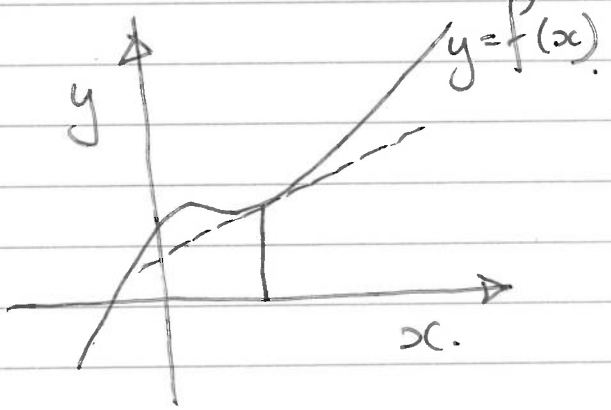
Lecture to GSS.

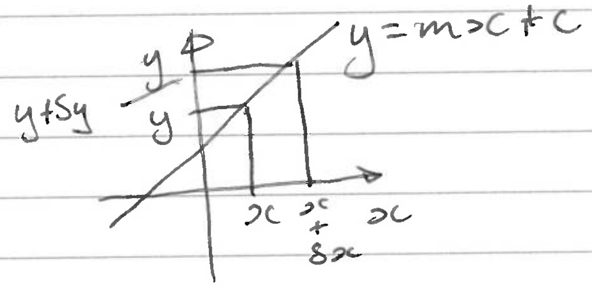
First need to start with some of the basic mathematical tools.

Differentiation



differentiation is just finding the gradient of a line at any point.

Consider simple case of a straight line.



$$\frac{\delta y}{\delta x} = \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$= \frac{m(x+\delta x) + c - (mx+c)}{\delta x}$$

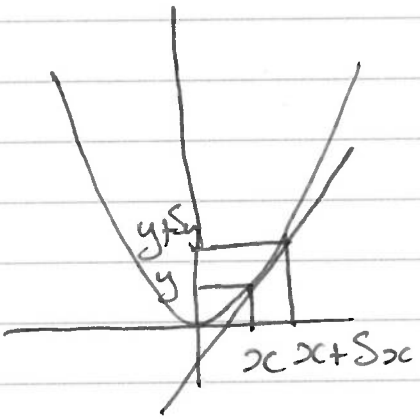
$$= \frac{\cancel{mx} + m\cancel{\delta x} + \cancel{c} - \cancel{mx} - \cancel{c}}{\cancel{\delta x}}$$

$$= m$$

No Surprise

Note constant has no effect.

Lets try $y = x^2$.



$$\frac{\delta y}{\delta x} = \frac{(x+\delta x)^2 - x^2}{\delta x}$$

$$= \frac{\cancel{x^2} + 2x\delta x + \delta x^2 - \cancel{x^2}}{\delta x}$$

$$= \frac{2x\cancel{\delta x} + \delta x^2}{\cancel{\delta x}}$$

$$= 2x + \delta x.$$

$$\text{in } \lim_{\delta x \to 0} \frac{\delta y}{\delta x} \to \frac{dy}{dx} = 2x \; (+\cancel{\delta x}^{\,0})$$

$$\therefore \frac{dy}{dx} = 2x \quad \text{for } y = x^2.$$

Can show that for $y = x^n$ (where n can be +ve or -ve or fractional)

$$\frac{dy}{dx} = n x^{n-1}$$

so $\dfrac{d x^3}{dx} = 3x^2$ etc.

$$\frac{d\sqrt{x}}{dx} = \frac{d x^{1/2}}{dx} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}!$$

$$\frac{d\,1/x}{dx} = \frac{d\,x^{-1}}{dx} = -1\,x^{-2} = \frac{-1}{x^2}.$$

Other functions

$$\frac{d \sin x}{dx} = \cos x.$$

$$\frac{d \cos x}{dx} = -\sin x.$$

Also $\frac{d \sin f(x)}{dx} = f'(x) \cos f(x)$ where $f'(x) = \frac{df(x)}{dx}$.

so $\frac{d \sin \omega x}{dx} = \omega \cos \omega x.$

Special example.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$\frac{d\, e^x}{dx} = e^x.$; $\frac{d\, e^{f(x)}}{dx} = f'(x) e^x.$

Note $\ln(e^x) = x$

Also $\frac{d \ln(x)}{dx} = \frac{1}{x}$.

Integration

Is the opposite to differentiation

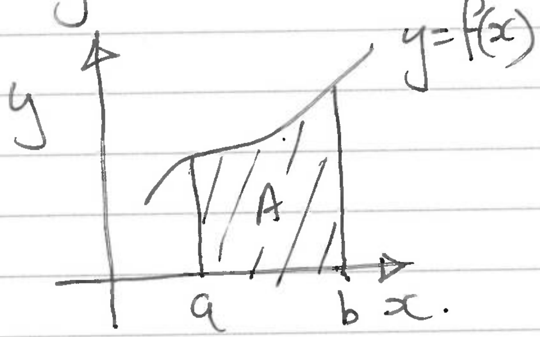
$$\frac{dy}{dx} = f(x)$$

$$\int dy = \int f(x)\,dx.$$

Old fashioned $\int$

$$y = \int f(x)\,dx. \quad \leftarrow \text{indefinite integral}$$

Actually a measure of the area.



$$A = \int_a^b f(x)\,dx.$$

Definite integral