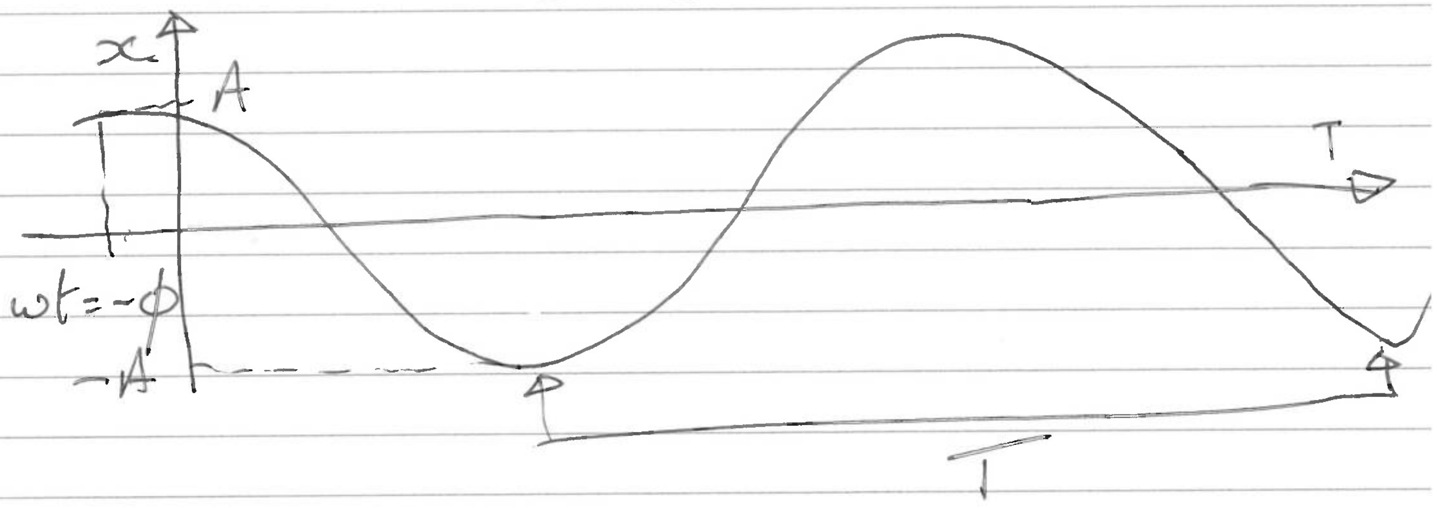
So what sort of motion is this?
Well, infact could have been
more general and had.

$$x = A \cos(\omega t + \phi)$$

amplitud (under A) phase (under ϕ)

as $$\frac{dx}{dt} = -A\omega \cos(\omega t + \phi)$$

So everything still works



$$T = \frac{2\pi}{\omega}$$
$$= 2\pi\sqrt{M/K}.$$

& $\nu = \omega/2\pi$

Note:- Doesn't depend on amplitude.

superposition
- Could add 2 solutions together.
eg. $x = A_1 \cos(\omega t + \phi_1) + A_2 \cos(\omega t + \phi_2)$

Now lets consider energy

if $x = A \cos(\omega t + \phi) = A \cos(\omega t)$

set to 0 for now

$$\frac{dx}{dt} = -A\omega \sin \omega t$$

$$K.E = \frac{1}{2} m v^2 = \frac{A^2 \omega^2 m \sin^2 \omega t}{2}$$

So K.E. change throughout motion.

Now lets consider P.E. ($U(x)$)

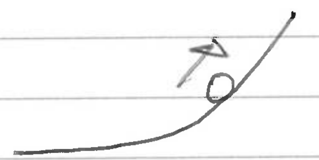
When is $U = 0$? Don't forget always offset

so really ask when is U minimum?

Clearly when @ equilibrium.

So calculate U

$F = -dU/dx$ ← discuss this in terms of a valley.



$$U = -\int_0^x F\,dx = +\int_0^x kx\,dx$$

$$\underline{\underline{U = \frac{1}{2}kx^2}} = \frac{1}{2}kA^2\cos^2(\omega t)$$

↓

$\boxed{\text{Parabolic potential}}$

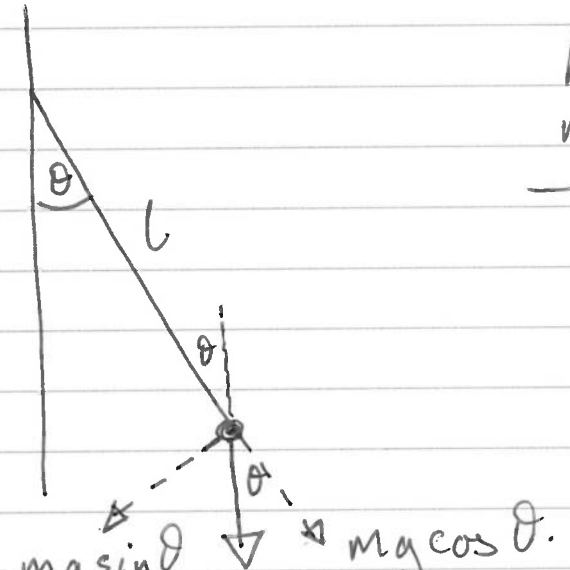
So $E_{Tot} = K.E + P.E$

$$= \frac{1}{2}A^2\omega^2 m \sin^2(\omega t) + \frac{1}{2}A^2 \overset{\omega^2 m}{k}\cos^2\omega t$$

$$= \frac{1}{2}A^2\omega^2 m \cancel{(\sin^2(\omega t) + \cos^2(\omega t))}$$

$$= \frac{1}{2}A^2\omega^2 m.$$

Now lets consider a pendulum



$F_\theta = -mg\sin\theta$

Note

Ask about resolving forces

From rotational movement equivalent of N. 2.

$\tau = I\ddot{\theta}$

torque

moment of Inertia

$I = ml^2$

$\tau = lF_\theta$

$$lF_\theta = ml^2\ddot{\theta}$$

$$-gm\sin\theta = ml\ddot{\theta}$$

$$-gm\theta = ml\ddot{\theta}$$

in small angle, $\sin\theta \approx \theta$, $\theta \ll 1$ rad.

$\therefore$ S. H. M. with $\omega = \sqrt{g/l}$

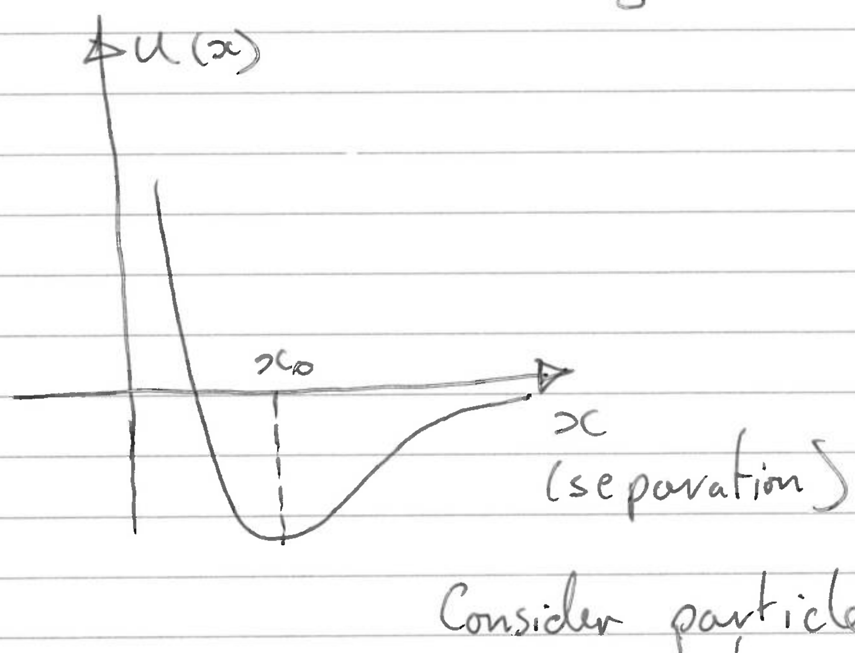
or $T = 2\pi\sqrt{l/g}$.

- Note independent of mass (and amplitude as long as $\sin\theta \approx \theta$)

So why is S.H.M Everywhere?

Consider any potential that has an equilibrium.

E.g. Attraction between neutral molecules in gas



Consider particle held by potential around x_0