Form Factors of the Delta Resonance

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Introduction
Motivation

- $\Delta(1232)$ resonance: important for Single Pion Production (SPP) in accelerator $\nu$ oscillation experiments. $\pi^0 \rightarrow \gamma\gamma$-background to $\nu_e$ appearance etc.

Vector part: rather well-known from pion photo- and electroproduction data.

Axial part: neutrino experiments, nuclear targets:

1. Strong modifications of $\Delta$ resonance properties in nuclear matter (e.g. E. Oset et al. 468, 631 (1987))
2. Nuclear FSI: $\pi$ from other channels, $\pi$ absorption, charge exchange, distortion...

- Initial $N \rightarrow \Delta$ transition obscured.
- Mismatch between MiniBooNE and Minerva SPP? (talk by S. Manly, Hyupwoo Lee)
Motivation

- Limited data for deuteron. Very old ANL and BNL experiments.
- So far: no usage of neutron channel data.
Motivation

- Δ vector form factors: extraction from coincidence cross sections. Dependence on Δ and nonresonant background model.


- MAID≠HNV.

Different Δ and background model → different vector form factors

Consistency requirement: vector form factors fit.

- Improvement of the picture by
  1. Fit of the vector form factors?
  2. Simultaneous fit to $p\pi^+$, $p\pi^0$ and $n\pi^+$ channels?
12-ft $^2H + ^1H$ bubble chamber at Argonne National Laboratory


$\langle E \rangle < 1$ GeV

$\delta_{flux} = 15\% (E < 1.5$ GeV$)$ and $25\%$ (above )

$\langle \frac{d\sigma}{dQ^2} \rangle_{ANL}, \nu_\mu + p \rightarrow \mu^- + p + \pi^+$ channel

$\langle \frac{dN}{dQ^2} \rangle_{ANL}, \nu_\mu + n \rightarrow \mu^- + p + \pi^0$ and $\nu_\mu + n \rightarrow \mu^- + \pi^+ + n$.

Experimental correction factors

$\sigma(E)$ (normalizations).

Kinematical cuts:

1. $0.5$ GeV$< E < 6.0$ GeV in $p\pi^+$ channel ("ANL1").
2. $0.5$ GeV$< E < 3.0$ GeV in $p\pi^0$ "ANL2", $n\pi^+$ "ANL3" channels.
3. $0.01$ GeV$^2 < Q^2 < 1$ GeV$^2$.
4. Data with $W < 1.4$ GeV
BNL experiment

- 7 foot $^2 H$ bubble chamber at Brookhaven National Laboratory.


- $\langle E \rangle = 1.6$ GeV

- $\delta_{\text{flux}} = 10\%$

- $\left\langle \frac{dN}{dQ^2} \right\rangle_{BNL}$ only $\nu\mu + p \rightarrow \mu^- + p + \pi^+$ channel with $W<1.4$ GeV.

- Kinematical cuts:
  
  1. $0.5 \text{ GeV} < E < 6.0 \text{ GeV}$.
  2. (efficiency) $0.1 \text{ GeV}^2 < Q^2 < 3\text{ GeV}^2$.

- $\sigma(E)$ (normalization), no cut in $W$.

- Neutron channels: no $W$ cut in $Q^2$ distributions. Distributions in $W$: $\Delta$ form factors almost $W$-independent. No use for them in this fit.
Theoretical framework
Low energy single pion production

- External weak/electromagnetic probe vertex: from Standard Model.
- Hadronic vertex: **strong** interactions. Low energies: interaction with hadrons. QCD $\rightarrow$ chiral perturbation theory (χPT).
- For T2K energy region: lowest order χPT.

Alltogether 7 currents: 2 from $\Delta$ resonance ($a$ and $b$), rest from χPT.
Non-resonant $\chi$PT amplitudes in HNV $\mathcal{J}_{\text{hadr.}}^\mu = \langle N'\pi | s^\mu | N \rangle$:

\[
\begin{align*}
  s^\mu_{NP} &= -iC_{NP} \frac{g_A}{\sqrt{2f_\pi}} k \gamma^5 (\not{p} + \not{q} + M) \left( \frac{1}{p^2 - M^2 + i\epsilon} j^\mu_{CCN} (q) F_\pi (k - q) \right) \\
  s^\mu_{CNP} &= -iC_{CNP} \frac{g_A}{\sqrt{2f_\pi}} j^\mu_{CCN} (q) \left( \frac{1}{p^2 - M^2 + i\epsilon} k \gamma^5 F_\pi (k - q) \right) \\
  s^\mu_{CT} &= -iC_{CT} \frac{1}{\sqrt{2f_\pi}} \gamma^\mu F_\pi (k - q) \left[ g_A F^V_{CT} (q^2) \gamma^5 - F_\rho ((q - k)^2) \right] \\
  s^\mu_{PIF} &= -iC_{PIF} \frac{g_A}{\sqrt{2f_\pi}} F^V_{PIF} (q^2) \left( \frac{2k - q)^\mu}{k^2 - m^2_\pi} 2M \gamma^5 F_\pi (k - q) \right) \\
  s^\mu_{PP} &= -iC_{PP} \frac{1}{\sqrt{2f_\pi}} F_\rho (k - q) \frac{q^\mu \not{q}}{q^2 - m^2_\pi} \]
\]

Two fundamental constants: pion decay $f_\pi \approx 92.4$ MeV and nucleon axial charge $g_A \approx 1.267$ ($\beta$-decay) plus nucleon electroweak form factors in the vertex $j^\mu_{CCN}$. Rest: CVC $\rightarrow F^V_{PIF} = F^V_{CT} = F^V_1$ and $\rho$-meson dominance hypothesis.
Main results
Our framework: coherent sum of QFT currents. Fit of vector $\Delta$ form factors to inelastic proton structure function $F_2^p$:

1. Up to $W = M_p + 2m_\pi$ Osipenko et al. CLAS data sets (arXiv:hep-ex/0309052) spanning the region 0.225 up to 2.025 GeV$^2$ each 0.05 GeV$^2$.

2. For invariant masses from $W = M_p + 2m_\pi$ up to 1.27 GeV- points generated with MAID2007 with Osipenko et al. errors.
Vector form factors

- Our form factor model:

\[
C_3^V (Q^2) = \frac{C_3^V (0) \cdot (1 + K_1 Q^2)}{1 + A Q^2 + B Q^4 + C Q^6}
\]

\[
C_4^V (Q^2) = -\frac{M}{W} C_3^V (Q^2) \cdot \frac{(1 + K_2 Q^2)}{(1 + K_1 Q^2)}
\]

\[
C_5^V (Q^2) = \frac{C_5^V (0)}{(1 + D \frac{Q^2}{M_V^2})^2}
\]

\[
C_6^V (Q^2) \equiv 0 \ (CVC).
\]

- inspired by SU(6)-symmetrical quark model (J. Liu et al. Phys. Rev. C 52, 1630 (1995))- simple relation between \( C_4^V (Q^2, W) = -\frac{M_p}{W} C_3^V (Q^2) \), \( C_5^V = 0 \).

- \( K_1, K_2 \) corrections to the above, analogous to the successful parametrization of proton e-m form factors from J. Kelly, Phys. Rev. C 70 (2004) 068202.

- Addition of non-zero \( C_5^V \).
Electromagnetic fit results

<table>
<thead>
<tr>
<th>$C_3^V(0)$</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>$C_5^V(0)$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1±0.1</td>
<td>4.7±0.7</td>
<td>-0.4±0.4</td>
<td>5.6±1.4</td>
<td>0.1±0.2</td>
<td>1.7±0.4</td>
<td>0.6±0.05</td>
<td>1.0±0.1</td>
</tr>
</tbody>
</table>

- Not perfect, but reasonable agreement with data.
- Comparable with MAID2007 up to the $\Delta$ peak.
- $Q^2$-dependence of $C_5^V$: driven by vector mass $M_V = 0.84$ GeV
- Best fit: $\frac{\chi^2_{W < M_p + 2m_\pi}}{D.O.F.} = 13.7$, MAID (unitarized, $\Delta$+Born+$\rho$+$\omega$)
- $\frac{\chi^2_{W < M_p + 2m_\pi}}{D.O.F.} = 12.1$. Data too accurate.
Electromagnetic fit results


- Blue curve: HNV model with Lalakulich-Paschos form factors, red curve: best fit. Still not perfect, but visible improvement.
Axial form factors

- **Axial part:** not so much available data. Nuclear targets: problem of many-body effects, FSI etc. Best ANL/BNL: deuteron bubble chambers.

- Lack of strong theoretical constraints on $C_i^A$. Leading form-factor in dipole ansatz:

  $$C_5^A(Q^2) = \frac{C_5^A(0)}{(1 + Q^2/M_A^2)^2}$$

- Off-diagonal Goldberger-Treiman relation $C_5^A(0) = \frac{f^*}{\sqrt{2}} \approx 1.2$. Delta axial mass-only from fits. Intuition- "axial charge radius" $M_A = \mathcal{O}(1 \text{ GeV})$. In general, both can be fitted.

- From PCAC:

  $$C_6^A(Q^2) = \frac{M^2C_5^A(Q^2)}{m_\pi^2 + Q^2}$$

- The rest: no real constraint, "Adler model", "handwaving":

  $$C_3^A(Q^2) = 0; \ C_4^A(Q^2) = -\frac{1}{4}C_5^A(Q^2)$$
Effective deuteron model (e.g. L. Alvarez-Ruso et al. Phys. Rev. C 59 (1999) 3386):

\[
\frac{d\sigma}{dQ^2} \text{ deuteron} = \int \frac{d^3p_N}{(2\pi)^3} \frac{f(|\vec{p}_N|)}{v_{rel.}} \frac{d\sigma(\vec{q}^\mu, \vec{p}_N)}{dQ^2} \text{ free}
\]

with \( f(p_N) \)- norm of momentum-space deuteron wave function ("Paris": M. Lacombe et al. Phys. Lett. B 101 (1981) 139), \( \vec{q}^\mu = (q^0 - B(|\vec{p}_N|), \vec{q}) \),

\[
B(|\vec{p}_N|) = M_D - 2\sqrt{\vec{p}_N^2 + M^2}
\]

and \( v_{rel.} \rightarrow \) flux correction due to nucleon movement.

Full \( \Delta + \) background computation: complicated numerical procedure. Usage of Wroclaw Centre for Networking and Supercomputing grid.
Preliminary results for ANL data

- **Top plot**: full model, free nucleon target
  best fit result for ANL data+ 1-σ
  contours:

- **Separate fits in ANL1 and ANL2**
  channels: consistent, ANL3: higher $C_5^A(0)$

- Preference of $C_5^A(0) \approx 0.95$, smaller from
  Goldberger-Treiman (1-2σ). $M_A \approx 0.8$
  GeV (all channels).

- **Bottom plot**: deuteron effects included
  $C_5^A(0) \approx 1.1$, $M_A \approx 0.85$ GeV
  statistical consistency with Goldberger-Treiman.

- Deuteron effects → higher $C_5^A$.

- **Global** $\chi^2_{ANL} / D.O.F. = \frac{63.37}{35}$ (free
  target) → $\frac{61.90}{35}$ (deuteron).
Comparison of total cross sections

- Cross sections with $W < 1.4$ GeV cut.
- Good in ANL $p\pi^+$ and $p\pi^0$ channels.
- Visible lack of cross section in $n\pi^+$ channel. Expected from the separate $C_5^A(0)$ fits.
- Same old problem with $n\pi^+$.

ANL $Q^2$ distributions

Again, biggest discrepancy in the $n\pi^+$ channel.
Inclusion of BNL

- BNL and ANL1 fits within $1\sigma$.
- Inclusion of BNL data in global fit $M_A=0.85$ GeV $\rightarrow$ 0.95 GeV.
- Problem: only $p\pi^+$ channel from BNL, $p\pi^+$ "double counting" $\rightarrow$ Restriction to ANL data.
- Need for more deuterium experiments.
Conclusions

- First fit of single pion production model to electromagnetic and weak data including all ANL channels has been performed.
- Tests of fitted model against inclusive electron data give reasonable, albeit not perfect agreement.
- For neutrino SPP each channel $p\pi^+$, $p\pi^0$ and $n\pi^+$ seems to fit nicely separately, but there is some tension between $n\pi^+$ channel and the rest (much higher $C_5^A(0)$). This tension is of the order of 2$\sigma$ level.
- Inclusion of $\chi PT$ did not resolve the ANL3 channel ($n\pi^+$) problem.
- ANL fit: $C_5^A(0)$ in good agreement with Goldberger-Treiman relation.
Most of numerical calculations were carried out in the Wroclaw Centre for Networking and Supercomputing (http://www.wcss.wroc.pl), grant No. 268. We would like to thank Luis Alvarez-Ruso and Juan Nieves for many fruitful discussions.
Backup
All fits so far: only $p\pi^+$ (ANL1) channel, flux-averaged $\frac{d\sigma}{dQ^2}$ with errors $\delta \frac{d\sigma}{dQ^2}$ (actually: $Q^2$-bin-averaged). Comparison of flux- and $Q^2$-averaged cross sections in each bin together with data normalization fit $p_{ANL}$ (statistical model from Phys. Rev. D 80, 093001 (2009)).

$$\chi^2_{ANL1} = \sum_i^{(Q^2 \text{bins})} \left( \frac{\sigma_i^{TH.} - p_{ANL} \cdot \sigma_i^{EXP.}}{p_{ANL} \cdot \delta\sigma_i^{EXP.}} \right)^2 + \left( \frac{p_{ANL} - 1}{\delta p_{ANL}} \right)^2$$
Statistical framework

- **ANL2** and **ANL3**: event distributions in $Q^2$ (distribution shapes).
- Experimental correction factors $C^{EXP}$ with errors $\delta C^{EXP}$. Normalization $p_{ANL}$.

$$
\chi^2_{ANL2,3} = \sum_i \left( \sigma_{TH.} \cdot \frac{\sum_j N_j^{EXP} \cdot C^{EXP}}{\sum_j \sigma_j^{TH.}} \cdot p_{ANL} - N_i^{EXP.} \right)^2 + \left( \frac{\sigma_{TH.}}{\sigma_{TOT.} \cdot p_{ANL}} - 1 \right)^2
$$

- First fit to all three channels.
Statistical framework

\[
\left\langle \frac{dN}{dQ^2} \right\rangle_{BNL} + \text{normalization } p_{BNL}. \text{ Problem: no cut in } W \text{ for } \sigma_{TOT}^{EXP}.
\]

\[\delta p_{BNL} = 10\% - \text{beam flux uncertainty from K. Graczyk et al. PRD 80, 093001 (2009).}\]

\[
\chi^2_{BNL} = \sum_i \left( \frac{\left( \frac{\sigma_{i}^{TH} \cdot \sum_j N_{j}^{EXP.}}{\sum_j \sigma_{j}^{TH}} \cdot \delta p_{BNL} - N_{i}^{EXP.}} \right)^2}{\sum_{j} \sigma_{j}^{TH}} \right) + \left( \frac{\sigma_{TOT}^{EXP.} \cdot \delta p_{BNL} - 1}{\delta p_{BNL}} \right)^2
\]
## Preliminary results table

<table>
<thead>
<tr>
<th>Data</th>
<th>bckgr.</th>
<th>deut.</th>
<th>$C_5^A(0)$</th>
<th>$M_A$</th>
<th>$p_{ANL}$</th>
<th>$p_{BNL}$</th>
<th>$\chi^2/D.O.F.$</th>
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</thead>
<tbody>
<tr>
<td>ANL1</td>
<td>0</td>
<td>0</td>
<td>1.1±0.3</td>
<td>0.95±0.2</td>
<td>1.04</td>
<td>-</td>
<td>0.71/8</td>
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<tr>
<td>ANL1</td>
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<td>0</td>
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<td>0.95±0.3</td>
<td>1.05</td>
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</tr>
<tr>
<td>ANL1</td>
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<td>1.0±0.2</td>
<td>1.06</td>
<td>-</td>
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<tr>
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<td>1.3±0.7</td>
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<td>0.93</td>
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<td>13.7/13</td>
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<td>0.94</td>
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<td>-</td>
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<td>114/65</td>
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