Ordinary

Solving O.D.E. (5)

Note indefinite integral only defined up to a constant

so if $\frac{dy}{dx} = x^2$

$$\int dy = \int x^2 dx$$

$$= \frac{x^3}{3} + \underline{\underline{c}}$$

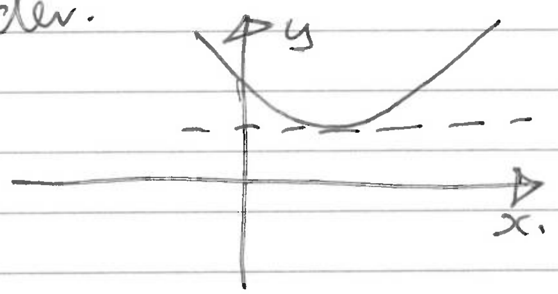
Second Order O.D.E

Differential of a differential.

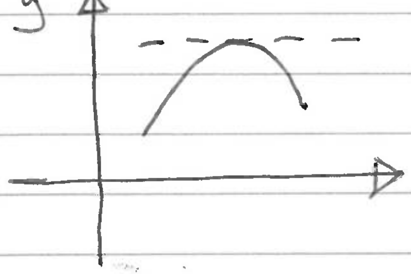
$$\frac{d}{dx}\left(\frac{df(x)}{dx}\right) = \frac{d^2 f(x)}{dx^2} = f''(x)$$

Maxima and minima

Consider.



or



o

6.

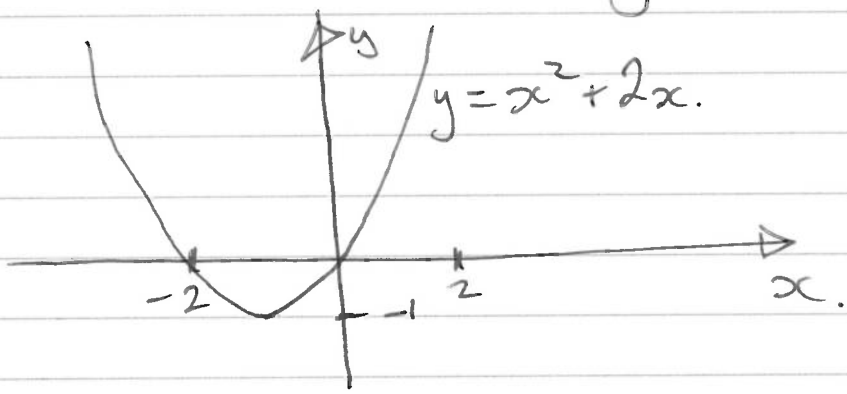
In both case $\frac{dy}{dx} = 0$

if $\frac{d^2y}{dx^2}$ is +ve it is a min

is −ve it is a max

is 0 can be max or min, or point of inflection.

For example $y = x^2 + 2x$



$\frac{dy}{dx} = 2x + 2 = 0$ @ min.

$\therefore \quad x = -1$.

$\left.\frac{d^2y}{dx^2}\right|_{x=-1} = 2$ ie +ve ie min

Phew

Now Consider a physical situation

Newton's Second Law

$$F = ma$$

Depends on the physics

Constant of the object

lets consider.

a = rate of change of vel^y.

$$= \frac{dv}{dt}$$

but v is rate of change of position.

$$v = dx/dt$$

$$\therefore \quad a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

$$\therefore \quad F = m\frac{d^2x}{dt^2} = m\ddot{x}$$

Aside

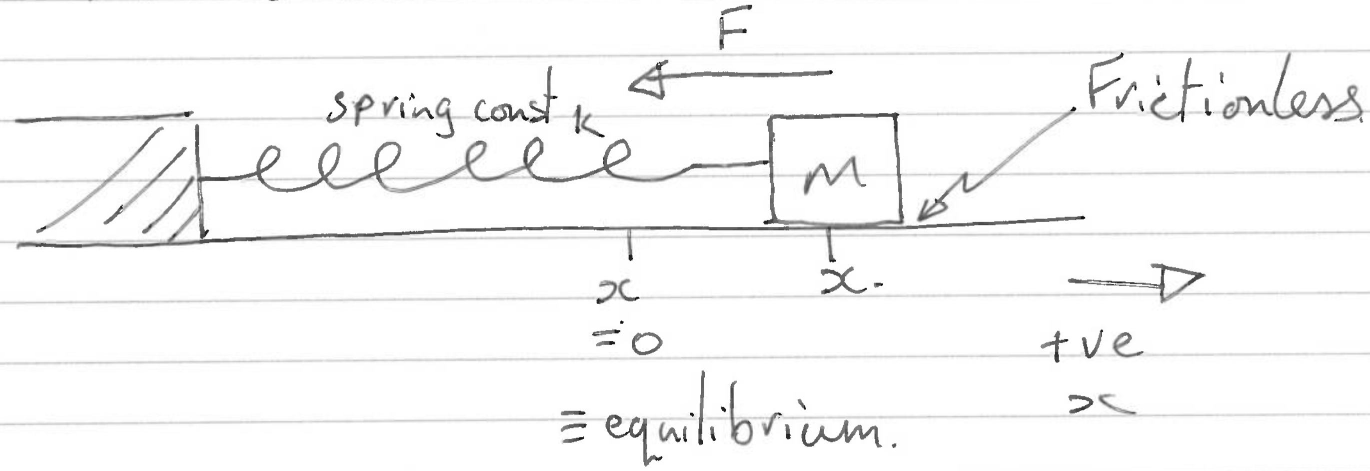
Can alway do this with second order differential equations

P.T.O

$$\frac{d^2x}{dt^2} = F(x)$$

$$\left.\begin{aligned} \frac{dV(x)}{dt} &= F(x) \\ \frac{dx}{dt} &= V(x) \end{aligned}\right\}$$

Now consider



$F = -kx.$

2nd Law $\qquad m\dfrac{d^2x}{dt^2} = F = -kx.$

So lets try $\quad x = \cos\omega t$

$$\frac{dx}{dt} = -\omega\sin\omega t$$

$$\frac{d^2x}{dt^2} = -\omega^2\cos\omega t$$

So

$$-m\omega^2\cos\omega t = -kx\cos\omega t$$

$$\Rightarrow \omega^2 = k/m$$

$$\text{or } \omega = \sqrt{k/m}.$$