Role of Correlations and Two-Body Currents in Quasielastic (QE) e/ν Scattering

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May 24, 2014

Outline:

- Nuclear interactions
- Correlations and short-range structure
- Nuclear electroweak currents
- Inclusive e/ν scattering and two-body currents
- Role of interference between 1b and 2b currents
- Summary

In collaboration with:

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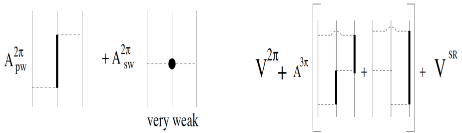
J. Carlson S.C. Pieper G. Shen R.B. Wiringa

Nuclear interactions

- $v = v_0(\text{static}) + v_p(\text{momentum dependent}) \rightarrow v(\text{OPE})$ fits large NN database with $\chi^2 \simeq 1$
- NN interactions alone fail to predict:
  - spectra of light nuclei
  - Intermediate-energy Nd and low-energy n-α scattering
  - nuclear matter $E_0(\rho)$
- Inclusion of 2π and 3π-NNN interactions leads to an excellent description of spectra of s- and p-shell nuclei (and n-α scattering)

Nuclear interactions



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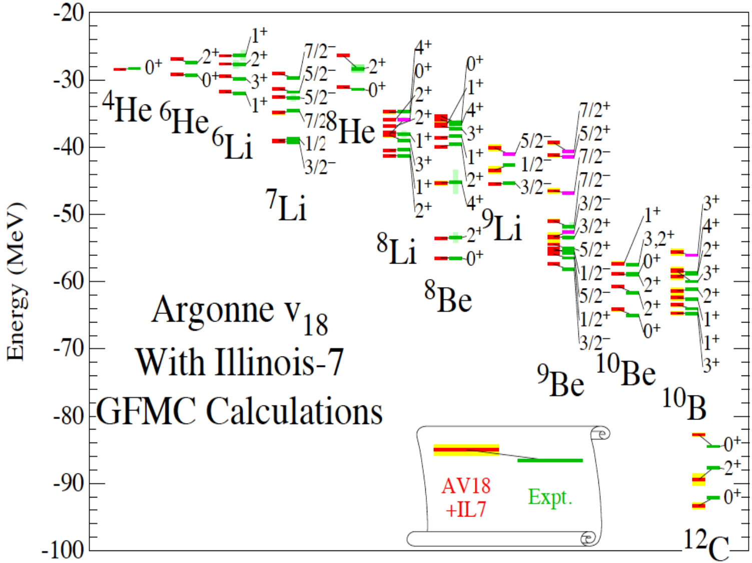
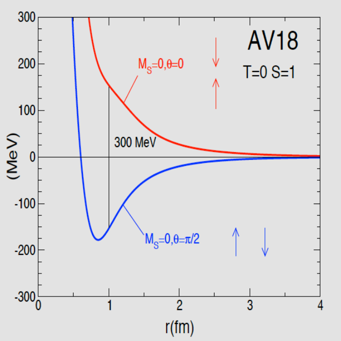
Spectra of light nuclei

Pieper and Wiringa, private communication

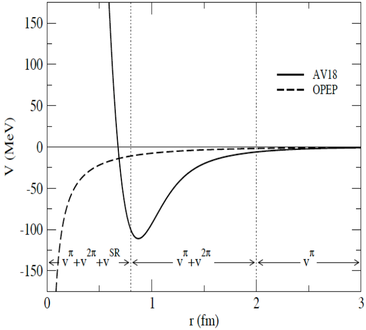


Dominant features of two-nucleon potential

v in T,S=0,1 (d-like)



v in T,S=1,0 (quasi-bound)

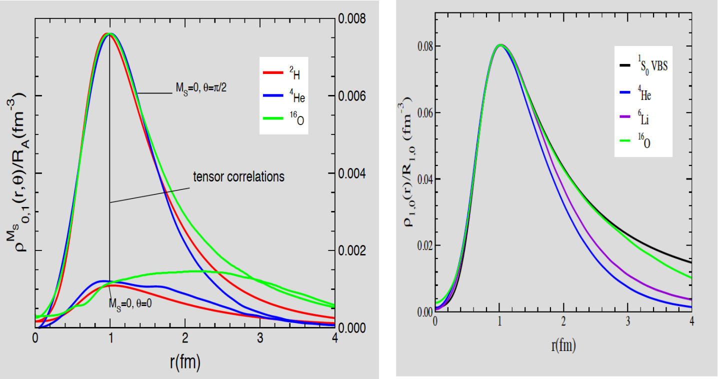


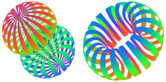
- Short-range repulsion (common to many systems)
- Intermediate to long-range tensor character (unique to nuclei)
- Strong spin and isospin-dependent correlations

Impact on pair spatial distributions

Forest *et al.* (1996)

T,S=0,1 (d-like) T,S=1,0 (quasi-bound)

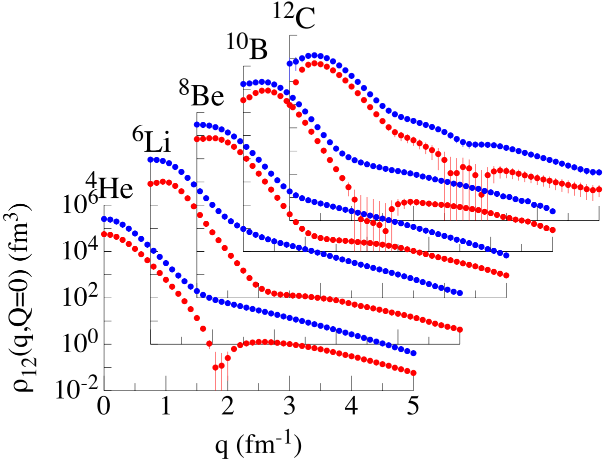




- $\rho^{M_S}_{T=0,S=1}(\mathbf{r})$ depends on $\mathbf{r}$ and M_S :
- Universality of short-range structure: scaling of $\rho^{M_S}_{T,S}(\mathbf{r})$

Impact on pair momentum distributions

Wiringa *et al.* (2014)



Impact on pair momentum distributions

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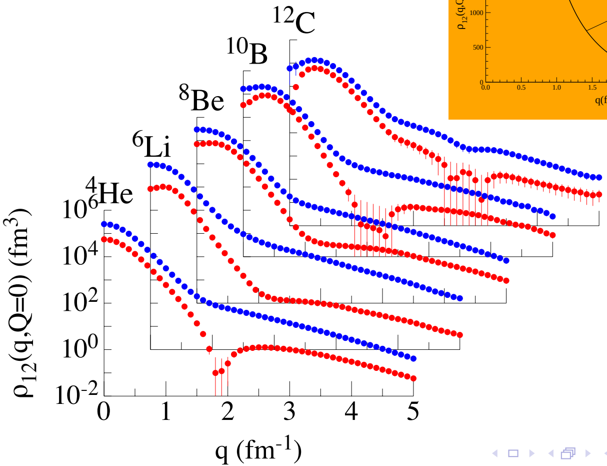
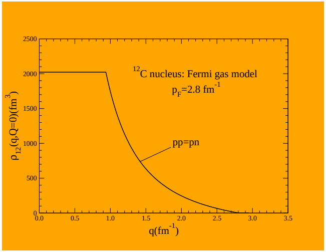
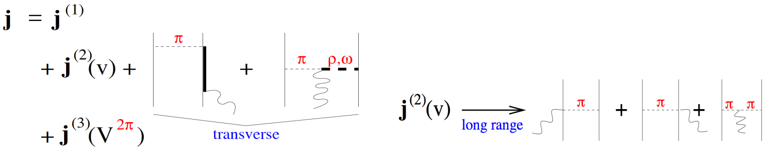




Electromagnetic current operators

- Set of (conserved) EM current operators



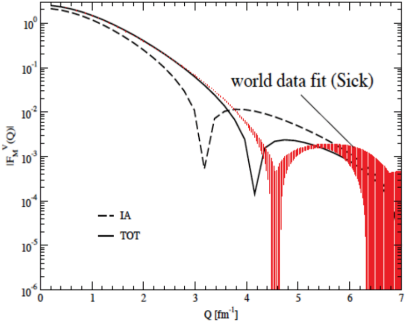
contain no free parameters and are consistent with short-range behavior of v and $V^{2\pi}$

- Many-body EM charge operators represent relativistic corrections to $\rho^{(1)}$, and lead to small corrections
- These many-body corrections are important to reproduce a variety of nuclear EM observables

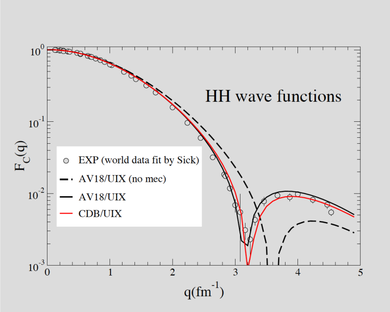
Elastic form factors of $A = 3$ and 4

Marcucci *et al.* (2005); Viviani *et al.* (2007)

^{3}He/^{3}H isovector magnetic FF



^{4}He charge FF



- Leading two-body currents are isovector
- Two-body charge contributions, while small, crucial for reproducing ^{2}H, ^{3}H, ^{3}He, and ^{4}He exp longitudinal f.f.'s

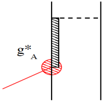
Weak current operators

- Charge-changing (CC) and neutral (NC) weak currents

$$j^{\mu}_{CC} = j^{\mu}_{\pm} + j^{\mu 5}_{\pm}$$

$$j^{\mu}_{NC} = -2\sin^2\theta_W \, j^{\mu}_{\gamma,S} + (1 - 2\sin^2\theta_W)\, j^{\mu}_{\gamma,z} + j^{\mu 5}_{z}$$

- Contributions to two-body axial currents from π and ρ exchange, $\rho\pi$ transition, and Δ-excitation (g^*_A)



- N-Δ axial coupling constant g^*_A fixed by fitting the GT m.e. in ^{3}H β-decay
- j^{μ}_{CC} reproduces well μ-capture rates in ^{2}H and ^{3}He
- Studies of weak transitions in light nuclei in progress . . .

Inclusive e/ν scattering

- Inclusive $\nu/\overline{\nu}$ $(-/+)$ cross section given in terms of five response functions

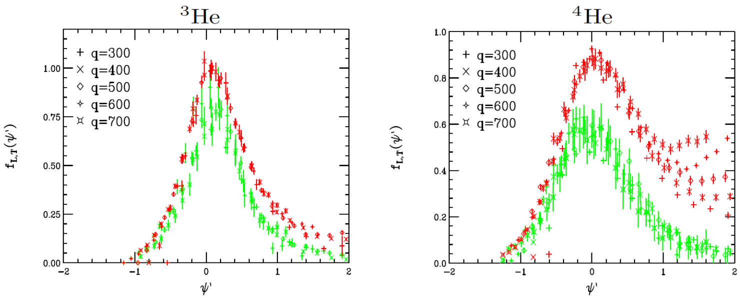
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\epsilon'\mathrm{d}\Omega} = \frac{G^2}{8\,\pi^2}\,\frac{k'}{\epsilon}\left[v_{00}\,R_{00}+v_{zz}\,R_{zz}-v_{0z}\,R_{0z}+v_{xx}\,R_{xx}\mp v_{xy}\,R_{xy}\right]$$

$$R_{\alpha\beta}(q,\omega)\sim\overline{\sum_i}\sum_f\delta(\omega+m_A-E_f)\langle f\,|\,j^\alpha(\mathbf{q},\omega)\,|i\rangle^*\langle f\,|\,j^\beta(\mathbf{q},\omega)\,|i\rangle$$

- In (e,e') scattering, interference $R_{xy}=0$, $j^z_\gamma\sim(\omega/q)j^0_\gamma$, and only $R_{00}=R_L$ and $R_{xx}=R_T$ are left

(e, e') inclusive response: scaling analysis

Donnelly and Sick (1999)



- Scaling variables: $\psi' \simeq y/k_F$ and $f_{L,T} = k_F\, R_{L,T}/G_{L,T}$
- Data at variance with PWIA expectation that $f_L \simeq f_T$
- Excess strength in transverse response
- Notion that the QE response is dominated by single-nucleon scattering is too simplistic

Sum rules for (e, e') inclusive scattering

Schiavilla *et al.* (1989); Carlson *et al.* (2002)

$$
\begin{aligned}
S_\alpha(q) &= C_\alpha \int_{\omega_{\rm th}^+}^\infty {\rm d}\omega \, \frac{R_\alpha(q,\omega)}{G_{Ep}^2(q,\omega)} \\
&= C_\alpha \left[\langle 0 | \, O_\alpha^\dagger(\mathbf{q}) \, O_\alpha(\mathbf{q}) \, | 0 \rangle - | \, \langle 0 | \, O_\alpha(\mathbf{q}) \, | 0 \rangle \, |^2 \right]
\end{aligned}
$$

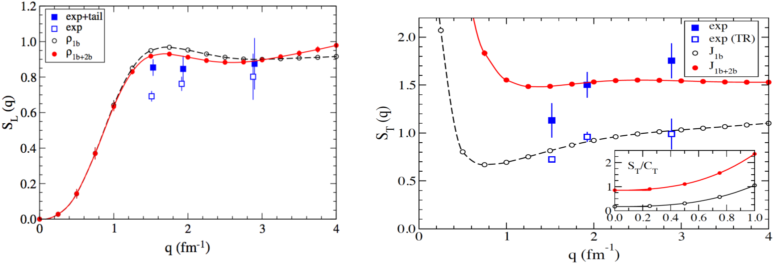
- C_α are normalization factors so as $S_\alpha(q \to \infty) = 1$ when only one-body terms are retained in O_α:

$$
C_L = \frac{1}{Z} \, , \qquad C_T = \frac{2\, m^2}{q^2} \frac{1}{Z \mu_p^2 + N \mu_n^2}
$$

- S_α can be calculated exactly with quantum Monte Carlo methods

Longitudinal and transverse sum rules in ^{12}C

Lovato *et al.* (2013)



- Direct comparison between theory and experiment problematic, especially for $R_T(q,\omega)$:
  - $R_\alpha(q,\omega)$ measured by (e,e') up to $\omega_{\max} \leq q$
  - present theory ignores explicit pion production mechanisms, crucial in the Δ-peak region of R_T
- Contribution for $\omega > \omega_{\max}$ estimated by assuming

$$R_\alpha^{\rm EXP}(q,\omega > \omega_{\max}; A) \propto R_\alpha(q,\omega;\text{deuteron})$$

Longitudinal and transverse sum rules in ^{12}C

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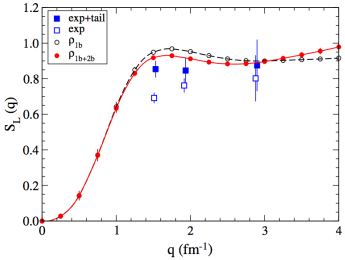

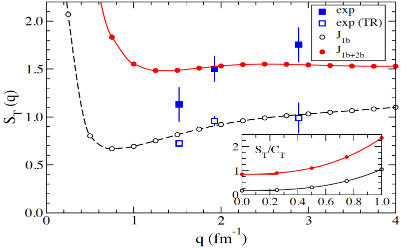

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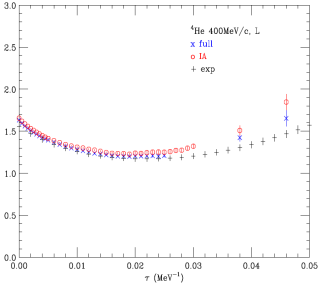
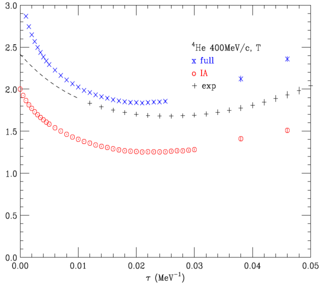
EM euclidean response functions in ^{4}He

Carlson and Schiavilla (1992, 1994); Carlson *et al.* (2002)

Longitudinal



Transverse



$$E_\alpha(q,\tau) = e^{\tau\, q^2/(2\, m)} \int_{\omega_{th}^+}^{\infty} d\omega\, e^{-\tau(\omega - E_0)} \frac{R_\alpha(q,\omega)}{G_{Ep}^2(q,\omega)}$$

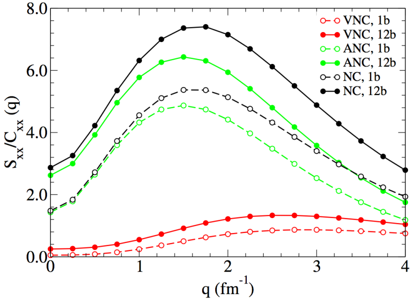
$$= e^{\tau\, q^2/(2\, m)} \langle 0 |\, O_\alpha^\dagger(\mathbf{q}) e^{-\tau(H-E_0)} O_\alpha(\mathbf{q})\, |0\rangle - (\text{elastic})$$

- At $\tau = 0$, $E_\alpha(q;0) \propto S_\alpha(q)$; as τ increases, $E_\alpha(q;\tau)$ probes strength in QE region

Weak NC $S_{xx}(q)$ (transverse) sum rule in ^{12}C

Lovato *et al.* (2014)

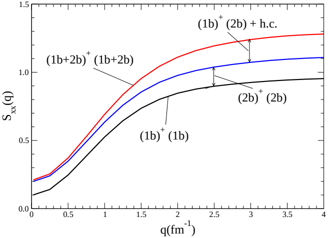
$$S_{xx}(q) = C_{xx} \int_{\omega_{\rm el}}^{\infty} {\rm d}\omega \, R_{xx}(q,\omega) = C_{xx} \langle 0 | \mathbf{j}_{NC}^{\perp\,\dagger}(\mathbf{q}) \, \mathbf{j}_{NC}^{\perp}(\mathbf{q}) | 0 \rangle$$



- Large increase ($\sim 30\%$) in all weak NC responses R_α, but for R_{00}, due to two-body (2b) terms in j^{μ}_{NC}
- Important interference effects between 1b and 2b (as well as among 2b) terms

1b-2b interference effects: sum rules

$$S - S^{1\mathrm{b}} = \langle A | \left(j_{1\mathrm{b}}^{\dagger}\, j_{2\mathrm{b}} + \mathrm{h.c.} \right) + j_{2\mathrm{b}}^{\dagger} j_{2\mathrm{b}} \, | A \rangle$$



$$\Delta S \simeq \langle A | \sum_{l<m} \left[(j_l^{\dagger} + j_m^{\dagger}) j_{lm} + \mathrm{h.c.} \right] + \sum_{l<m} j_{lm}^{\dagger} j_{lm} + \cdots | A \rangle$$

$$= \int \mathrm{d}\mathbf{x}\, j(\mathbf{x})\, \rho^{A}(\mathbf{x}; pn)$$

- Enhancement ΔS driven by pn pairs
- Scaling law for ΔS follows from scaling of $\rho^{A}(\mathbf{x}; pn)$

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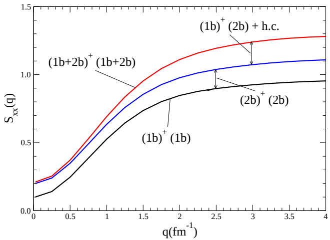

$$\Delta S \simeq \langle A | \sum_{l<m} \left[(j^{\dagger}_{l} + j^{\dagger}_{m}) j_{lm} + \mathrm{h.c.} \right] + \sum_{l<m} j^{\dagger}_{lm} j_{lm} + \cdots \, | A \rangle$$

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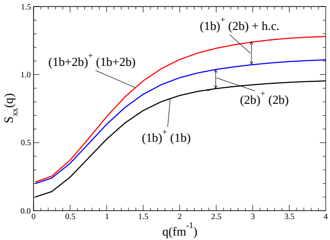

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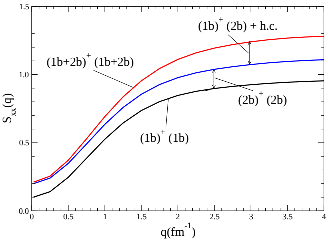

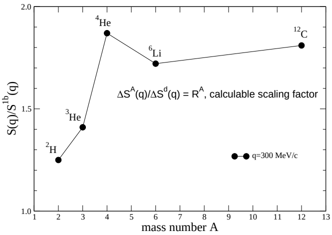

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Baseline: the Fermi gas response

Van Orden and Donnelly (1981)

- Excited states of the Fermi gas (up to $2ph$ states):

$$|\mathbf{ph}\rangle = a^\dagger_{\mathbf{p}} a_{\mathbf{h}} \,|0\rangle \ \text{with} \ p > k_F;\, h < k_F$$
$$|\mathbf{p}_1\mathbf{p}_2\mathbf{h}_1\mathbf{h}_2\rangle = a^\dagger_{\mathbf{p}_1} a^\dagger_{\mathbf{p}_2} a_{\mathbf{h}_2} a_{\mathbf{h}_1} \,|0\rangle \ \text{with} \ p_1, p_2 > k_F;\, h_1, h_2 < k_F$$

- One-body operator $j_{1b} = \sum_{\mathbf{k}\mathbf{k}'} j^{\mathbf{k}'}_{\mathbf{k}}\, a^\dagger_{\mathbf{k}'} a_{\mathbf{k}}$ and

$$\langle \mathbf{ph} \mid j_{1b} \,|0\rangle = j^{\mathbf{p}}_{\mathbf{h}} \,; \qquad \langle \mathbf{p}_1\mathbf{p}_2\mathbf{h}_1\mathbf{h}_2 \mid j_{1b} \,|0\rangle = 0$$

- Two-body operator $j_{2b} = 1/2 \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}'_1\mathbf{k}'_2} j^{\mathbf{k}'_1,\mathbf{k}'_2}_{\mathbf{k}_1,\mathbf{k}_2}\, a^\dagger_{\mathbf{k}'_1} a^\dagger_{\mathbf{k}'_2} a_{\mathbf{k}_2} a_{\mathbf{k}_1}$ and

$$\langle \mathbf{ph} \mid j_{2b} \,|0\rangle = \sum_{\mathbf{k}} \left(j^{\mathbf{p},\mathbf{k}}_{\mathbf{h},\mathbf{k}} - j^{\mathbf{p},\mathbf{k}}_{\mathbf{k},\mathbf{h}} \right) \theta(k_F - k) \,; \qquad \langle \mathbf{p}_1\mathbf{p}_2\mathbf{h}_1\mathbf{h}_2 \mid j_{2b} \,|0\rangle = j^{\mathbf{p}_1,\mathbf{p}_2}_{\mathbf{h}_1,\mathbf{h}_2} - j^{\mathbf{p}_1,\mathbf{p}_2}_{\mathbf{h}_2,\mathbf{h}_1}$$

- Fermi gas response:

$$R(\omega) = \sum_{\mathbf{ph}} |\langle \mathbf{ph} \mid j_{1b} + j_{2b} \,|0\rangle|^2 \,\delta(\omega + E_{1ph}) + \sum_{\mathbf{p}_1\mathbf{p}_2\mathbf{h}_1\mathbf{h}_2} |\langle \mathbf{p}_1\mathbf{p}_2\mathbf{h}_1\mathbf{h}_2 \mid j_{2b} \,|0\rangle|^2 \delta(\omega + E_{2ph})$$

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- $1ph$ contribution involves interference between 1b and 2b currents

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Van Orden and Donnelly (1981)

- Excited states of the Fermi gas (up to $2ph$ states):

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- One-body operator $j_{1\mathrm{b}} = \sum_{\mathbf{k}\mathbf{k}'} j^{\mathbf{k}'}_{\mathbf{k}} \, a^\dagger_{\mathbf{k}'} a_{\mathbf{k}}$ and

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Summary

- Enhancement of 1b response due to 2b currents driven by strongly correlated pn pairs
- Presence of these correlated pairs leads to important interference effects between 1b and 2b currents:
  - 1b currents can knock-out two nucleons from a correlated ground state (with amplitude A_{1b})
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