07 ± 0.15	1.0 ± 0.3	1 ± 4	-2 ± 4	0.44

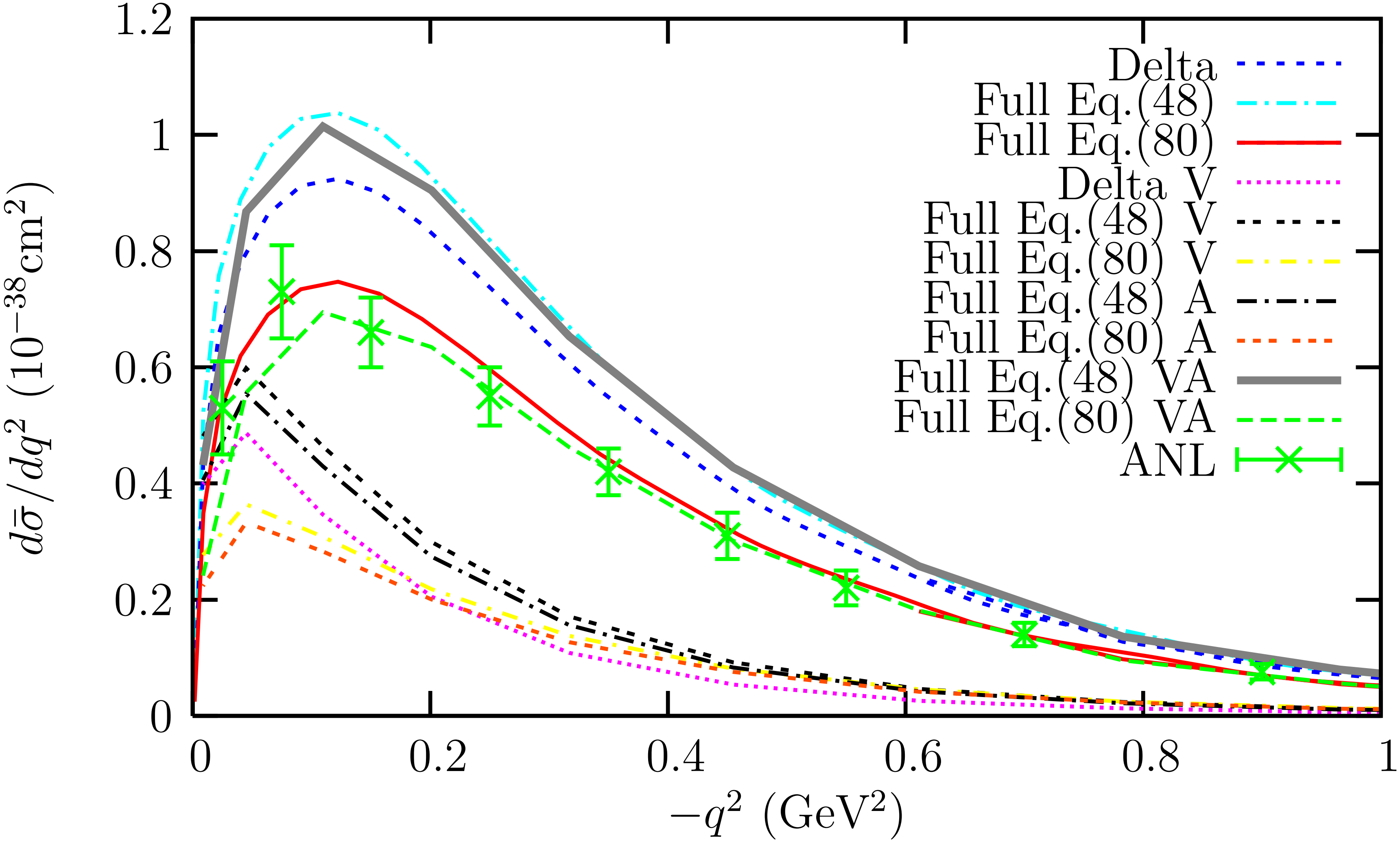
* No deuteron effects included.

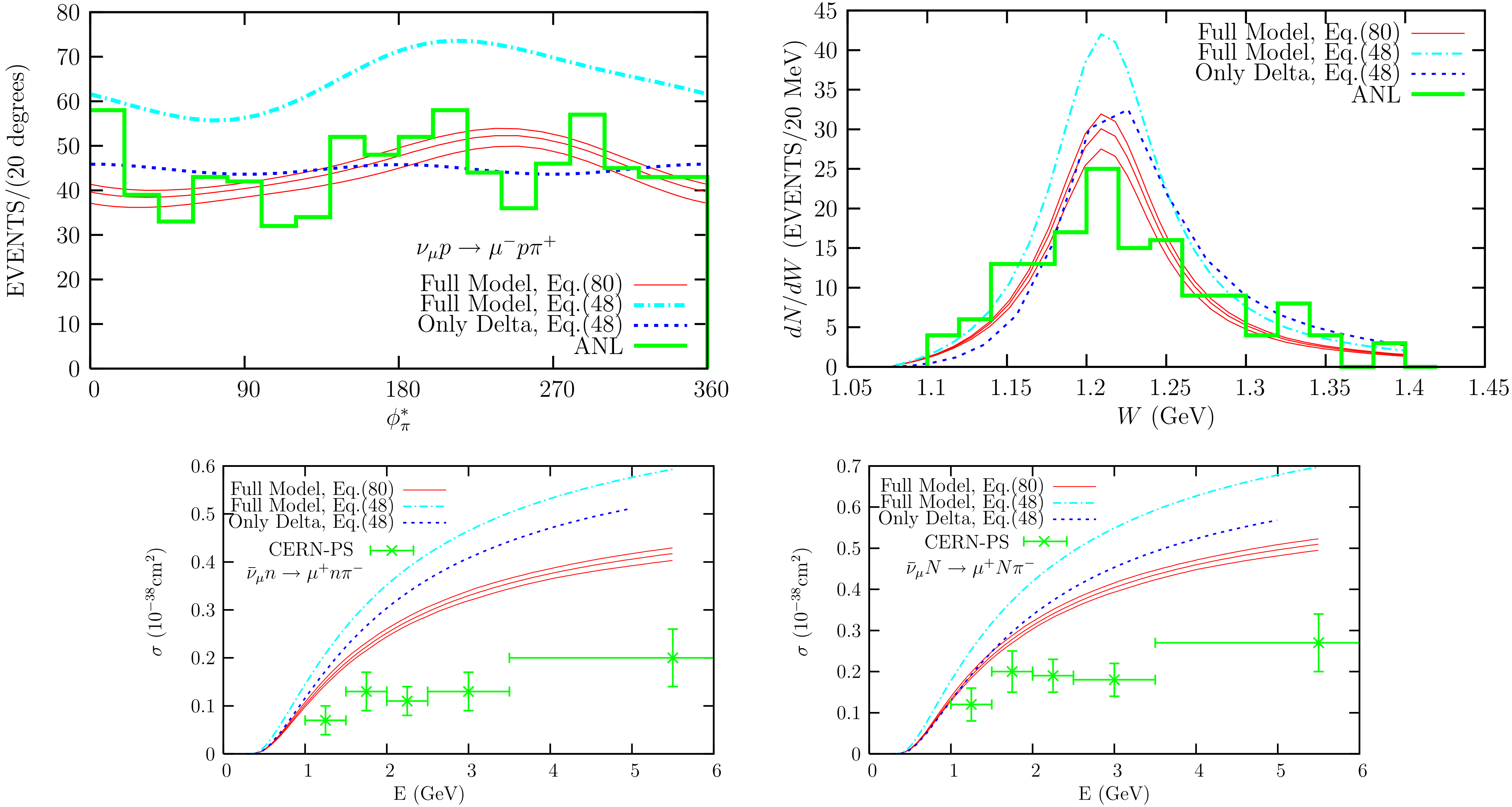

W+ n → p
W+ N → Δ, N*
W+NN → NN
W+N → N π, Nρ, ...
W+
q
p
p+q
n
p
Δ,N*
N
π,ρ,...
Δ, N*
+ ...
n < F
2


(e,e'X)
12C
EINC=620 MeV
θ=60°
d²σ/dΩdE (nb/sr)*MeV⁻¹
ω (MeV)
Total pion produc.
(producc. pion)

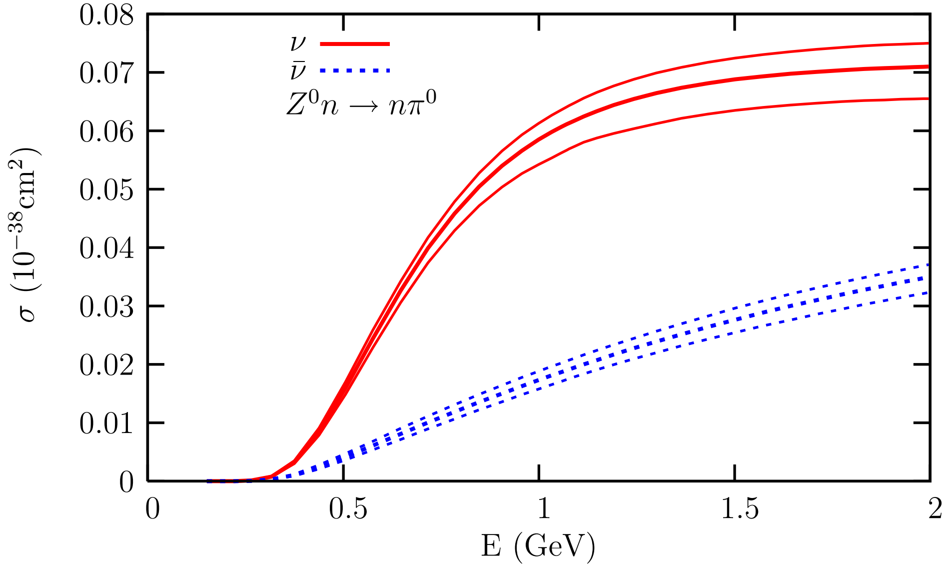

W+
Δ
π
N
N'
π

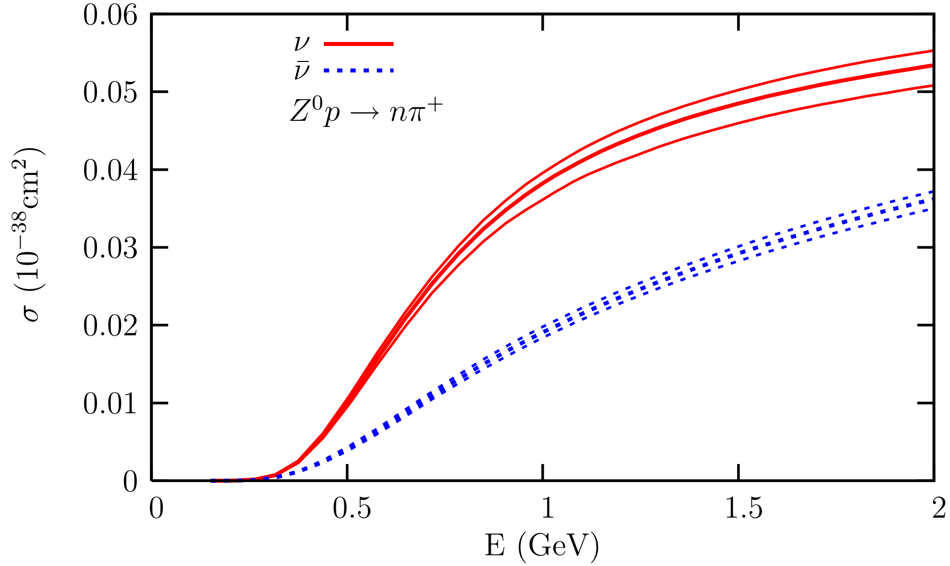

Full Model, Eq.(48)
Eq.(80) no W cut
Full Model, Eq.(48)
Only Delta, Eq.(48)
ANL
$\nu n \to \nu p \pi^-$
σ (10^{-38}cm^2)
E (GeV)
Full Model, Eq.(80)
Full Model, Eq.(48)
Only Delta, Eq.(48)
$\nu n \to \nu p \pi^-$
$E = 1$ GeV
$d\sigma/dW$ (10^{-38}cm^2/GeV)
W (GeV)
$g_S = -0.15$
$g_S = 0.00$
$\nu p \to \nu p \pi^0$
σ (10^{-38}cm^2)
E (GeV)
$\bar\nu n \to \bar\nu p \pi^-$
$\bar\nu p \to \bar\nu n \pi^+$
σ (10^{-38}cm^2)
E (GeV)

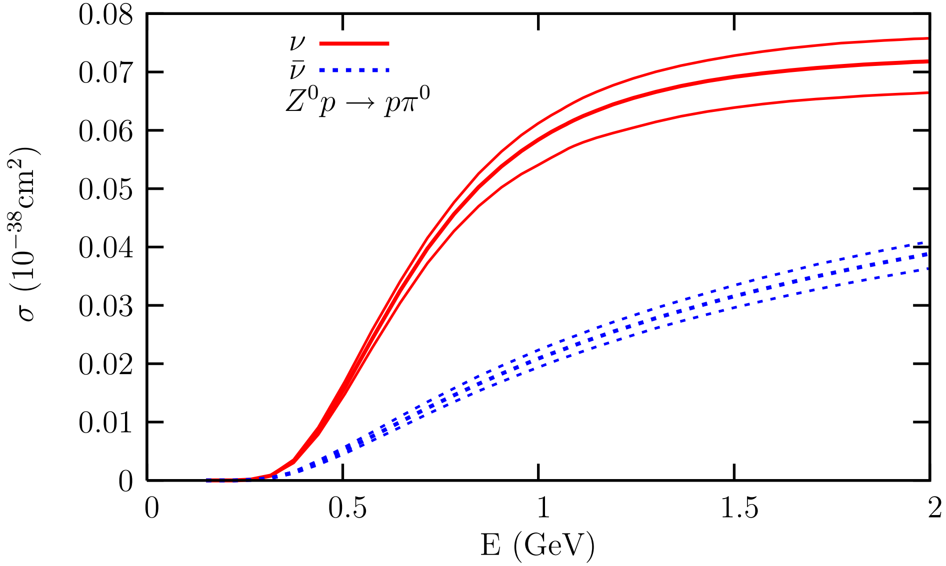

1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
$d\bar{\sigma}/dq^2$ (10^{-38}cm^2)
$-q^2$ (GeV^2)
Delta
Full Eq.(48)
Full Eq.(80)
Delta V
Full Eq.(48) V
Full Eq.(80) V
Full Eq.(48) A
Full Eq.(80) A
Full Eq.(48) VA
Full Eq.(80) VA
ANL

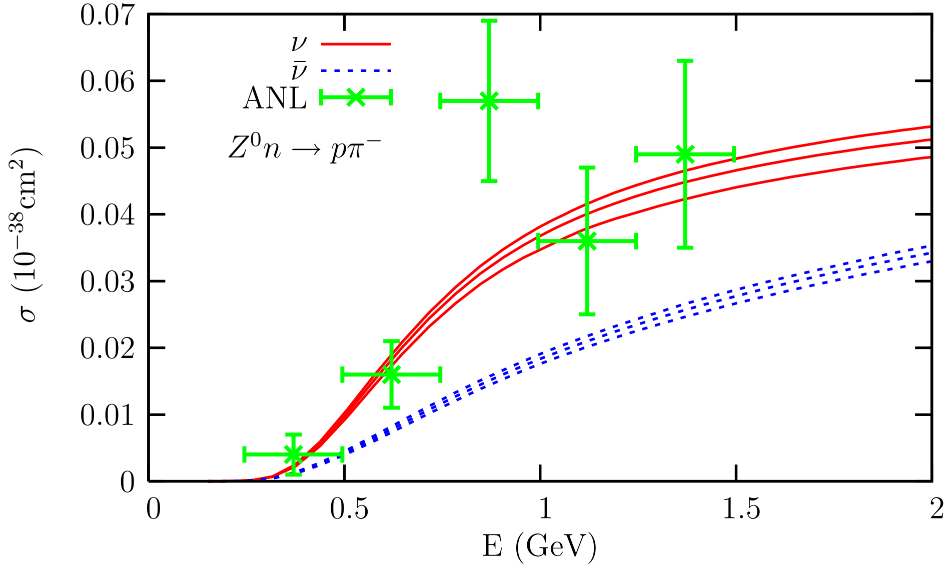


Gargamelle chamber: propane + small admixture freon CF3Br









Below the τ prod. threshold, Distinguish ν_τ from $\bar{\nu}_\tau$?

$\sigma_{\rm NC}/\sigma_{\rm CC}$ ANL cross sections at $E = 0.6 - 1.2$ GeV

	ANL	Our results
$R_+ = \sigma(\nu p \to \nu n \pi^+)/\sigma(\nu p \to \mu^- p \pi^+)$	0.12 ± 0.04	0.12 – 0.10
$R_0 = \sigma(\nu p \to \nu p \pi^0)/\sigma(\nu p \to \mu^- p \pi^+)$	0.09 ± 0.05	0.18 – 0.14
$R_- = \sigma(\nu n \to \nu p \pi^-)/\sigma(\nu p \to \mu^- p \pi^+)$	0.11 ± 0.022	0.12 – 0.09

NC: Cross sections (10^{-38}cm^2) for $\langle E \rangle = 2.2$ GeV (no cut in W)

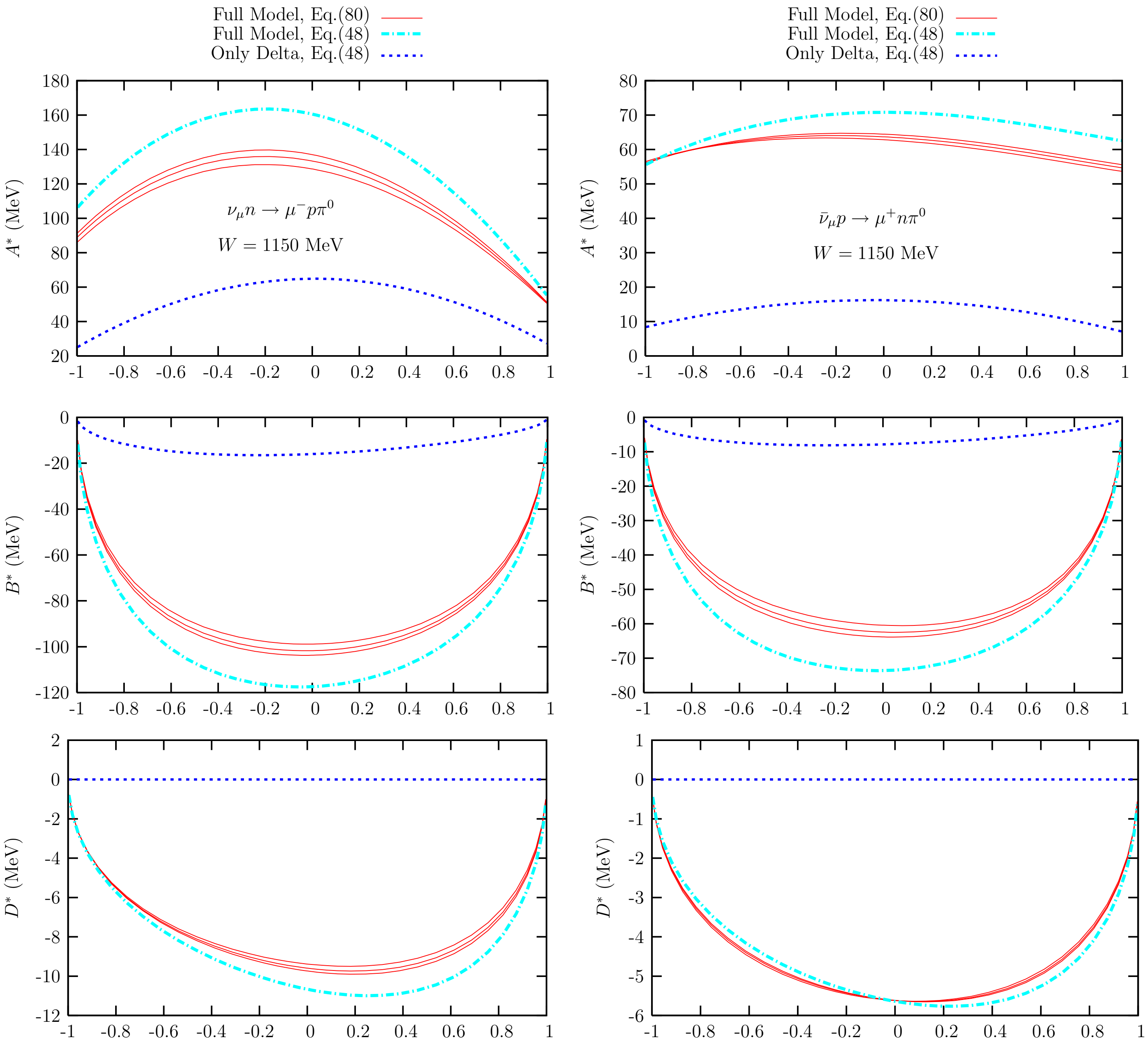
	CERN	Our results
$\sigma(\nu p \to \nu p \pi^0)$	0.130 ± 0.020	0.105 ± 0.006
$\sigma(\nu p \to \nu n \pi^+)$	0.080 ± 0.020	0.091 ± 0.003
$\sigma(\nu n \to \nu n \pi^0)$	0.080 ± 0.020	0.104 ± 0.006
$\sigma(\nu n \to \nu p \pi^-)$	0.110 ± 0.030	0.082 ± 0.003

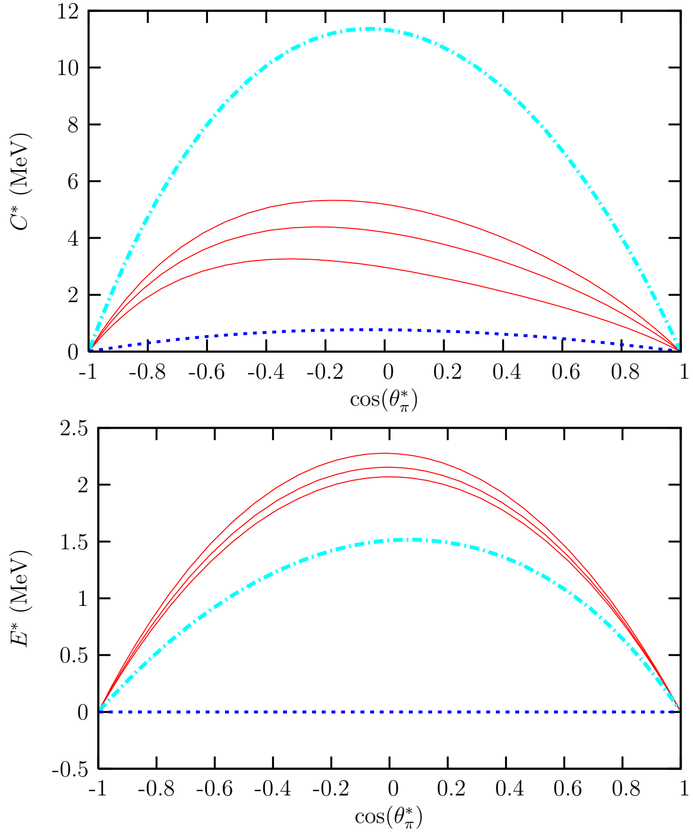
Dependence on θ^*_π (CM πN pion polar angle). Lorentz invariance (f.i., CC) $\Rightarrow$

$$\frac{d^5\sigma_{\nu_l l}}{d\Omega(\hat{k}')dE'd\Omega^*(\hat{k}_\pi)} = \frac{|\vec{k}'|}{|\vec{k}\,|}\frac{G^2}{4\pi^2}\Bigg\{\overbrace{A^* + B^*\cos\phi^*_\pi + C^*\cos 2\phi^*_\pi}^{\text{Similar to } eN\to e'N\pi}$$

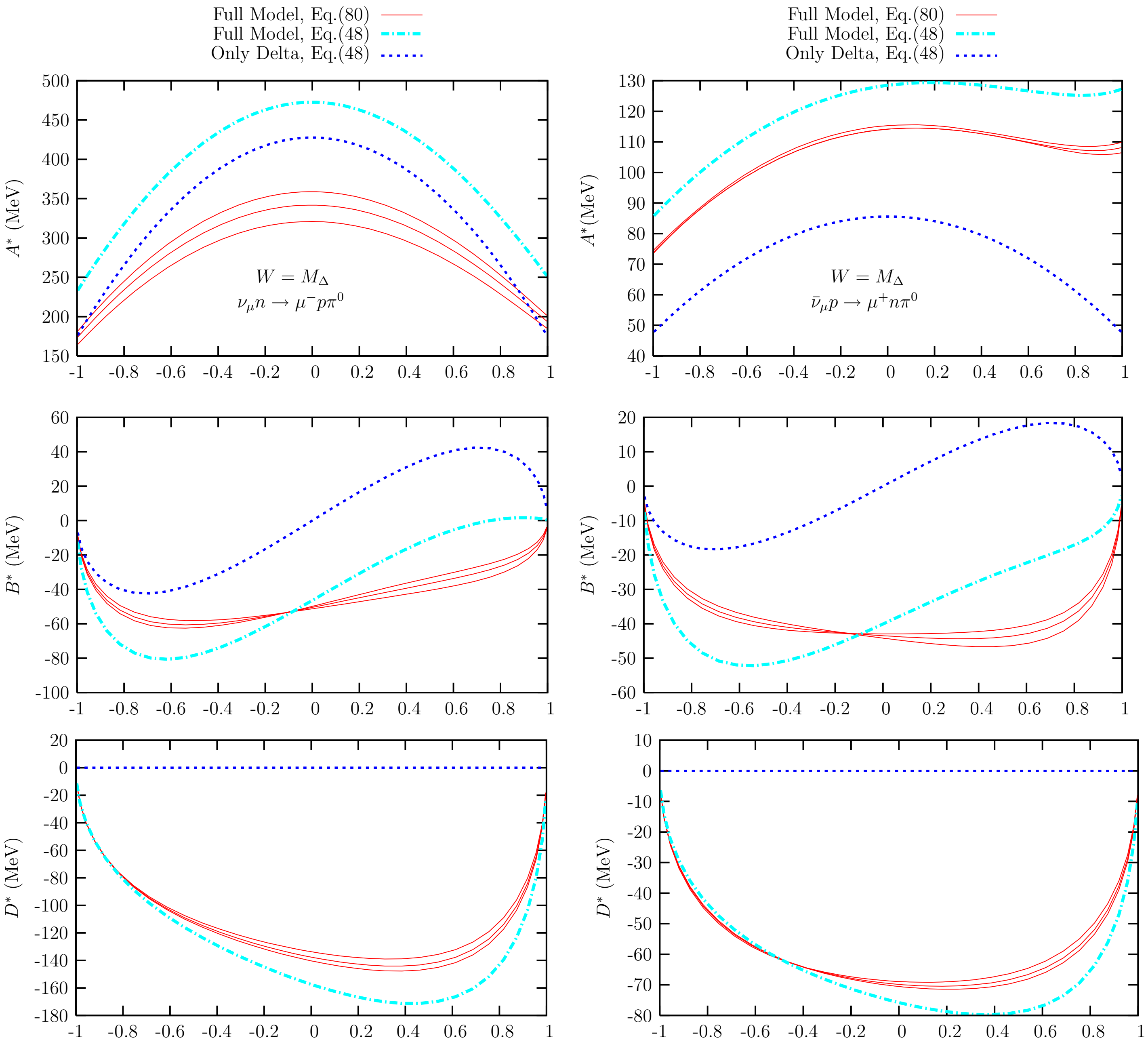
$$+ \underbrace{D^*\sin\phi^*_\pi + E^*\sin 2\phi^*_\pi}_{\text{parity violating}}\Bigg\}$$

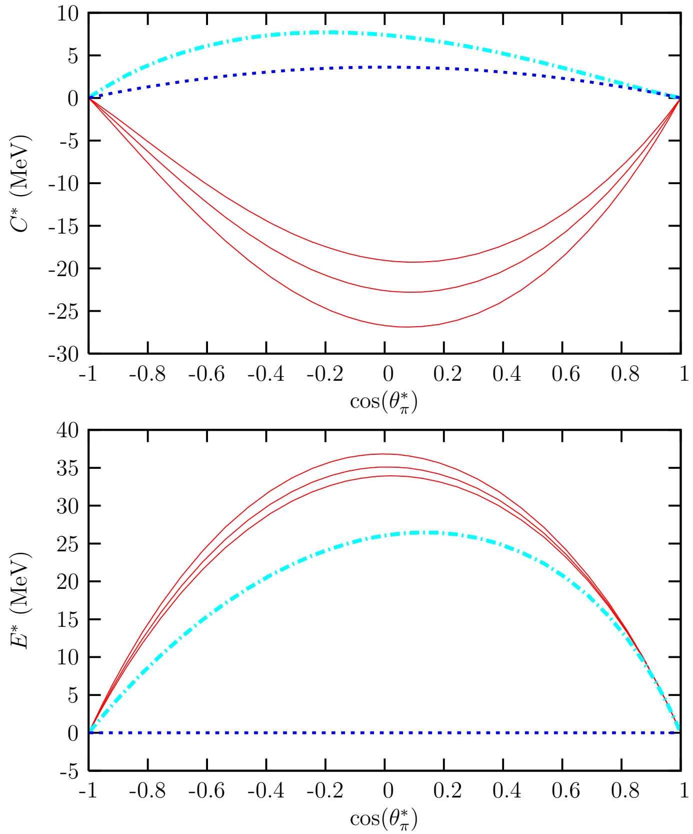
- **Explicit** ϕ_π dependence
- A^*, B^*, C^*, D^*, E^* functions of E, q^2, W, θ^*_π
- C^* and E^* are the same ν and $\bar{\nu}$, when $(W^{\mu\sigma})^{(\bar{\nu})} = (W^{\mu\sigma})^{(\nu)}$





A^*, B^*, C^*, D^*, E^* **vs** $\cos\theta^*_\pi$, E = 1.5 GeV, W = 1.15 GeV and $q^2 = -0.5$ GeV^2. Reactions $\nu_\mu n \to \mu^- p \pi^0$ and $\bar{\nu}_\mu p \to \mu^+ n \pi^0$





A^*, B^*, C^*, D^*, E^* **vs** $\cos\theta^*_\pi$**,** $E = 1.5$ GeV, $W = M_\Delta$ and $q^2 = -0.5$ GeV^2. Reactions $\nu_\mu n \to \mu^- p \pi^0$ and $\bar\nu_\mu p \to \mu^+ n \pi^0$

... new NC neutrino–antineutrino asymmetries

$$\frac{1}{2}\left(\frac{d\sigma(\phi_\pi)}{d\phi_\pi} - \frac{d\sigma(\phi_\pi+\pi)}{d\phi_\pi}\right)\Big|_{\nu} = (\mathcal{B}_s+\mathcal{B}_a)\cos\phi_\pi + (\mathcal{D}_s+\mathcal{D}_a)\sin\phi_\pi$$

$$\frac{1}{2}\left(\frac{d\sigma(\phi_\pi)}{d\phi_\pi} - \frac{d\sigma(\phi_\pi+\pi)}{d\phi_\pi}\right)\Big|_{\bar\nu} = (\mathcal{B}_s-\mathcal{B}_a)\cos\phi_\pi + (\mathcal{D}_s-\mathcal{D}_a)\sin\phi_\pi$$