Energy Scaling of a Neutrino Factory Decay Ring

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Energy Scaling

- Start from high energy decay ring
  - Around 25 GeV
- Scale down to low energy decay ring
  - Around 4 GeV

Basic Equations

- Angular divergence requirement in straights

$$\sigma_{\theta S} = \sqrt{\frac{\epsilon_N}{\beta_S \gamma}} = \frac{k}{\gamma} \qquad \beta_S = \frac{\epsilon_N \gamma}{k^2}$$

- Arc $\beta_A = \kappa \beta_S$, κ energy-independent
- Arc phase advance μ_A fixed (disp. suppression)
- Arc length L_A proportional to energy:

$$L_A = \mu_A \beta_A = \frac{\kappa \epsilon_N \mu_A}{k^2} \gamma$$

Quadrupole Equations

- Focal length

$$\frac{1}{F_A} = \frac{eB'L_Q}{\gamma mc}$$

- $F_A = \lambda\beta_A$ for fixed λ, thus $B'L_Q$ indep. of γ:

$$B'L_Q = \frac{k^2 mc}{e\lambda\kappa\epsilon_N}$$

Quadrupole Equations

- Peak quad field $B'\sigma_R$

$$B'\sqrt{\frac{\beta_A \epsilon_N}{\gamma}} = B'\sqrt{\kappa}\frac{\epsilon_N}{k}$$

- Peak quad field independent of γ
- Thus L_Q independent of γ, since $B'L_Q$ constant
- L_A proportional to γ
  - Dipole fields increasing when γ reduced
  - Space problem

Corrected Scaling Law

- Keep $\beta_S \propto \gamma$
- Assume $\beta_A \propto \gamma^{-m}\beta_S$
  - $m \geqslant 0$ so design isn't harder
- Make L_Q and L_A have same γ scaling
- Resulting design scaling
  - $L_A \propto \gamma^{1/3}$, same for total circumference
  - Straight quad count $\propto \gamma^{-2/3}$
  - Constant arc quad count, bends lower field

Compromise Scaling

- Arc quads to occupy greater fraction of arc
- Bends get shorter, fields higher
  - Fields still less than higher energy lattice
  - Avoid going to bad aspect ratio

Study II Scaling

- Extremely aggressive lattice
  - High dipole fields
  - Very compact
- 53 m arc for 20 GeV
- Keep dipole fields same
- Keep max quad fields same
- New 4 GeV arc 13.56 m long: 26% of size
- $\gamma^{1/3}$ scaling: 58% of size (31 m)

Bunch Trains

- Circumference may be determined by bunch train structure
- Currently, three trains must fit
  - Minimum circumference $\approx$360 m
- Determined by bunch structure from proton driver
  - Can’t spread out more: can’t replace target fast enough
  - Could try for fewer proton bunches per cycle