LANCASTER UNIVERSITY

DIS/Hadronization

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NuInt14

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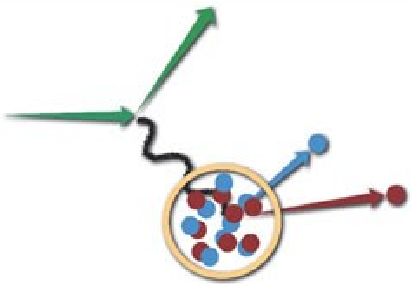
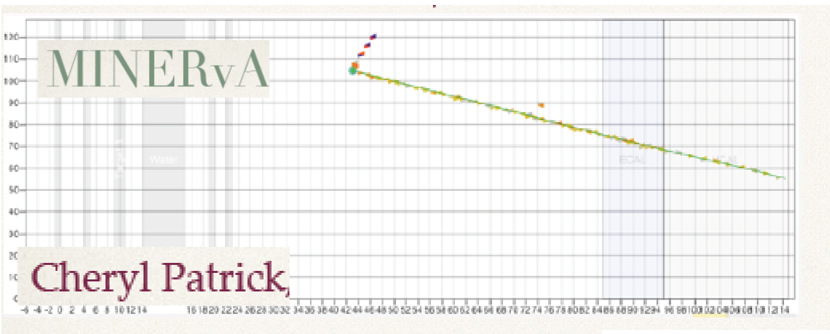
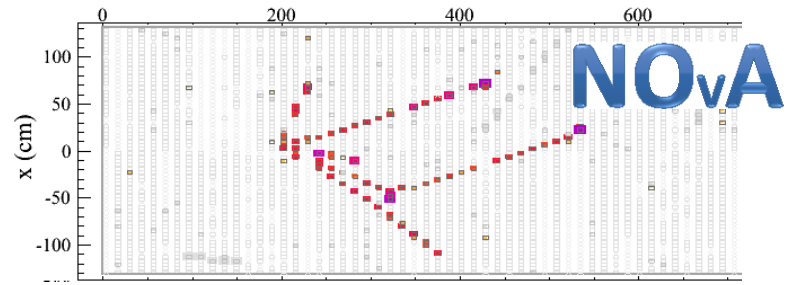


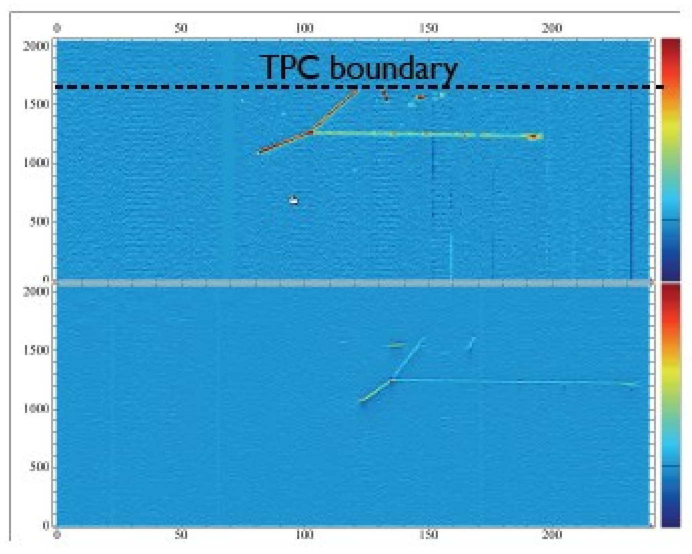
Outline

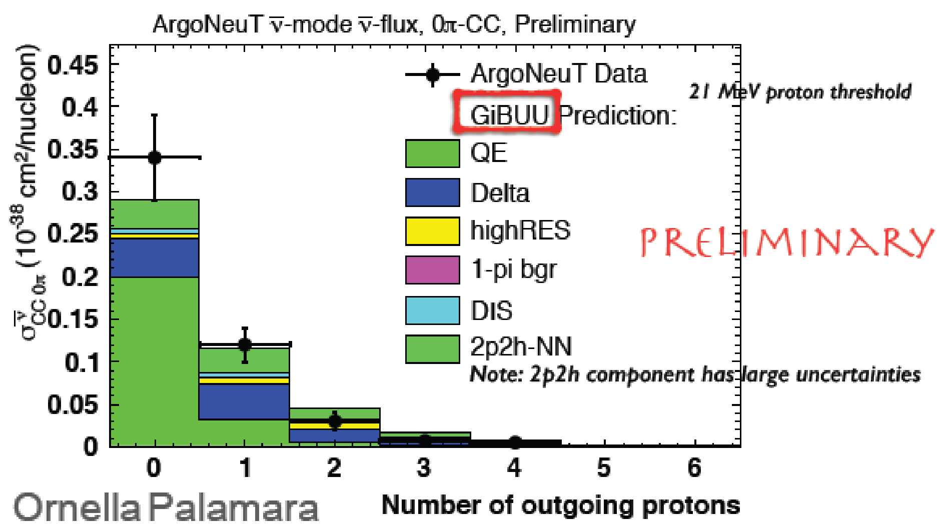
- Deep Inelastic Scattering
  - Cross section
  - Parton Distribution Functions
- Hadronization
  - Phenomenological approach (KNO scaling)
  - Theoretical approach (Lund model)
  - Resonance excitation – DIS transition region
- Monte Carlo implementation
  - NuWro
  - GENIE

Why do we care about hadronization?



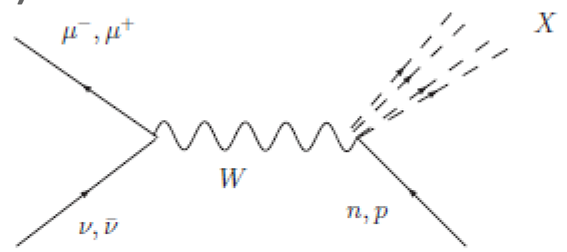






- Deep inelastic scattering (DIS)

$$\begin{aligned}
\nu_l(k) + N(p) &\rightarrow l(k') + X(p'), \\
\bar{\nu}_l(k) + N(p) &\rightarrow \bar{l}(k') + X(p'), \\
\nu_l(k) + N(p) &\rightarrow \nu_l(k') + X(p'), \\
\bar{\nu}_l(k) + N(p) &\rightarrow \bar{\nu}_l(k') + X(p'),
\end{aligned}$$



- Cross section is given by contraction of leptonic and hadronic tensors

$$\frac{d^2\sigma(\nu\bar{\nu})}{dxdy} = \frac{G_F^2 y}{16\pi} \frac{1}{\left(1 + \frac{Q^2}{M_{W,Z}^2}\right)^2} L_{\mu\nu} W^{\mu\nu},$$

- Kinematic variables
  - Four momentum transfer

$$Q^2 = -q^2 = -(k-k')^2 \stackrel{LAB}{=} -m_l^2 + 2E_\nu(E' - |\vec{k}'|\cos\theta_\mu)$$

  - Energy transfer

$$\nu = \frac{q \cdot p}{M} \stackrel{LAB}{=} E_\nu - E'$$

  - Hadronic invariant mass

$$W^2 = (q+p)^2 \stackrel{LAB}{=} M^2 + 2M\nu - Q^2.$$

  - inelasticity

$$y = \frac{p \cdot q}{p \cdot k} \stackrel{LAB}{=} \frac{\nu}{E_\nu}$$

  - B. scaling variable

$$x = \frac{-q^2}{2p \cdot q} \stackrel{LAB}{=} \frac{Q^2}{2M\nu}$$

Cross section

- Leptonic tensor

$$\begin{aligned} L_{\mu\nu} &= 2Tr[(\not{k}' + m_l)\gamma_\mu(1-\gamma_5)\,(\not{k} + m_\nu)\,\gamma_\nu] \\ &= 8\left(k'_\mu k_\nu + k_\mu k'_\nu - g_{\mu\nu}k'\cdot k \mp i\epsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta\right) \end{aligned}$$

- Hadronic tensor
  - Unpolzarized target

$$\begin{aligned} W^{\mu\nu} &= -g^{\mu\nu}W_1 + \frac{p^\mu p^\nu}{M^2}W_2 - i\epsilon^{\mu\nu\rho\sigma}\frac{p_\rho q_\sigma}{2M^2}W_3 + \frac{q^\mu q^\nu}{M^2}W_4 \\ &+ \frac{(p^\mu q^\nu + p^\nu q^\mu)}{2M^2}W_5 + i\frac{(p^\mu q^\nu + p^\nu q^\mu)}{2M^2}W_6. \end{aligned}$$

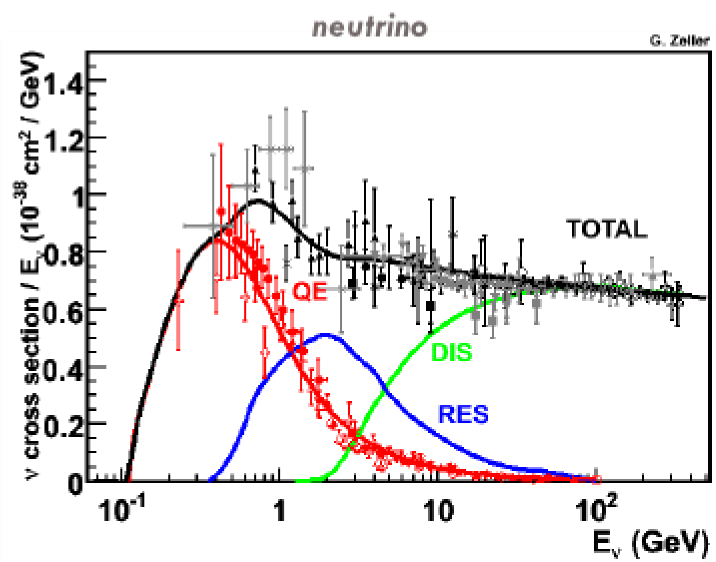
W_i are real functions describing structure of nucleons

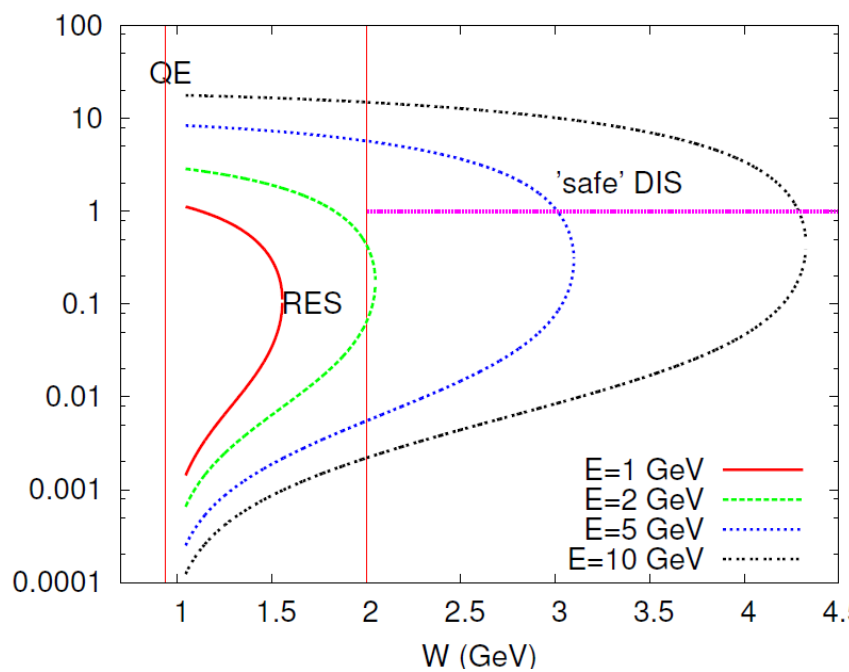
$$\begin{aligned} \frac{d^2\sigma}{dxdy} &= \frac{G_F^2 y M E_\nu}{\pi(1+Q^2/M_{W,Z}^2)^2}\left[\left(xy + \frac{m_l^2}{2E_\nu M}\right)W_1 + \left(\frac{E}{M} - \frac{yE}{M} - \frac{xy}{2} - \frac{m_l^2}{4ME_\nu}\right)W_2\right. \\ &+ \left.\left(\frac{xyE}{M}\left(1-\frac{y}{2}\right) - y\frac{m_l^2}{4M^2}\right)W_3 + \left(xy\frac{m_l^2}{2M^2} + \frac{m_l^4}{4M^3E_\nu}\right)W_4 - \frac{m_l^2}{2yME_\nu}W_5\right], \qquad (2.46) \end{aligned}$$

  - W_6 vanishes during contraction of two tensors
  - W_4 and W_5 are proportional to lepton mass (negligible)
  - W_j are general structure function which can describe all types of interactions (QE, RES, COH, DIS – added incohorently)

Final state hadrons

- The W_j are not sufficient for the experiments
  - Only kinematics of the final lepton
  - Remaining invariant mass has to be distributed to final state hadrons





Parton model

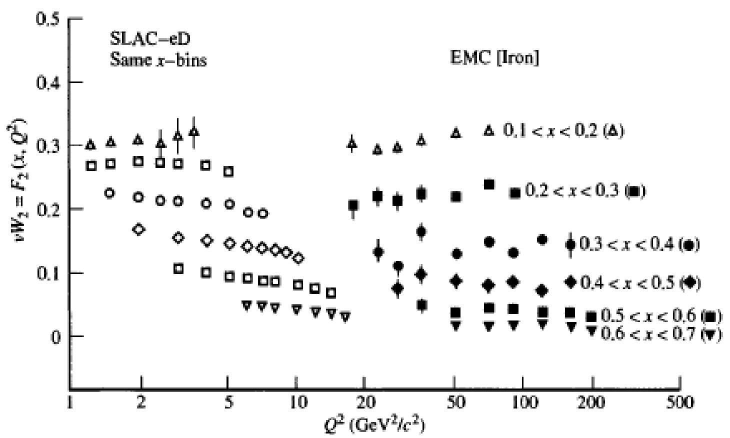
- Usually we rewrite the W_j in the dimensionless structure functions

$$F_1(x,Q^2) = W_1(x,Q^2), \quad F_i(x,Q^2) = W_i(x,Q^2)\cdot \nu/M \quad (i=2,...,5),$$

$$F_1\left(x,Q^2\right) = \sum_j \left[q_j\left(x,Q^2\right) + \bar{q}_j\left(x,Q^2\right)\right]$$

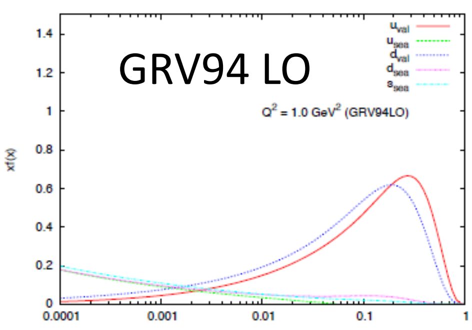
$$F_3\left(x,Q^2\right) = 2\sum_j \left[q_j\left(x,Q^2\right) - \bar{q}_j\left(x,Q^2\right)\right]$$

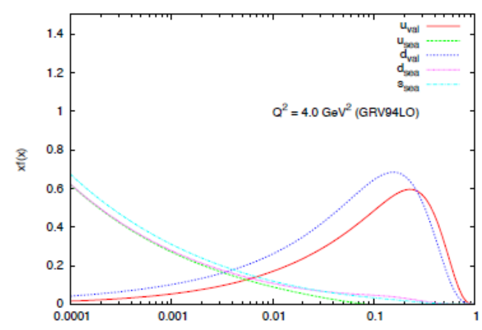
$$F_2\left(x,Q^2\right) = 2xF_1\left(x,Q^2\right)$$



Parton Distribution Functions

$$u_p(x) = d_n(x), \quad \bar{u}_p(x) = \bar{d}_n(x)$$
$$d_p(x) = u_n(x), \quad \bar{d}_p(x) = \bar{u}_n(x)$$





Hadronization model

- DIS gives inclusive cross section
  - **Problem:** in event generator we need a hadronization model to obtain exclusive channels
  - **Solution:** → model based on LUND string fragmentation implemented in PYTHIA generator

[Sjostrand et al, Comput. Phys. Commun. **135** (2001)], [Sjostrand et al, hep-ph/0108264]

- BUT
  - Pythia can generate events only for E > 10GeV
  - In fragmentation routines of PYTHIA there are no limits for energy of the considered system
  - We need method of selection interacting quark

PYTHIA hadronization

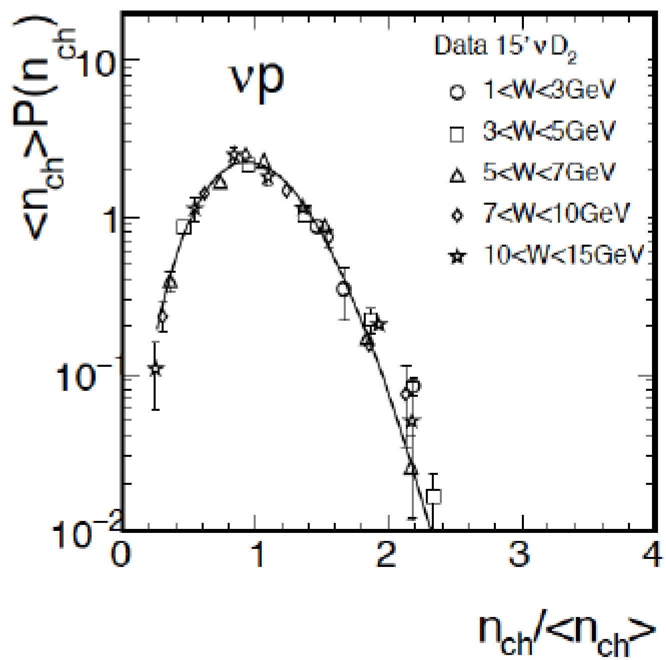
- All generators use PYTHIA for hadronization at high invariant masses. (LUND string model)
  - GENIE: Transition window from 2.3 to 3.0 GeV/c^2
  - GiBUU: Uses PYTHIA for hard scattering and hadronization, transition window from 2.0 to 2.4 GeV/c^2
  - NEUT: above 2.0 GeV/c^2
  - NuWro: above 1.21 GeV/c^2
- For a transitions regions MC generator use various approaches
  - Phenomenological (KNO scaling)
  - LUND model

KNO scaling – phenomenological approach

- Distribution of charged hadrons is predicted by the KNO scaling

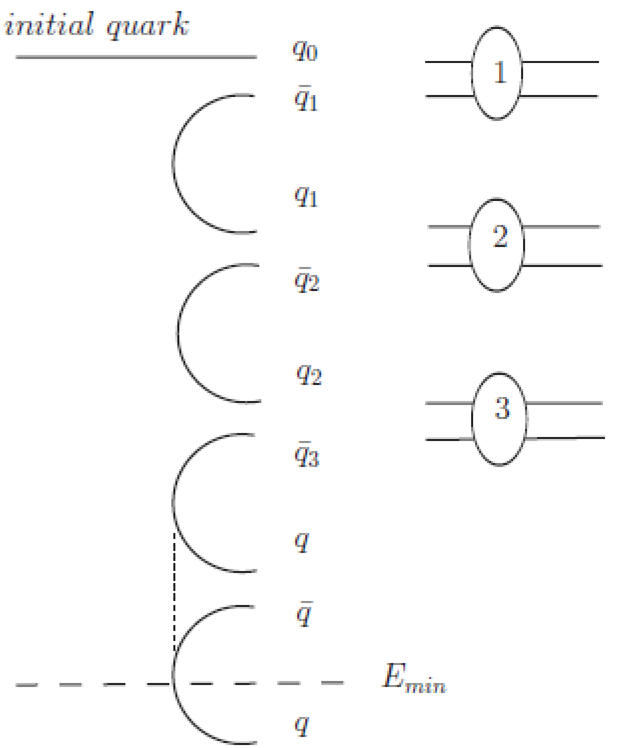
$$\langle n_{ch}\rangle P(n_{ch}) = 2\frac{e^{-c}c^{cz+1}}{\Gamma\left(cz+1\right)},$$

- MC implementation
  - From available W calculate $<n_{ch}>$
  - Select multiplicity from KNO distribution
  - Select baryon and mesons and assign them 4-momenta



$$z = \frac{n_{ch}}{\langle n_{ch}\rangle}.$$

Independent fragmentation (LUND model)



$$f(z) = \frac{(1-z)^a}{z} \cdot \exp\left(\frac{-bm_\perp^2}{z}\right)$$

- The fragmentation starts with struck quark q_0 which creates a meson (baryon) with quark (diquark) with z_0 of system energy. Quark q_1 gets

$$(1 - z_0) = z_1$$

- The fragmentation is stopped at $E<E_{min}$
- Last step is cluster fragmentation (two hadrons) or cluster collapse (one hadron)

also GENIE except for

Parameter	Value (Wroclaw)	Value (NUX)	Description
PARJ(2)	-	0.21	(D=0.30) is P(s)/P(u), the suppression of s quark pair production in the field compared with u or d pair production.
PARJ(21)	-	0.44	(D=0.36 GeV) corresponds to the width σ in the Gaussian px and py transverse momentum distributions for primary hadrons. See also PARJ(22) -PARJ(24).
PARJ(23)	-	0.01	PARJ(23-24) : (D=0.01, 2.) a fraction PARJ(23) of the Gaussian transverse momentum distribution is taken to be a factor PARJ(24) larger than input in PARJ(21). This gives a simple parametrization of non-Gaussian tails to the Gaussian shape assumed above.
PARJ(32)	0.1 GeV	-	(D=1. GeV) is, with quark masses added, used to define the minimum allowable energy of a colour-singlet jet system.
PARJ(33)	0.5 GeV	0.2 GeV	(D=0.8 GeV, 1.5 GeV) are, together with quark masses, used to define the remaining energy below which the fragmentation of a jet system is stopped and two final hadrons formed. PARJ(33) is normally used, except for MSTJ(11)=2, when PARJ(34) is used.
PARJ(34)	1.0 GeV	-	
PARJ(36)	0.3 GeV	-	(D=2.) represents the dependence on the mass of the final quark pair for defining the stopping point of the fragmentation. Is strongly correlated to the choice of PARJ(33)

also MSTJ(17)=3 in NuWRO

J. Nowak (NuWRO), Phys.Scripta T127 (2006) 70-72

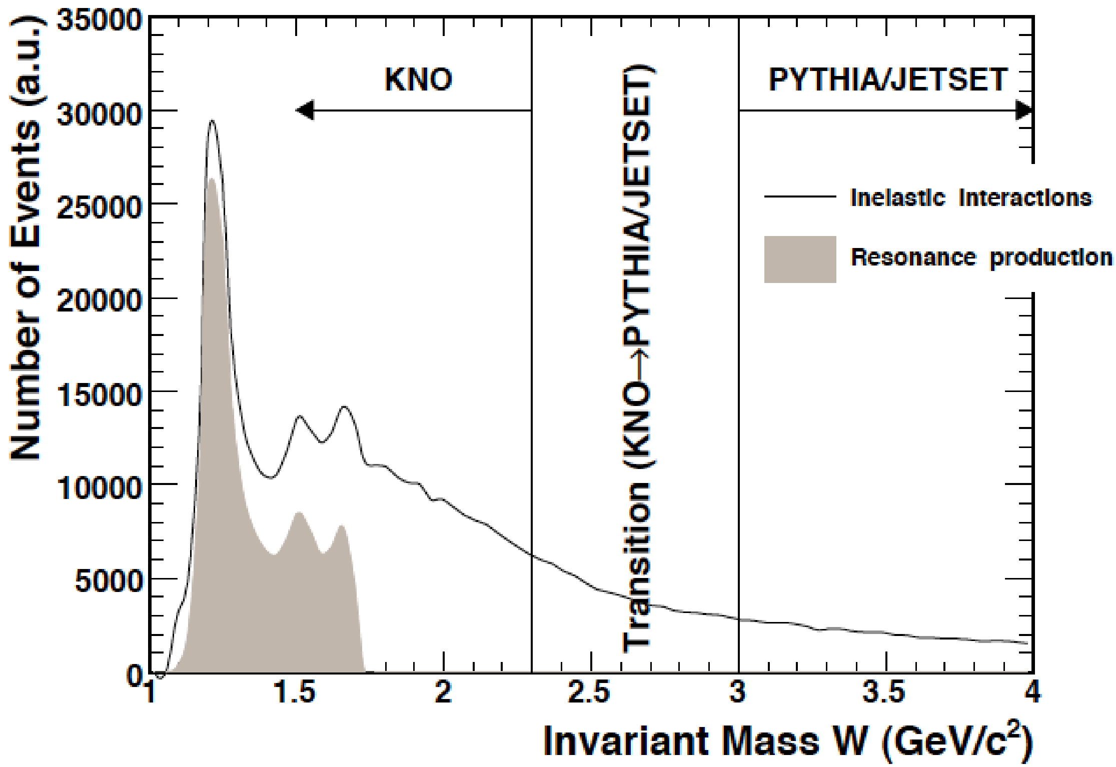
PYTHIA parameters

- Parameters were fine-tuned for NuWro generator to reproduce charged hadron multiplicity

- MSTJ(17) was set to be 3 rather then 2 (number of tries to find two hadrons with masses lower than claster mass).

Transition region in GENIE

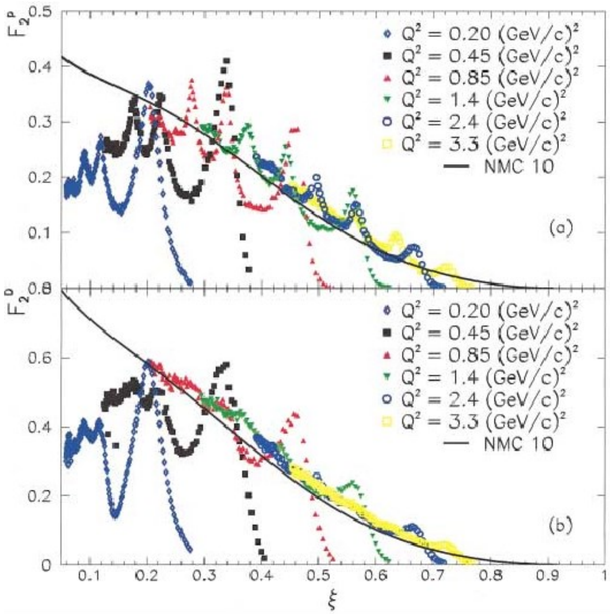
- The model for the cross section will affect many things.
- DIS vs. ‘non-resonant background’ in the resonance region.
- The KNO parameters obtained from fit to electron scattering data



T. Yang et al, Eur.Phys.J. C63 (2009) 1-10.
T. Yang, Ph. D Thesis, Stanford U (2009)

Hadronization in NuWro

- NuWro is a MC generator which uses PYTHIA from about W=1.21 GeV/c^2
- It was necessary as in NuWro we have only Δ resonance and remaining cross section is describe in terms of DIS



Quark-hadron duality

Bloom and Gilman showed that for electron scattering: structure functions averaged over resonances are approximately equal to leading twist contributions.

Transition region in NuWro

- There is a smooth transition between Δ and DIS regions for the single pion production

$$\frac{d\sigma^{SPP}}{dW} = \frac{d\sigma^{\Delta}}{dW}(1-\alpha(W)) + \frac{d\sigma^{DIS}}{dW}F^{SPP}(W)\alpha(W)$$

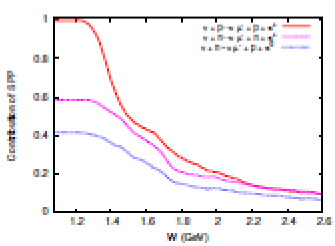
$$\begin{aligned} \alpha(W) &= \Theta(W_{min}-W)\frac{W-W_{th}}{W_{min}-W_{th}}\alpha_0 \\ &+ \Theta(W_{max}-W)\Theta(W-W_{min})\frac{W-W_{min}+\alpha_0(W_{max}-W)}{W_{max}-W_{min}} \\ &+ \Theta(W-W_{max}) \end{aligned}$$

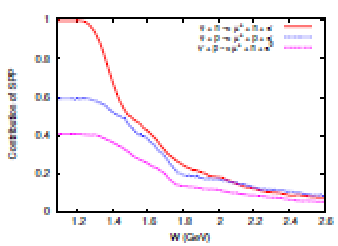
$$W_{min} = 1.3GeV, \; W_{max} = 1.6GeV$$

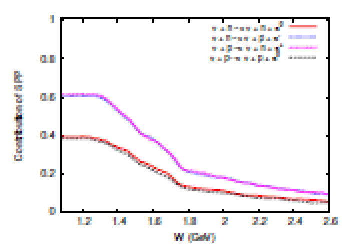
α_0 values

$\nu p \rightarrow \mu^- p \pi^+$ 0.0	$\nu n \rightarrow \mu^- n \pi^+$ 0.2	$\nu n \rightarrow \mu^- p \pi^0$ 0.3
$\bar{\nu} n \rightarrow \mu^+ n \pi^-$ 0.0	$\bar{\nu} p \rightarrow \mu^+ p \pi^-$ 0.2	$\bar{\nu} p \rightarrow \mu^+ n \pi^0$ 0.3

- Single pion production functions were obtained from LUND

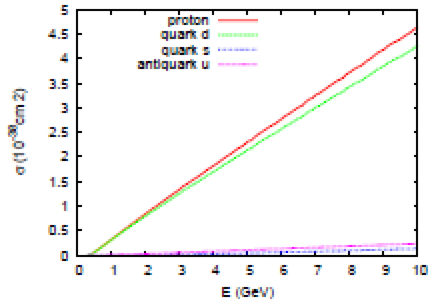






Probability of scattering on parton

- Fragmentation algorithm
  - Cross section is approximately sum of contributions from separate quarks, where cross section for scattering on quark q_i (valance or sea quark)



$$\frac{d^2\sigma^{\nu q_i \to \mu q_j}}{dxdy} \sim q_i K_i$$

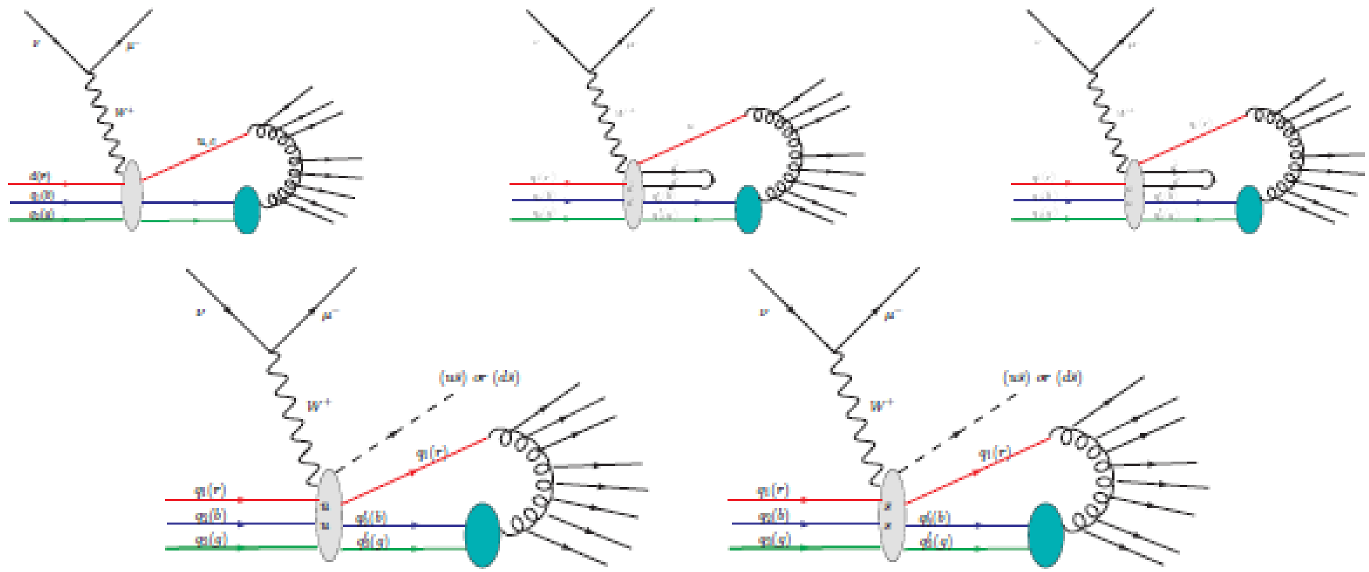
  - Kinematic factor for quark q_i.
- Probability of scattering on a quark is given

$$P(q_i) = \frac{\frac{d^2\sigma^{q_i}}{dxdy}}{\sum_{j=q,\bar{q}} \frac{d^2\sigma^{q_j}}{dxdy}}$$

[Juszczak, Nowak, Sobczyk, hep-ph/0512365]

Scattering on given quark

- The process $\nu N \to l^- X$ one can split into 5 cases corresponding to scattering off separate parton $d_{val}, d_{sea}, \quad \bar{u}_{sea} \to \bar{d}, \bar{u}_{sea} \to \bar{s}, s_{sea}$



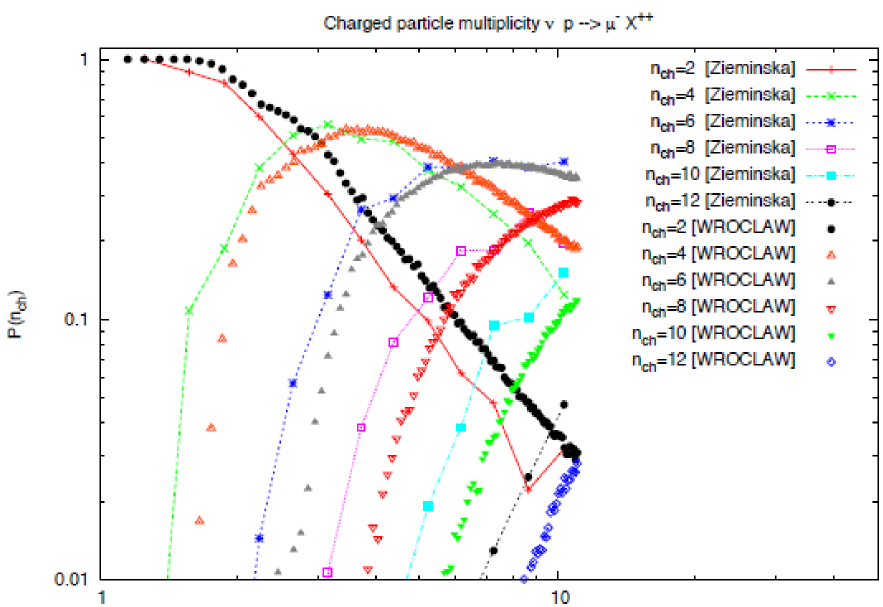
- Fragmentation of system quark-diquark is performed by PYTHIA6 routines. However, in case of $\bar{u}_{sea} \to \bar{s}, s_{sea}$ first step, when strange or charm meson is produced, is performed in NuWro.

Reconstruction of final states

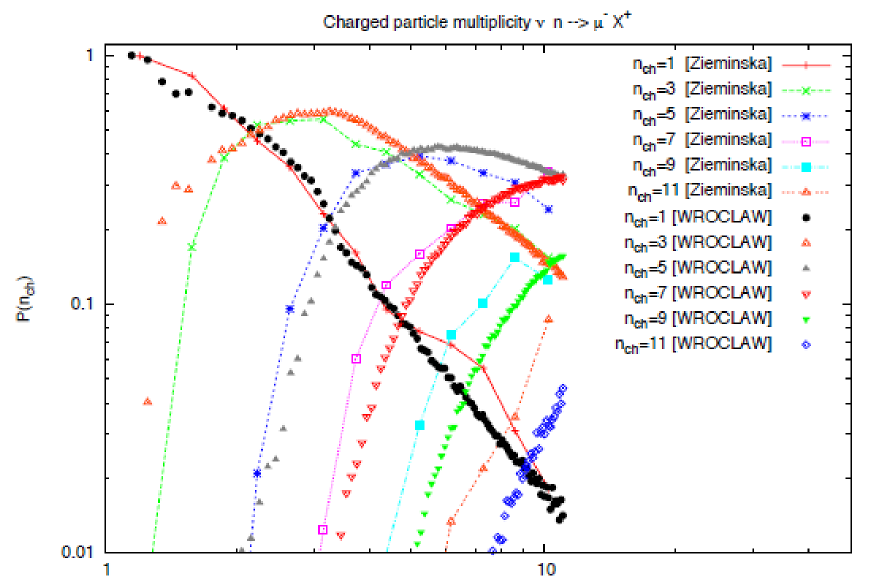
- Charged hadron multiplicity

(D. Zieminska et al. Phys. Rev. D27, 47(1983))

$$P(n_{ch}) = \frac{\sigma(n_{ch})}{\sum_{n_{ch}} \sigma(n_{ch})}$$



W(GeV)



W(GeV)

Hadron multiplicities

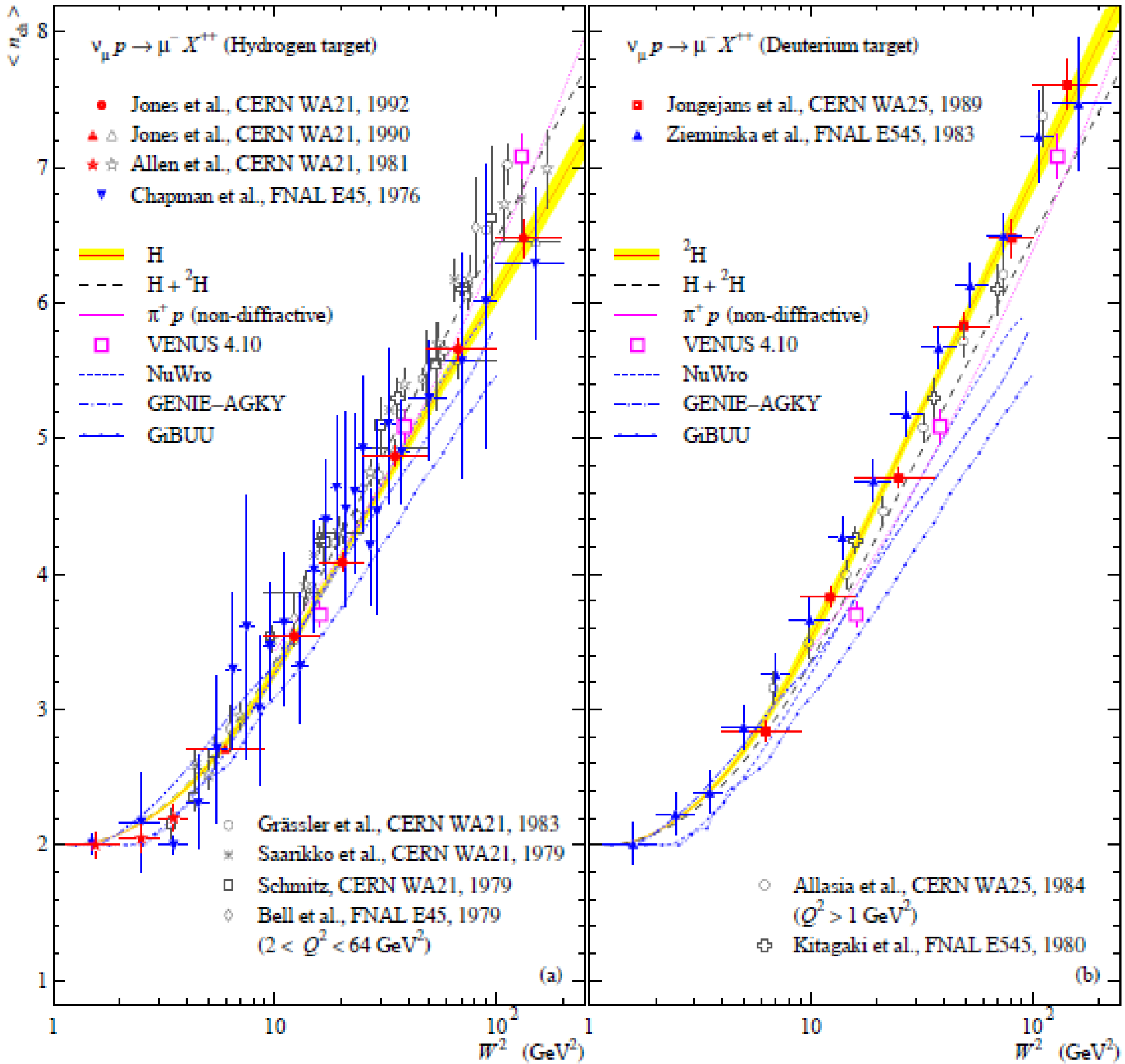
- All generators were check against data
- Use experimental data for charge hadron multiplicities
- Usually following fit was used

$$< n_{ch} >= A + B \ln W^2$$

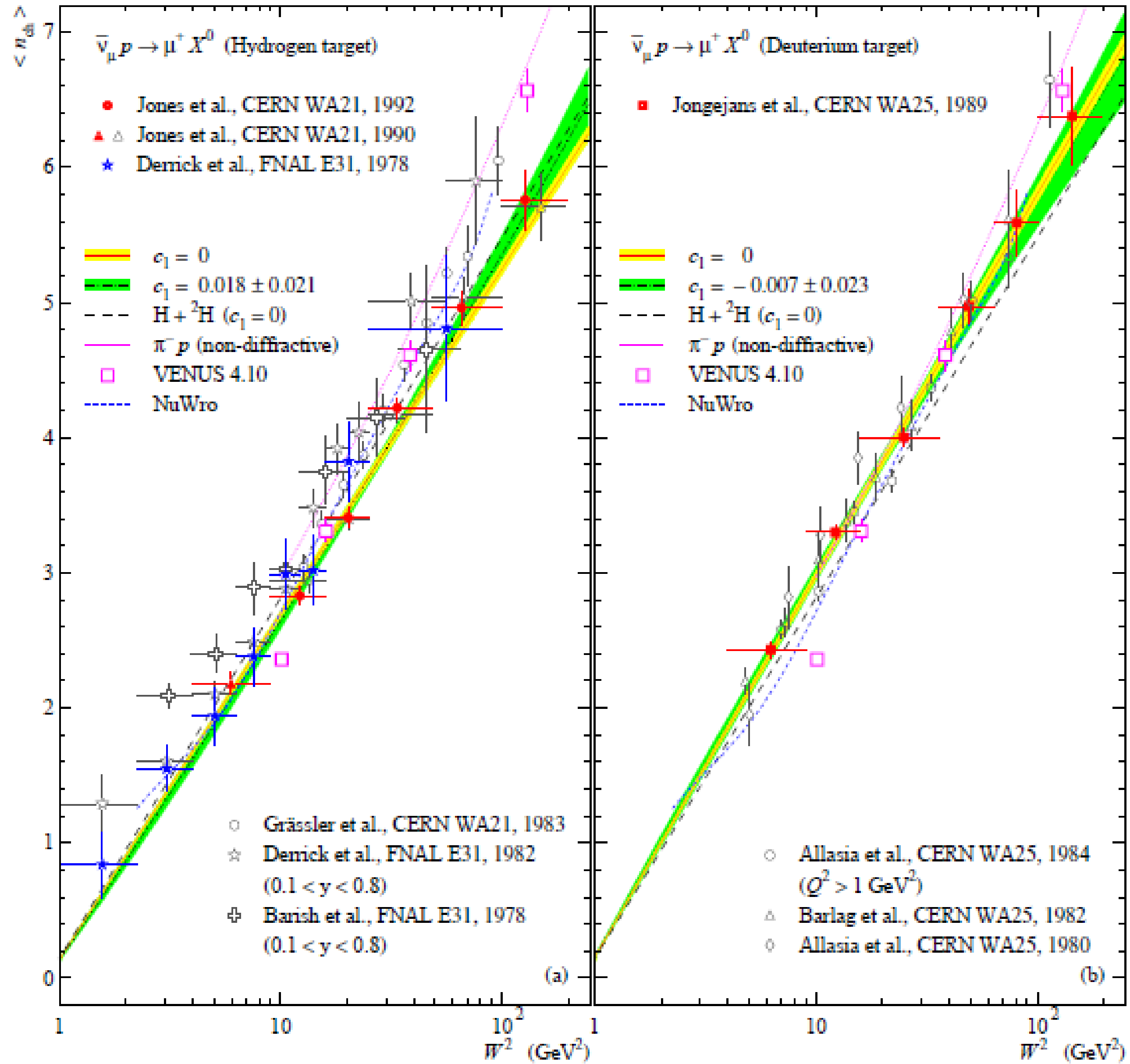
- Kuzmin and Naumov proposed recently (arXiv:1311.4047)

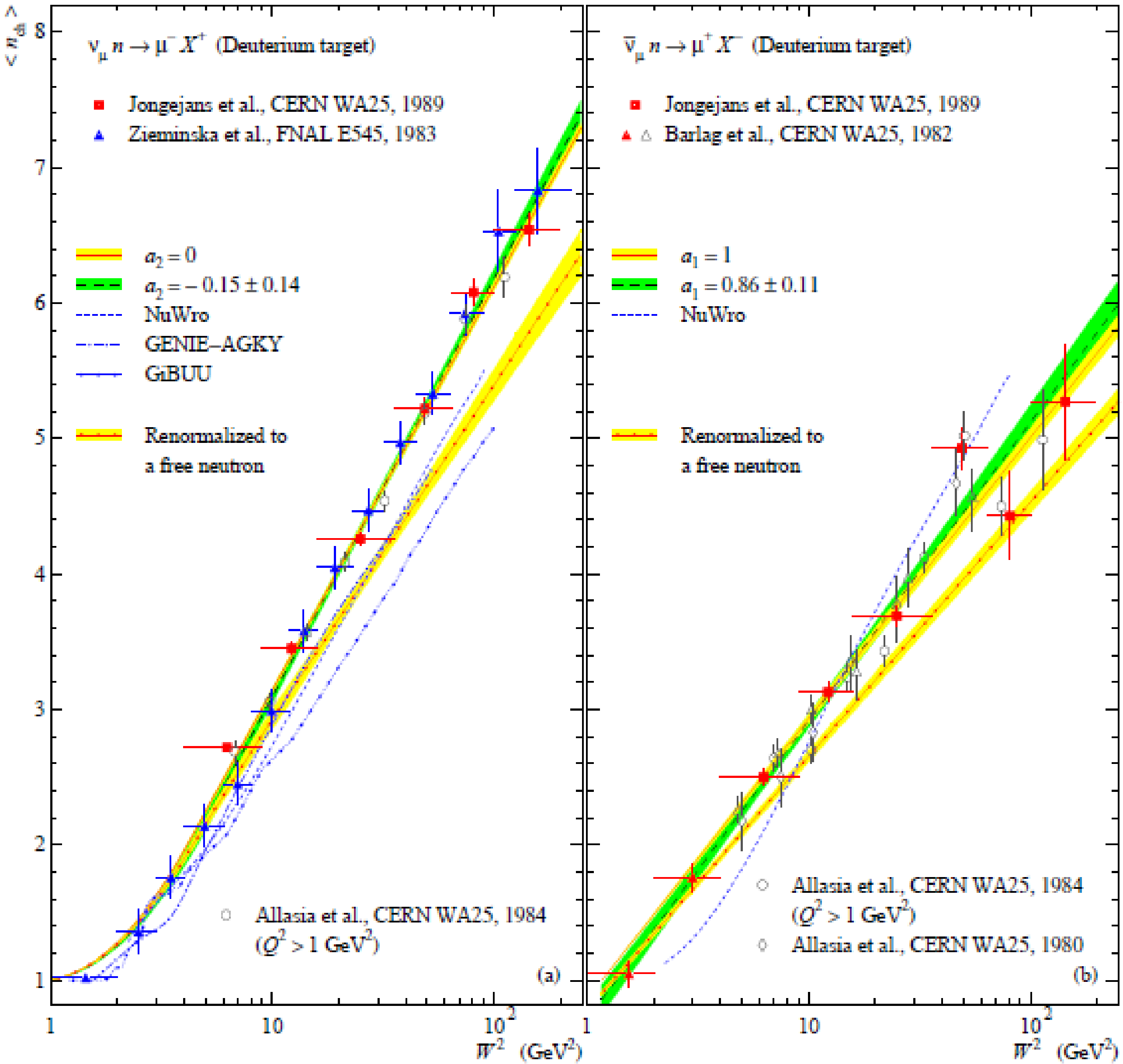
$$< n_{ch} >= \begin{cases} a_1 + b_1 \ln X + c_1 \ln^2 X & \text{if } X \leq X_0 \\ a_2 + b_2 \ln X + c_2 \ln^2 X & \text{if } X > X_0 \end{cases}$$

$X = \frac{W^2}{(M+m_\pi)^2}$, $X_0 = \frac{W_0^2}{(M+m_\pi)^2}$, W_0 of the order of 3 GeV.



$\nu_\mu p \to \mu^- X^{++}$ (Hydrogen target)
Jones et al., CERN WA21, 1992
Jones et al., CERN WA21, 1990
Allen et al., CERN WA21, 1981
Chapman et al., FNAL E45, 1976
H
H + ^{2}H
$\pi^+ p$ (non-diffractive)
VENUS 4.10
NuWro
GENIE–AGKY
GiBUU
Grässler et al., CERN WA21, 1983
Saarikko et al., CERN WA21, 1979
Schmitz, CERN WA21, 1979
Bell et al., FNAL E45, 1979
($2 < Q^2 < 64$ GeV^2)
(a)
$\nu_\mu p \to \mu^- X^{++}$ (Deuterium target)
Jongejans et al., CERN WA25, 1989
Zieminska et al., FNAL E545, 1983
^{2}H
H + ^{2}H
$\pi^+ p$ (non-diffractive)
VENUS 4.10
NuWro
GENIE–AGKY
GiBUU
Allasia et al., CERN WA25, 1984
($Q^2 > 1$ GeV^2)
Kitagaki et al., FNAL E545, 1980
(b)
$< n_{ch} >$
W^2 (GeV^2)


$\bar{\nu}_\mu p \to \mu^+ X^0$ (Hydrogen target)
Jones et al., CERN WA21, 1992
Jones et al., CERN WA21, 1990
Derrick et al., FNAL E31, 1978
$c_1 = 0$
$c_1 = 0.018 \pm 0.021$
H + ^{2}H ($c_1 = 0$)
$\pi^- p$ (non-diffractive)
VENUS 4.10
NuWro
Grässler et al., CERN WA21, 1983
Derrick et al., FNAL E31, 1982
(0.1 < y < 0.8)
Barish et al., FNAL E31, 1978
(0.1 < y < 0.8)
(a)
$< n_{ch} >$
W^2 (GeV^2)
$\bar{\nu}_\mu p \to \mu^+ X^0$ (Deuterium target)
Jongejans et al., CERN WA25, 1989
$c_1 = 0$
$c_1 = -0.007 \pm 0.023$
H + ^{2}H ($c_1 = 0$)
$\pi^- p$ (non-diffractive)
VENUS 4.10
NuWro
Allasia et al., CERN WA25, 1984
(Q^2 > 1 GeV^2)
Barlag et al., CERN WA25, 1982
Allasia et al., CERN WA25, 1980
(b)
W^2 (GeV^2)



$\langle n_{ch} \rangle$
$\nu_\mu n \to \mu^- X^+$ (Deuterium target)
Jongejans et al., CERN WA25, 1989
Zieminska et al., FNAL E545, 1983
$a_2 = 0$
$a_2 = -0.15 \pm 0.14$
NuWro
GENIE–AGKY
GiBUU
Renormalized to
a free neutron
Allasia et al., CERN WA25, 1984
($Q^2 > 1$ GeV^2)
(a)
$\bar{\nu}_\mu n \to \mu^+ X^-$ (Deuterium target)
Jongejans et al., CERN WA25, 1989
Barlag et al., CERN WA25, 1982
$a_1 = 1$
$a_1 = 0.86 \pm 0.11$
NuWro
Renormalized to
a free neutron
Allasia et al., CERN WA25, 1984
($Q^2 > 1$ GeV^2)
Allasia et al., CERN WA25, 1980
(b)
W^2 (GeV^2)

Conclusion

- Hadronization in difficult
- Many decisions have to be made based on limited data.
- New experiments soon enough will check assumption made in the Monte Carlo generators.