DIS/Hadronization

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Outline

• Deep Inelastic Scattering
  – Cross section
  – Parton Distribution Functions
• Hadronization
  – Phenomenological approach (KNO scaling)
  – Theoretical approach (Lund model)
  – Resonance excitation — DIS transition region
• Monte Carlo implementation
  – NuWro
  – GENIE
Why do we care about hadronization?
• Deep inelastic scattering (DIS)

\[ \nu_l(k) + N(p) \rightarrow l(k') + X(p'), \]
\[ \bar{\nu}_l(k) + N(p) \rightarrow \bar{l}(k') + X(p'), \]
\[ \nu_l(k) + N(p) \rightarrow \nu_l(k') + X(p'), \]
\[ \bar{\nu}_l(k) + N(p) \rightarrow \bar{\nu}_l(k') + X(p'), \]

\[ \int \frac{d^2\sigma(\nu \bar{\nu})}{dxdy} \propto G_F^2 y \frac{1}{16\pi} \left( 1 + \frac{Q^2}{M_{W,Z}^2} \right)^{-2} L_{\mu\nu} W^{\mu\nu}, \]

• Cross section is given by contraction of leptonic and hadronic tensors

• Kinematic variables
  – Four momentum transfer
  \[ Q^2 = -q^2 = -(k - k')^2 \]
  \[ = L_{AB} - m_l^2 + 2E_\nu(E' - |k'| \cos \theta_\mu) \]
  \[ \nu = \frac{q \cdot p}{M} = \frac{L_{AB}}{E_\nu} \]
  \[ = E_\nu - E' \]

  – Hadronic invariant mass
  \[ W^2 = (q + p)^2 \]
  \[ - L_{AB} M^2 + 2M_\nu - Q^2. \]
  \[ y = \frac{p \cdot q}{p \cdot k} = \frac{L_{AB}}{E_\nu} \]
  \[ = \frac{\nu}{E_\nu} \]

  – inelasticity

  – B. scaling variable
  \[ x = \frac{-q^2}{2p \cdot q} = \frac{L_{AB}}{2M_\nu} = \frac{Q^2}{2M_\nu} \]
Cross section

- Leptonic tensor
  \[
  L_{\mu\nu} = 2T\gamma[(k + m_\ell)\gamma_\mu(1 - \gamma_5)(k + m_\nu)\gamma_\nu] = 8 \left( k_\ell k_\nu + k_\mu k_\nu' - g_{\mu\nu}k^\alpha k_{\alpha\beta} \right)
  \]

- Hadronic tensor
  - Unpolzarized target

\(W_i\) are real functions describing structure of nucleons

\[
\frac{d^2\sigma}{dx dy} = \frac{G_F^2 y M E_\nu}{\pi(1 + Q^2/M^2_{W,\cdots})^2} \left[ \left( xy + \frac{m_\ell^2}{2E_\nu M} \right) W_1 + \left( \frac{E}{M} - \frac{y E}{M} - \frac{xy}{2} - \frac{m_\ell^2}{4M E_\nu} \right) W_2 
  + \left( \frac{xyE}{M} \left( 1 - \frac{y}{2} - \frac{m_\ell^2}{4M^2} \right) \right) W_3 + \left( \frac{xy m^2_\ell}{2M^2} + \frac{m^4_\ell}{4M^3 E_\nu} \right) W_4 - \frac{m^2_\ell}{2yM E_\nu} W_5 \right], \quad (2.46)
\]

- \(W_6\) vanishes during contraction of two tensors
- \(W_4\) and \(W_5\) are proportional to lepton mass (negligible)
- \(W_j\) are general structure function which can describe all types of interactions (QE, RES, COH, DIS – added incoherently)
Final state hadrons

- The $W_j$ are not sufficient for the experiments
  - Only kinematics of the final lepton
  - Remaining invariant mass has to be distributed to final state hadrons
Parton model

- Usually we rewrite the $W_j$ in the dimensionless structure functions

\[
F_1(x, Q^2) = W_1(x, Q^2), \quad F_i(x, Q^2) = W_i(x, Q^2) \cdot \nu/M \quad (i = 2, \ldots, 5),
\]

\[
F_1(x, Q^2) = \sum_j \left[ q_j(x, Q^2) + \bar{q}_j(x, Q^2) \right]
\]
\[
F_3(x, Q^2) = 2 \sum_j \left[ q_j(x, Q^2) - \bar{q}_j(x, Q^2) \right]
\]
\[
F_2(x, Q^2) = 2xF_1(x, Q^2)
\]

Parton Distribution Functions

\[
\begin{align*}
  u_p(x) &= d_n(x), & \bar{u}_p(x) &= \bar{d}_n(x) \\
  d_p(x) &= u_n(x), & \bar{d}_p(x) &= \bar{u}_n(x)
\end{align*}
\]
Hadronization model

• DIS gives inclusive cross section
  – **Problem:** in event generator we need a hadronization model to obtain exclusive channels
  – **Solution:** model based on LUND string fragmentation implemented in PYTHIA generator

• **BUT**
  – Pythia can generate events only for \( E > 10 \text{GeV} \)
  – In fragmentation routines of PYTHIA there are no limits for energy of the considered system
  – We need method of selection interacting quark
PYTHIA hadronization

• All generators use PYTHIA for hadronization at high invariant masses. (LUND string model)
  – GENIE: Transition window from 2.3 to 3.0 GeV/c²
  – GiBUU: Uses PYTHIA for hard scattering and hadronization, transition window from 2.0 to 2.4 GeV/c²
  – NEUT: above 2.0 GeV/c²
  – NuWro: above 1.21 GeV/c²
• For a transitions regions MC generator use various approaches
  – Phenomenological (KNO scaling)
  – LUND model
KNO scaling – phenomenological approach

• Distribution of charged hadrons is predicted by the KNO scaling

\[ \langle n_{ch} \rangle P(n_{ch}) = 2 \frac{e^{-c \langle n_{ch} \rangle}}{\Gamma(cz + 1)}, \]

• MC implementation
  – From available W calculate \( \langle n_{ch} \rangle \)
  – Select multiplicity from KNO distribution
  – Select baryon and mesons and assign them 4-momenta
Independent fragmentation (LUND model)

- The fragmentation starts with struck quark $q_0$ which creates a meson (baryon) with quark (diquark) with $z_0$ of system energy. Quark $q_1$ gets
  \[
  (1 - z_0) = z_1
  \]
- The fragmentation is stopped at $E < E_{\text{min}}$
- Last step is cluster fragmentation (two hadrons) or cluster collapse (one hadron)

\[
f(z) = \frac{(1 - z)^a}{z} \cdot \exp \left( -\frac{bm_q^2}{z} \right)
\]
### PYTHIA parameters

- Parameters were fine-tuned for NuWro generator to reproduce charged hadron multiplicity.

- MSTJ(17) was set to be 3 rather than 2 (number of tries to find two hadrons with masses lower than cluster mass).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (Wroclaw)</th>
<th>Value (NUX)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARJ(2)</td>
<td>-</td>
<td>0.21</td>
<td>(D=0.30) is P(s)/P(u), the suppression of s quark pair production in the field compared with u or d pair production.</td>
</tr>
<tr>
<td>PARJ(21)</td>
<td>-</td>
<td>0.44</td>
<td>(D=0.36 GeV) corresponds to the width of the Gaussian px and py transverse momentum distributions for primary hadrons. See also PARJ(22) - PARJ(24).</td>
</tr>
<tr>
<td>PARJ(23)</td>
<td>-</td>
<td>0.01</td>
<td>PARJ(23-24): (D=0.01, 2.) a fraction of the Gaussian transverse momentum distribution is taken to be a factor PARJ(24) larger than input in PARJ(21). This gives a simple parametrization of non-Gaussian tails to the Gaussian shape assumed above.</td>
</tr>
<tr>
<td>PARJ(32)</td>
<td>0.1 GeV</td>
<td>-</td>
<td>(D=1. GeV) is, with quark masses added, used to define the minimum allowable energy of a colour-singlet jet system.</td>
</tr>
<tr>
<td>PARJ(33)</td>
<td>0.5 GeV</td>
<td>0.2 GeV</td>
<td>(D=0.8 GeV, 1.5 GeV) are, together with quark masses, used to define the remaining energy below which the fragmentation of a jet system is stopped and two final hadrons formed. PARJ(33) is normally used, except for MSTJ(11)=2, when PARJ(34) is used.</td>
</tr>
<tr>
<td>PARJ(34)</td>
<td>1.0 GeV</td>
<td>-</td>
<td>(D=2.) represents the dependence on the mass of the final quark pair for defining the stopping point of the fragmentation. Is strongly correlated to the choice of PARJ(33).</td>
</tr>
</tbody>
</table>

*also MSTJ(17)=3 in NuWRO*

Transition region in GENIE

- The model for the cross section will affect many things.
- DIS vs. ‘non-resonant background’ in the resonance region.
- The KNO parameters obtained from fit to electron scattering data

Hadronization in NuWro

- NuWro is a MC generator which uses PYTHIA from about $W=1.21$ GeV/c$^2$
- It was necessary as in NuWro we have only $\Delta$ resonance and remaining cross section is describe in terms of DIS

Quark-hadron duality

Bloom and Gilman showed that for electron scattering: structure functions averaged over resonances are approximately equal to leading twist contributions.
Transition region in NuWro

• There is a smooth transition between $\Delta$ and DIS regions for the single pion production

\[
\frac{d\sigma^{SPP}}{dW} = \frac{d\sigma^\Delta}{dW} (1 - \alpha(W)) + \frac{d\sigma^{DIS}}{dW} F^{SPP}(W) \alpha(W)
\]

\[
\alpha(W) = \Theta(W_{\text{min}} - W) \frac{W - W_{th}}{W_{\text{min}} - W_{th}} \alpha_0
\]

+ \Theta(W_{\text{max}} - W) \Theta(W - W_{\text{min}}) \frac{W - W_{\text{min}} + \alpha_0(W_{\text{max}} - W)}{W_{\text{max}} - W_{\text{min}}}

+ \Theta(W - W_{\text{max}})

$W_{\text{min}} = 1.3\text{GeV}$, $W_{\text{max}} = 1.6\text{GeV}$

• Single pion production functions were obtained from LUND

$\alpha_0$ values

<table>
<thead>
<tr>
<th>$\nu p \rightarrow \mu^- \pi^+$</th>
<th>$\nu n \rightarrow \mu^- n\pi^+$</th>
<th>$\nu n \rightarrow \mu^- p\pi^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>$\bar{\nu} n \rightarrow \mu^+ n\pi^-$</td>
<td>$\bar{\nu} p \rightarrow \mu^+ p\pi^-$</td>
<td>$\bar{\nu} p \rightarrow \mu^+ n\pi^0$</td>
</tr>
<tr>
<td>0.0</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Probability of scattering on parton

- Fragmentation algorithm
  - Cross section is approximately sum of contributions from separate quarks, where cross section for scattering on quark $q_i$ (valance or sea quark)
    \[
    \frac{d^2\sigma^{\nu q_i \rightarrow \mu q_j}}{dx dy} \sim q_i K_i
    \]
  - Kinematic factor for quark $q_i$.
- Probability of scattering on a quark is given
  \[
P(q_i) = \frac{d^2\sigma^{q_i}}{dx dy} \sum_{j=q,\bar{q}} \frac{d^2\sigma^{q_j}}{dx dy}
  \]

[Juszczak, Nowak, Sobczyk, hep-ph/0512365]
Scattering on given quark

- The process $\nu N \rightarrow l^- X$ can be split into 5 cases corresponding to scattering off separate parton $d_{val}, d_{sea}, \bar{u}_{sea} \rightarrow \bar{d}, \bar{u}_{sea} \rightarrow \bar{s}, s_{sea}$.

- Fragmentation of system quark-diquark is performed by PYTHIA6 routines. However, in case of $\bar{u}_{sea} \rightarrow \bar{s}, s_{sea}$ first step, when strange or charm meson is produced, is performed in NuWro.
Reconstruction of final states

- Charged hadron multiplicity

\[
P(n_{ch}) = \frac{\sigma(n_{ch})}{\sum_{n_{ch}} \sigma(n_{ch})}
\]
Hadron multiplicities

- All generators were check against data
- Use experimental data for charge hadron multiplicities
- Usually following fit was used

\[ <n_{ch}> = A + B \ln W^2. \]

- Kuzmin and Naumov proposed recently (arXiv:1311.4047)

\[ <n_{ch}> = \begin{cases} a_1 + b_1 \ln X + c_1 \ln^2 X & \text{if } X \leq X_0 \\ a_2 + b_2 \ln X + c_2 \ln^2 X & \text{if } X > X_0 \end{cases} \]

\[ X = \frac{w^2}{(M+m_\pi)^2}, \quad X_0 = \frac{w_0^2}{(M+m_\pi)^2}, \quad W_0 \text{ of the order of 3 GeV.} \]
Conclusion

• Hadronization in difficult

• Many decisions have to be made based on limited data.

• New experiments soon enough will check assumption made in the Monte Carlo generators.