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# Form Factors of the Delta Resonance

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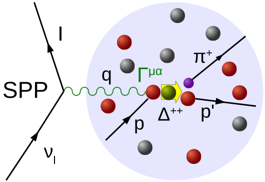
# Introduction

Introduction

# Motivation

- $\Delta(1232)$ resonance: important for Single Pion Production (SPP) in accelerator $\nu$ oscillation experiments. $\pi^0 \to \gamma\gamma$-background to $\nu_e$ appearance etc.

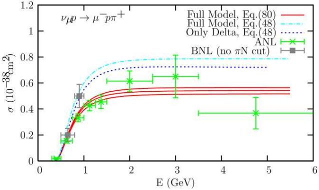



$$\Gamma^{\mu\alpha} = \left[\frac{C_3^V}{M}(g^{\alpha\mu}\not{q} - q^{\alpha}\gamma^{\mu}) + \frac{C_4^V}{M^2}(g^{\alpha\mu}q\cdot p_\Delta - q^{\alpha}p_\Delta^{\mu}) + \frac{C_5^V}{M^2}(g^{\alpha\mu}q\cdot p - q^{\alpha}p^{\mu}) + g^{\alpha\mu}C_6^V\right]\gamma^5 + \left[\frac{C_3^A}{M}(g^{\alpha\mu}\not{q} - q^{\alpha}\gamma^{\mu}) + \frac{C_4^A}{M^2}(g^{\alpha\mu}q\cdot p_\Delta - q^{\alpha}p_\Delta^{\mu}) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2}q^{\alpha}q^{\mu}\right]$$

- Vector part: rather well-known from pion photo- and electroproduction data.
- Axial part: neutrino experiments, nuclear targets:
  1. Strong modifications of $\Delta$ resonance properties in nuclear matter (*e.g.* E. Oset *et al.* 468, 631 (1987))
  2. Nuclear FSI: $\pi$ from oter channels, $\pi$ absorption, charge exchange, distortion...
- Initial $N \to \Delta$ transition obscured.
- Mismatch between MiniBooNE and Minerva SPP? (talk by S. Manly, Hyupwoo Lee)

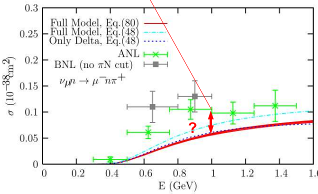
# Motivation

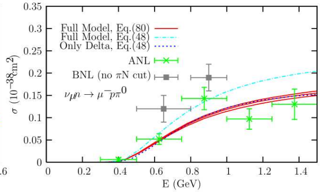
- Limited data for deuteron. Very old **ANL** and **BNL** experiments
- Statistical analyses by K. M. Graczyk *et al.* Phys. Rev. D 80, 093001 (2009) ($\Delta$ resonance) and E. Hernandez *et al.* Phys. Rev. D 81, 085046 (2010) (nonresonant background) in the $\Delta^{++} \to p\pi^+$ channel.
- So far: no usage of neutron channel data.
- Tension between ANL and BNL data (resolved in K. M. Graczyk *et al.* Phys. Rev. D 80, 093001 (2009) for $p\pi^+$).
- Tension between theory and experiment, E. Hernandez *et al.*, Phys. Rev. D 76, 033005 (2007):




Hernandez et al, Phys. Rev. D76 033005 (2007)

pπ+ and pπ 0 channels in fair agreement, nπ+ below data



# Motivation

- $\Delta$ vector form factors: extraction from coincidence cross sections. Dependence on $\Delta$ and nonresonant background model.
- HNV: vector form factors from O. Lalakulich, E. Paschos and G. Piranishvili, Phys. Rev. D 74 014009 (2006)$\leftarrow$ MAID pion electroproduction analysis (see e.g. D. Drechsel, S.S. Kamalov, L. Tiator Nucl. Phys. A645 (1999) 145-174).
- MAID$\neq$HNV.

**Different $\Delta$ and background model $\rightarrow$ different vector form factors**

Consistency requirement: vector form factors fit.

- Improvement of the picture by
  1. Fit of the vector form factors?
  2. Simultaneous fit to $p\pi^+$, $p\pi^0$ and $n\pi^+$ channels?

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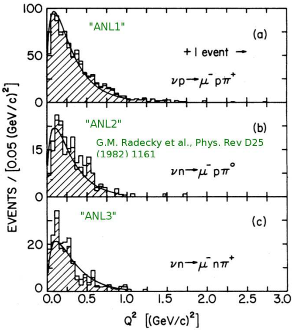
# ANL experiment

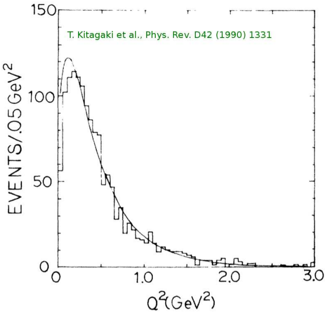
- 12-ft $^2H + ^1H$ bubble chamber at Argonne National Laboratory
- **S. J. Barish *et. al.*, Phys. Rev. D19(1979) 2521**, **G. M. Radecky Phys. Rev. D25 (1982) 1161**.
- $\langle E \rangle < 1$ GeV
- $\delta_{flux} = 15\%$ ($E < 1.5$ GeV) and $25\%$ (above )
- $\left\langle \frac{d\sigma}{dQ^2} \right\rangle_{ANL}$, $\nu_\mu + p \to \mu^- + p + \pi^+$ channel
- $\left\langle \frac{dN}{dQ^2} \right\rangle_{ANL}$, $\nu_\mu + n \to \mu^- + p + \pi^0$ and $\nu_\mu + n \to \mu^- + \pi^+ + n$.
- Experimental correction factors
- $\sigma(E)$ (normalizations).

- Kinematical cuts:
  1. $0.5$ GeV$< E < 6.0$ GeV in $p\pi^+$ channel ("ANL1").
  2. $0.5$ GeV$< E < 3.0$ GeV in $p\pi^0$ "ANL2", $n\pi^+$ "ANL3" channels.
  3. $0.01$ GeV$^2 < Q^2 < 1$GeV$^2$.
  4. Data with W $<$ 1.4 GeV



# BNL experiment

- 7 foot $^2H$ bubble chamber at Brookhaven National Laboratory.
- **T. Kitagaki et al. Phys. Rev. D 34 (1986) 2554**, **T. Kitagaki et al., Phys. Rev. D42 (1990) 1331**.
- $\langle E \rangle = 1.6$ GeV
- $\delta_{flux} = 10\%$
- $\left\langle \frac{dN}{dQ^2} \right\rangle_{BNL}$ only $\nu_\mu + p \to \mu^- + p + \pi^+$ channel with W<1.4 GeV.
- Kinematical cuts:
  1. $0.5$ GeV$< E < 6.0$ GeV.
  2. (efficiency)$0.1$ GeV$^2 < Q^2 <$ 3GeV$^2$.
- $\sigma(E)$ (normalization), no cut in W.



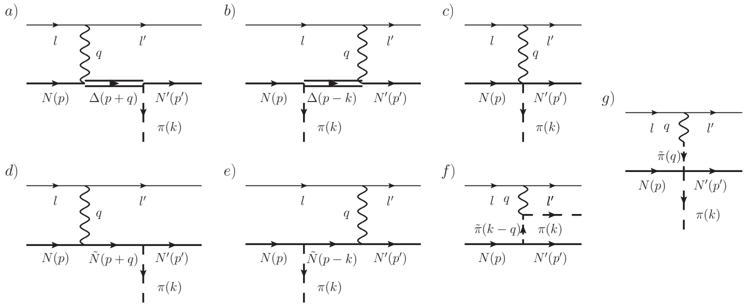

- Neutron channels: no W cut in $Q^2$ distributions. Distributions in W: $\Delta$ form factors almost W-independent. No use for them in this fit.

# Theoretical framework

Theoretical framework

# Low energy single pion production

- External weak/electromagnetic probe vertex: from Standard Model.
- Hadronic vertex: **strong** interactions. Low energies: interaction with hadrons. QCD→ chiral perturbation theory ($\chi$PT).
- For T2K energy region: lowest order $\chi$PT.
- Hernandez, Nieves, Valverde (HNV) model, Phys. Rev. D 76, 033005 (2007):




- Alltogether 7 currents: 2 from $\Delta$ resonance ($a$) and $b$)), rest from $\chi$PT.

# $\chi$PT background

- Non-resonant $\chi$PT amplitudes in HNV $\mathcal{J}^{\mu}_{hadr.} = \langle N'\pi \,| s^{\mu} |\, N \rangle$:

$$
\begin{aligned}
s^{\mu}_{NP} &= -iC_{NP}\frac{g_A}{\sqrt{2}f_\pi}\not{k}\gamma^5\frac{(\not{p}+\not{q}+M)}{(p+q)^2-M^2+i\epsilon}j^{\mu}_{CCN}(q)F_\pi(k-q) \\
s^{\mu}_{CNP} &= -iC_{CNP}\frac{g_A}{\sqrt{2}f_\pi}j^{\mu}_{CCN}(q)\frac{(\not{p}-\not{k}+M)}{(p-k)^2-M^2+i\epsilon}\not{k}\gamma^5F_\pi(k-q) \\
s^{\mu}_{CT} &= -iC_{CT}\frac{1}{\sqrt{2}f_\pi}\gamma^{\mu}F_\pi(k-q)\left[g_AF^V_{CT}(q^2)\gamma^5-F_\rho((q-k)^2)\right] \\
s^{\mu}_{PIF} &= -iC_{PIF}\frac{g_A}{\sqrt{2}f_\pi}F^V_{PIF}(q^2)\frac{(2k-q)^{\mu}}{(k-q)^2-m_\pi^2}2M\gamma^5F_\pi(k-q) \\
s^{\mu}_{PP} &= -iC_{PP}\frac{1}{\sqrt{2}f_\pi}F_\rho(k-q)\frac{q^{\mu}\not{q}}{q^2-m_\pi^2}
\end{aligned}
$$

- Two fundamental constants pion decay $f_\pi \approx 92.4$ MeV and nucleon axial charge $g_A \approx 1.267$ ($\beta$-decay) plus nucleon electroweak form factors in the vertex $j^{\mu}_{CCN}$. Rest: CVC $\rightarrow F^V_{PIF} = F^V_{CT} = F^V_1$ and $\rho$-meson dominance hypothesis.

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# Main results

Main results

# Vector form factors

- Our framework: coherent sum of QFT currents. Fit of vector $\Delta$ form factors to inelastic proton structure function $F_2^p$:
  1. Up to W=$M_p + 2m_\pi$ Osipenko et al. CLAS data sets (arXiv:hep-ex/0309052) spanning the region 0.225 up to 2.025 GeV$^2$ each 0.05 GeV$^2$.
  2. For invariant masses from W=$M_p + 2m_\pi$ up to 1.27 GeV- points generated with MAID2007 with Osipenko et al. errors.

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# Vector form factors

- Our form factor model:

$$
\begin{aligned}
C_3^V(Q^2) &= \frac{C_3^V(0)\cdot(1+K_1Q^2)}{1+AQ^2+BQ^4+CQ^6}\\
C_4^V(Q^2) &= -\frac{M}{W}C_3^V(Q^2)\cdot\frac{(1+K_2Q^2)}{(1+K_1Q^2)}\\
C_5^V(Q^2) &= \frac{C_5^V(0)}{\left(1+D\frac{Q^2}{M_V^2}\right)^2}\\
C_6^V(Q^2) &\equiv 0\ (CVC).
\end{aligned}
$$

- inspired by SU(6)-symmetrical quark model (J. Liu *et al.* Phys. Rev. C 52, 1630 (1995))- simple relation between $C_4^V(Q^2,W)=-\frac{M_p}{W}C_3^V(Q^2)$, $C_5^V=0$.
- $K_1, K_2$ corrections to the above, analogous to the successful parametrization of proton e-m form factors from J. Kelly, Phys. Rev. C 70 (2004) 068202.
- Addition of non-zero $C_5^V$.

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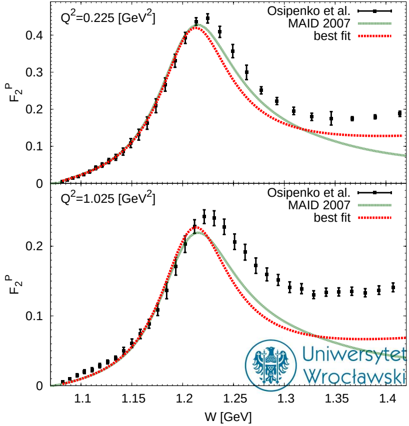
# Electromagnetic fit results

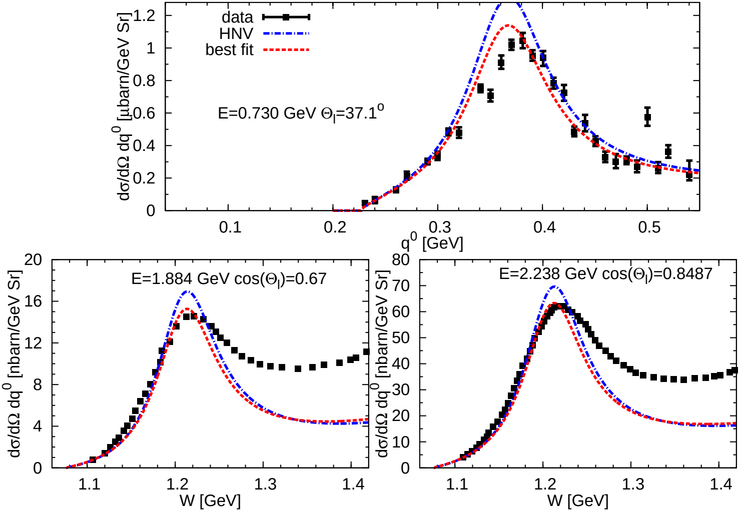
| $C_3^V(0)$ | $A$ | $B$ | $C$ | $K_1$ | $K_2$ | $C_5^V(0)$ | $D$ |
|---|---|---|---|---|---|---|---|
| 2.1±0.1 | 4.7±0.7 | -0.4±0.4 | 5.6±1.4 | 0.1±0.2 | 1.7±0.4 | 0.6±0.05 | 1.0±0.1 |

- Not perfect, but reasonable agreement with data.
- Comparable with MAID2007 up to the $\Delta$ peak.
- $Q^2$-dependence of $C_5^V$: driven by vector mass $M_V = 0.84$ GeV
- Best fit: $\frac{\chi^2_{W<M_p+2m_\pi}}{D.O.F.} = 13.7$, MAID (unitarized, $\Delta$+Born+$\rho$+$\omega$) $\frac{\chi^2_{W<M_p+2m_\pi}}{D.O.F.} = 12.1$. Data too accurate



# Electromagnetic fit results

- Comparison to J.S. O'Connell *et al.* Phys. Rev. Lett. 53, 1627 (1984) (top) and The Jefferson Lab Hall C E94-110 Collaboration: arXiv:nucl-ex/0410027v2 (bottom) inclusive electron-proton data.
- Bleue curve: HNV model with Lalakulich-Paschos form factors, red curve: best fit. Still not perfect, but visible improvement.



# Axial form factors

- Axial part: not so much available data. Nuclear targets: problem of many-body effectss, FSI etc. Best ANL/BNL: deuteron bubble chambers.
- Lack of strong theoretical constraints on $C_i^A$. Leading form-factor in dipole ansatz:

$$C_5^A(Q^2) \quad = \quad \frac{C_5^A(0)}{(1+Q^2/{M_A}^2)^2}$$

- Off-diagonal Goldberger-Treiman relation $C_5^A(0) = \frac{f^*}{\sqrt{2}} \approx 1.2$. Delta axial mass-only from fits. Intuition- "axial charge radius" $M_A = \mathcal{O}(1\ \mathrm{GeV})$. In general, both can be fitted.
- From PCAC:

$$C_6^A(Q^2) \quad = \quad \frac{M^2 C_5^A(Q^2)}{m_\pi^2 + Q^2}$$

- The rest: no real constraint, "Adler model", "handwaving":

$$C_3^A(Q^2) = 0; \;\; C_4^A(Q^2) = -\frac{1}{4} C_5^A(Q^2)$$

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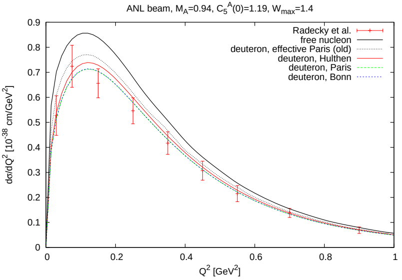
# Deuteron model

- Effective deuteron model (e.g. L.Alvarez-Ruso *et al.* Phys. Rev. C 59 (1999) 3386):

$$\frac{d\sigma}{dQ^2}^{deuteron} = \int \frac{d^3p_N}{(2\pi)^3}\frac{f(|\vec{p}_N|)}{v_{rel.}}\frac{d\sigma(\tilde{q}^\mu,\vec{p}_N)}{dQ^2}^{free}$$

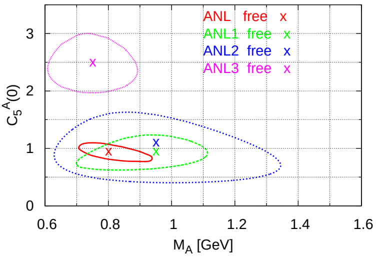
  with $f(p_N)$- norm of momentum-space deuteron wave function ("Paris": M. Lacombe *et al.* Phys. Lett. B 101 (1981) 139),$\tilde{q}^\mu = (q^0 - B(|\vec{p}_n|),\vec{q})$, $B(|\vec{p}_N|) = M_D - 2\sqrt{\vec{p}_N^{\,2} + M^2}$ and $v_{rel.}\to$ flux correction due to nucleon movement.

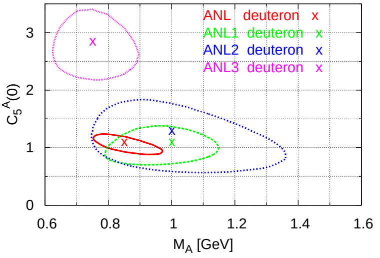



- Full $\Delta$+background computation: complicated numerical procedure. Usage of Wroclaw Centre for Networking and Supercomputing grid.

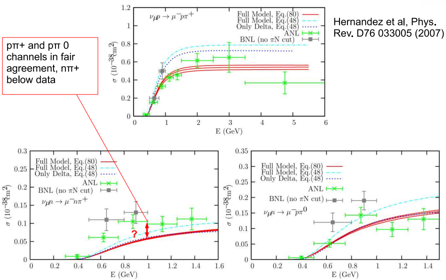
# Preliminary results for ANL data

- Top plot: full model, free nucleon target best fit result for ANL data+ 1-$\sigma$ contours:
- Separate fits in ANL1 and ANL2 channels: consistent, ANL3: higher $C_5^A(0)$
- Prefference of $C_5^A(0)$ ≈0.95, smaller from Goldbgerger-Treiman (1-2$\sigma$). $M_A$ ≈0.8 GeV (all channels).
- Bottom plot: deuteron effects included $C_5^A(0)$ ≈1.1, $M_A$ ≈0.85 GeV statistical consistency with Goldberger-Treiman.
- Deuteron effects→ higher $C_5^A$.
- Global $\frac{\chi^2_{ANL}}{D.O.F.} = \frac{63.37}{35}$(free target)→ $\frac{61.90}{35}$ (deuteron).

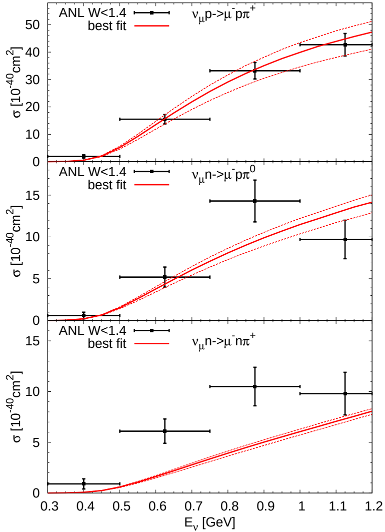


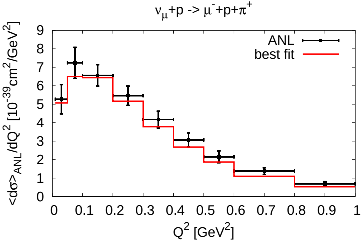
- Cross sections with $W < 1.4$ GeV cut.
- Good in ANL $p\pi^+$ and $p\pi^0$ channels.
- Visible lack of cross section in $n\pi^+$ channel. Expected from the separate $C_5^A(0)$ fits.
- Same old problem with $n\pi^+$.

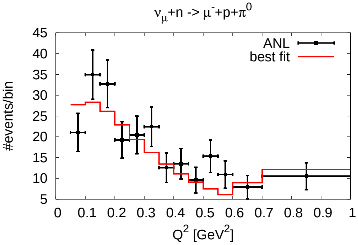


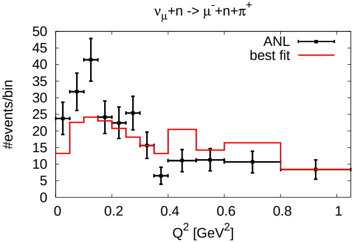

Hernandez et al, Phys. Rev. D76 033005 (2007)



# ANL $Q^2$ distributions



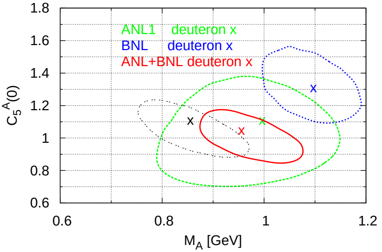




- Again, biggest discrepancy in the $n\pi^+$ channel.

# Inclusion of BNL

- BNL and ANL1 fits within $1\sigma$.
- Inclusion of BNL data in global fit $M_A$=0.85 GeV$\rightarrow$ 0.95 GeV.
- Problem: only $p\pi^+$ channel from BNL, $p\pi^+$ "double counting" $\rightarrow$ Restriction to ANL data.
- Need for more deuterium experments.

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# Conclusions

- First fit of single pion production model to electromagnetic and weak data including all ANL channels has been performed.
- Tests of fitted model against inclusive electron data give reasonable, albeit not perfect agreement.
- For neutrino SPP each channel $p\pi^+$, $p\pi^0$ and $n\pi^+$ seems to fit nicely separately, but there is some tension between $n\pi^+$ channel and the rest (much higher $C_5^A(0)$). This tension is of the order of $2\sigma$ level.
- Inclusion of $\chi PT$ did not resolve the ANL3 channel ($n\pi^+$) problem.
- ANL fit: $C_5^A(0)$ in good agreement with Goldberger-Treiman relation.

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# Acknowledgements


- Most of numerical calculations were carried out in the Wroclaw Centre for Networking and Supercomputing (http://www.wcss.wroc.pl), grant No. 268
- We would like to thank Luis Alvarez-Ruso and Juan Nieves for many fruitful discussions.


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# Backup

Backup

# Statistical framework

- All fits so far: only $p\pi^+$ (ANL1) channel, flux-averaged $\frac{d\sigma}{dQ^2}$ with errors $\delta\frac{d\sigma}{dQ^2}$ (actually: $Q^2$-bin-averaged). Comparison of flux- and $Q^2$- averaged cross sections in each bin together with data normalization fit $p_{ANL}$ (statistical model from Phys. Rev. D 80, 093001 (2009)).

$$\chi^2_{ANL1} = \sum_i^{(Q^2 bins)} \left( \frac{\sigma_i^{TH.} - p_{ANL} \cdot \sigma_i^{EXP.}}{p_{ANL} \cdot \delta\sigma_i^{EXP.}} \right)^2 + \left( \frac{p_{ANL} - 1}{\delta p_{ANL}} \right)^2$$

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# Statistical framework

- ANL2 and ANL3: event distributions in $Q^2$ (distribution shapes).
- Experimental correction factors $C^{EXP.}$ with errors $\delta C^{EXP.}$. Normalization $p_{ANL}$.

$$\chi^2_{ANL2,3} = \sum_i^{(Q^2 bins)} \frac{\left(\sigma_i^{TH.} \cdot \frac{\sum_j N_j^{EXP.} \cdot C^{EXP}}{\sum_j \sigma_j^{TH.}} \cdot p_{ANL} - N_i^{EXP.}\right)^2}{N_i^{EXP.} \cdot C^{EXP.}(1+\delta C^{EXP.})} + \left(\frac{\frac{\sigma_{TOT.}^{TH.}}{\sigma_{TOT.}^{EXP.} \cdot p_{ANL}} - 1}{\delta p_{ANL}}\right)^2$$

- First fit to all three channels.

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# Statistical framework

- $\left\langle \frac{dN}{dQ^2} \right\rangle_{BNL}$ + normalization $p_{BNL}$. Problem: no cut in $W$ for $\sigma^{EXP.}_{TOT.}$.
  $\delta p_{BNL} = 10\%$- beam flux uncertainty from K. Graczyk *et al.* PRD 80, 093001 (2009).

$$\chi^2_{BNL} = \sum_i^{(Q^2 bins)} \frac{\left( \sigma_i^{TH.} \cdot \frac{\sum_j N_j^{EXP.}}{\sum_j \sigma_j^{TH.}} \cdot p_{BNL} - N_i^{EXP.} \right)^2}{N_i^{EXP.}} + \\ + \left( \frac{\frac{\sigma^{TH.}_{TOT.}}{\sigma^{EXP.}_{TOT.} \cdot p_{BNL}} - 1}{\delta p_{BNL}} \right)^2$$

# Preliminary results table

| Data | bckgr. | deut. | $C_5^A(0)$ | $M_A$ | $p_{ANL}$ | $p_{BNL}$ | $\chi^2/D.O.F.$ |
|---|---|---|---|---|---|---|---|
| ANL1 | 0 | 0 | 1.1±0.3 | 0.95±0.2 | 1.04 | - | 0.71/8 |
| ANL1 | 1 | 0 | 0.95±0.3 | 0.95±0.3 | 1.05 | - | 0.95/8 |
| ANL1 | 1 | 1 | 1.1±0.3 | 1.0±0.2 | 1.06 | - | 1.28/8 |
| ANL2 | 0 | 0 | 1.45±0.5 | 0.95±0.2 | 0.93 | - | 14.6/13 |
| ANL2 | 1 | 0 | 1.1±0.7 | 0.95±0.3 | 0.93 | - | 14.0/13 |
| ANL2 | 1 | 1 | 1.3±0.7 | 1.0±0.3 | 0.93 | - | 13.7/13 |
| ANL3 | 0 | 0 | 2.7±0.7 | 0.75±0.1 | 0.93 | - | 14.0/12 |
| ANL3 | 1 | 0 | 2.5±0.5 | 0.75±0.15 | 0.94 | - | 14.0/12 |
| ANL3 | 1 | 1 | 2.85±0.6 | 0.75±0.15 | 0.94 | - | 13.3/12 |
| ANL | 0 | 0 | 1.15±0.1 | 0.8±0.1 | 0.91 | - | 57.9/35 |
| ANL | 1 | 0 | 0.95±0.15 | 0.8±0.1 | 0.89 | - | 63.4/35 |
| ANL | 1 | 1 | 1.1±0.2 | 0.85±0.2 | 0.90 | - | 61.9/35 |
| BNL | 1 | 0 | 1.1±0.2 | 0.95±0.1 | - | 0.98 | 24.2/29 |
| BNL | 1 | 1 | 1.3±0.25 | 1.1±0.1 | - | 0.97 | 34.2/29 |
| ANL+BNL | 1 | 0 | 0.9±0.2 | 0.9±0.1 | 0.92 | 0.97 | 93.8/65 |
| ANL+BNL | 1 | 1 | 1.05±0.2 | 0.95±0.1 | 0.95 | 0.95 | 114/65 |

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