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Consequences for theory

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Glasgow, 18th to the 20th of April

THE INTERNATIONAL DESIGN STUDY FOR THE NEUTRINO FACTORY

Neutrino Factory International Design Study
8th Plenary Meeting

History of Neutrino Mixing (98-)

- ☑ Atmospheric ν_μ disappear, large θ_{23} (SK) (98)
- ☑ Solar ν_e disappear, large θ_{12} (H/S,Ga,SK) (02)
- ☑ Solar ν_e are converted to $\nu_\mu+\nu_\tau$ (SNO) (02)
- ☑ Reactor anti-ν_e disappear/reappear (KamLAND) (04)
- ☑ Accelerator ν_μ disappear (K2K 04 , MINOS 06)
- ☑ Accelerator ν_μ converted to ν_τ (OPERA 10)
- ☑ Accelerator ν_μ converted to ν_e , θ_{13} hint(T2K, MINOS,DC) (11)
- ☑ Reactor anti-ν_e disappear, θ_{13} meas.(Daya Bay, RENO) (12)

Three Neutrino Mixing

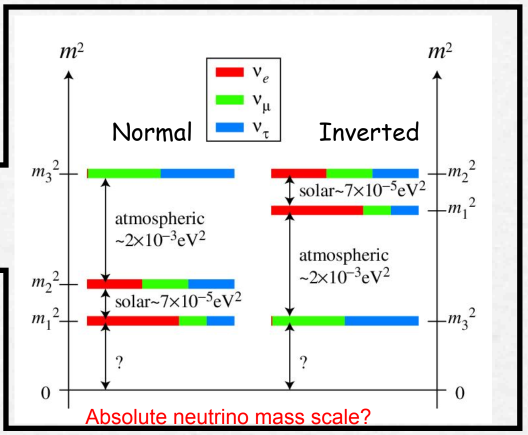
Standard Model states

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L$$

Neutrino mass states

Pontecorvo
Maki
Nakagawa
Sakata

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$



$$U_{PMNS} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}e^{i\alpha_1/2} & 0 & 0\\ 0 & e^{i\alpha_2/2} & 0\\ 0 & 0 & 1\end{pmatrix}$$

$s_{ij} = \sin\theta_{ij}$
$c_{ij} = \cos\theta_{ij}$

Atmospheric | Reactor | Solar | Majorana

Oscillation phase δ

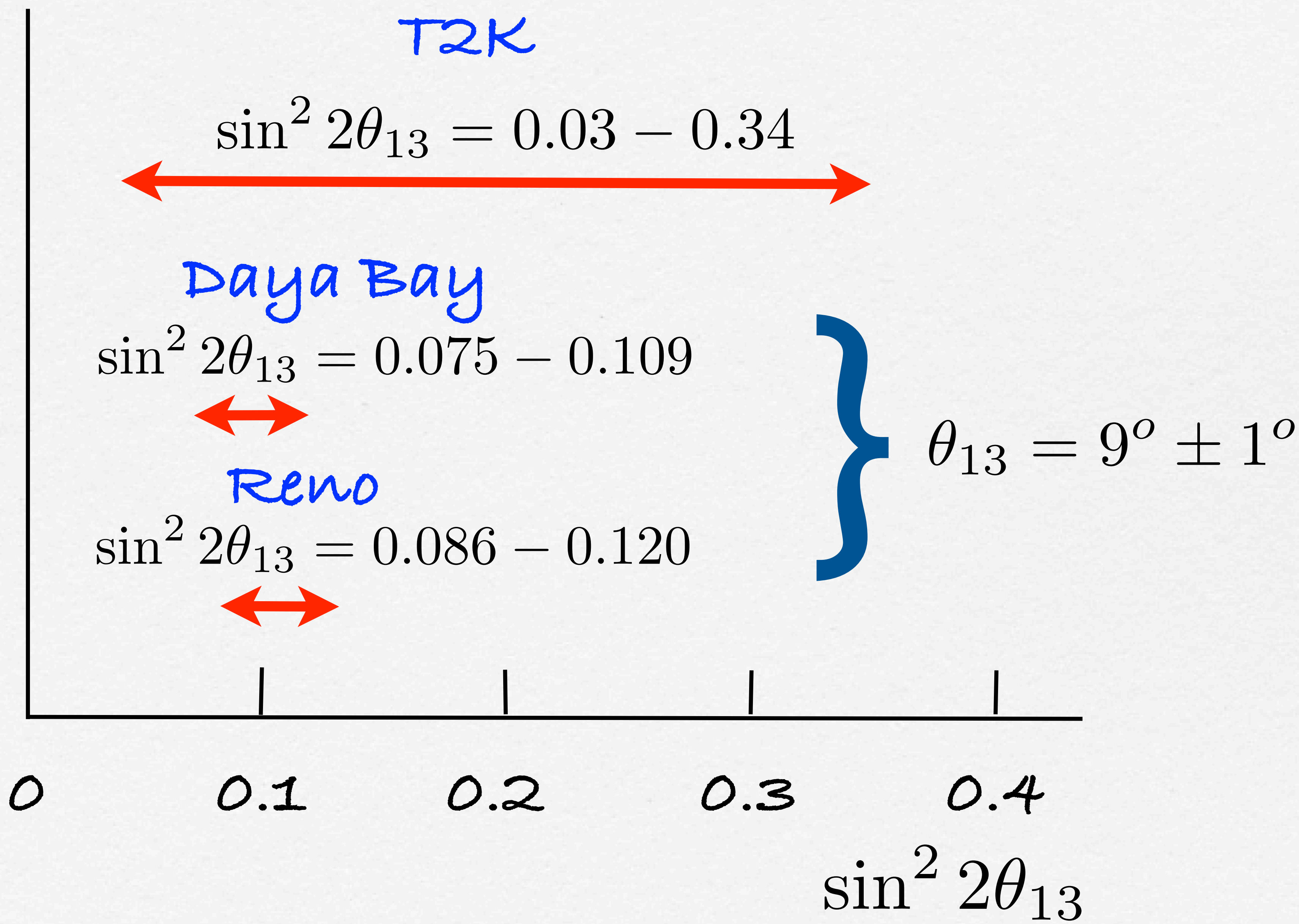
Majorana phases α_1, α_2

3 masses + 3 angles + 1 (or 3) phase(s)
= 7 (or 9) new parameters for SM

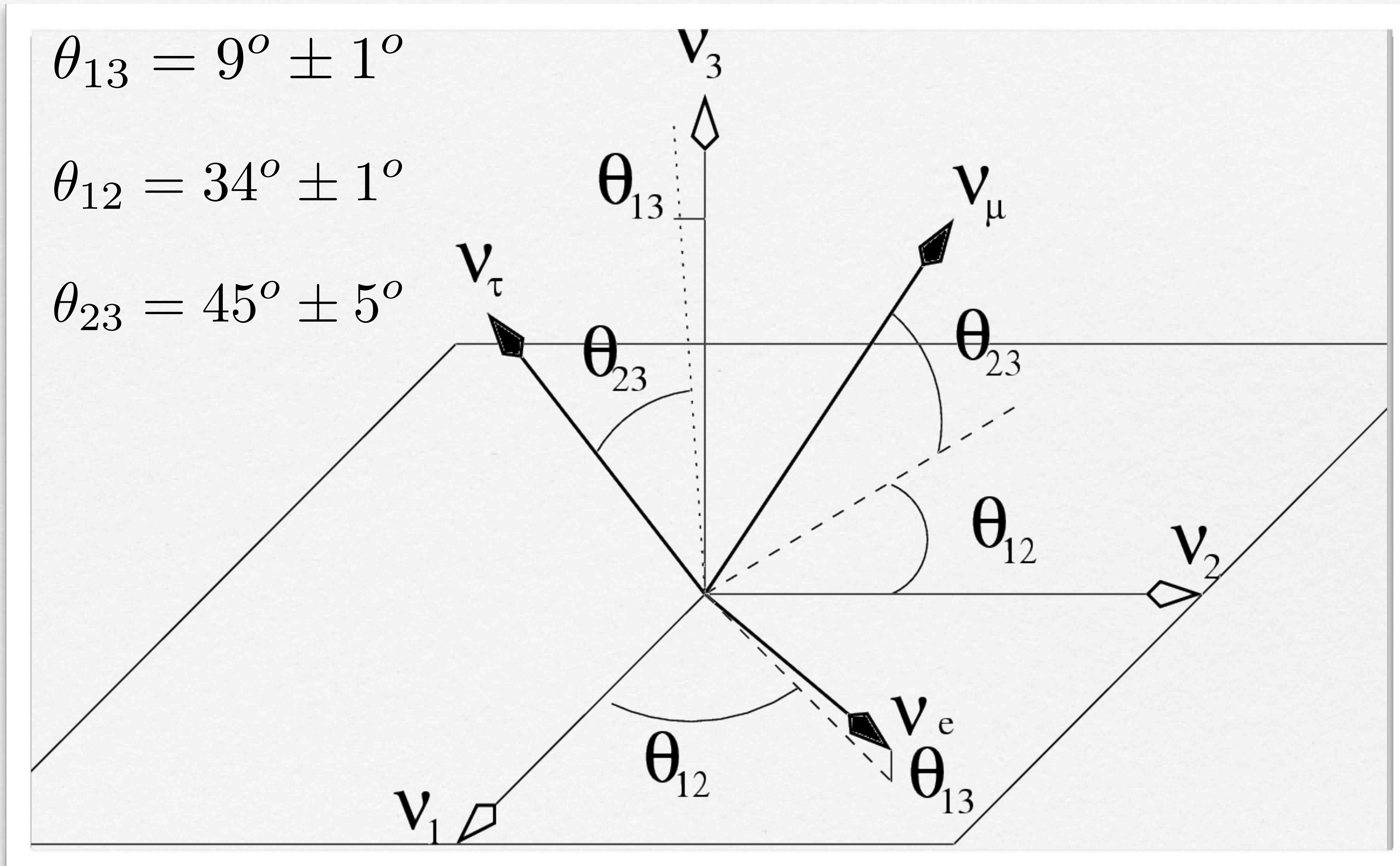
Global Fits 2011

parameter	best fit $\pm 1\sigma$ (Schwetz, Tortola, Valle '11)	best fit $\pm 1\sigma$ (Fogli, Lisi, Marrone, Palazzo, Rotunno '11)
Δm^2_{21} $[10^{-5}\text{eV}^2]$	$7.59^{+0.20}_{-0.18}$	$7.58^{+0.22}_{-0.26}$
Δm^2_{31} $[10^{-3}\text{eV}^2]$	$2.50^{+0.09}_{-0.16}$ $-(2.40^{+0.08}_{-0.09})$	$2.35^{+0.12}_{-0.09}$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	$0.312^{+0.17}_{-0.16}$
$\sin^2\theta_{23}$	$0.52^{+0.06}_{-0.07}$ 0.52 ± 0.06	$0.42^{+0.08}_{-0.03}$

Theta13 in 2011/12



Neutrino Mixing



Simple Mixing Patterns RULED OUT
(since all have max. atm. and zero reactor angle)

- Bimaximal

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 45^o$$

- Tri-bimaximal

Harrison, Perkins, Scott

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

$$\theta_{12} = 35.26^o$$

- Golden ratio

Kajirama, Raidal, Strumia; Everett, Stuart

$$U_{BM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

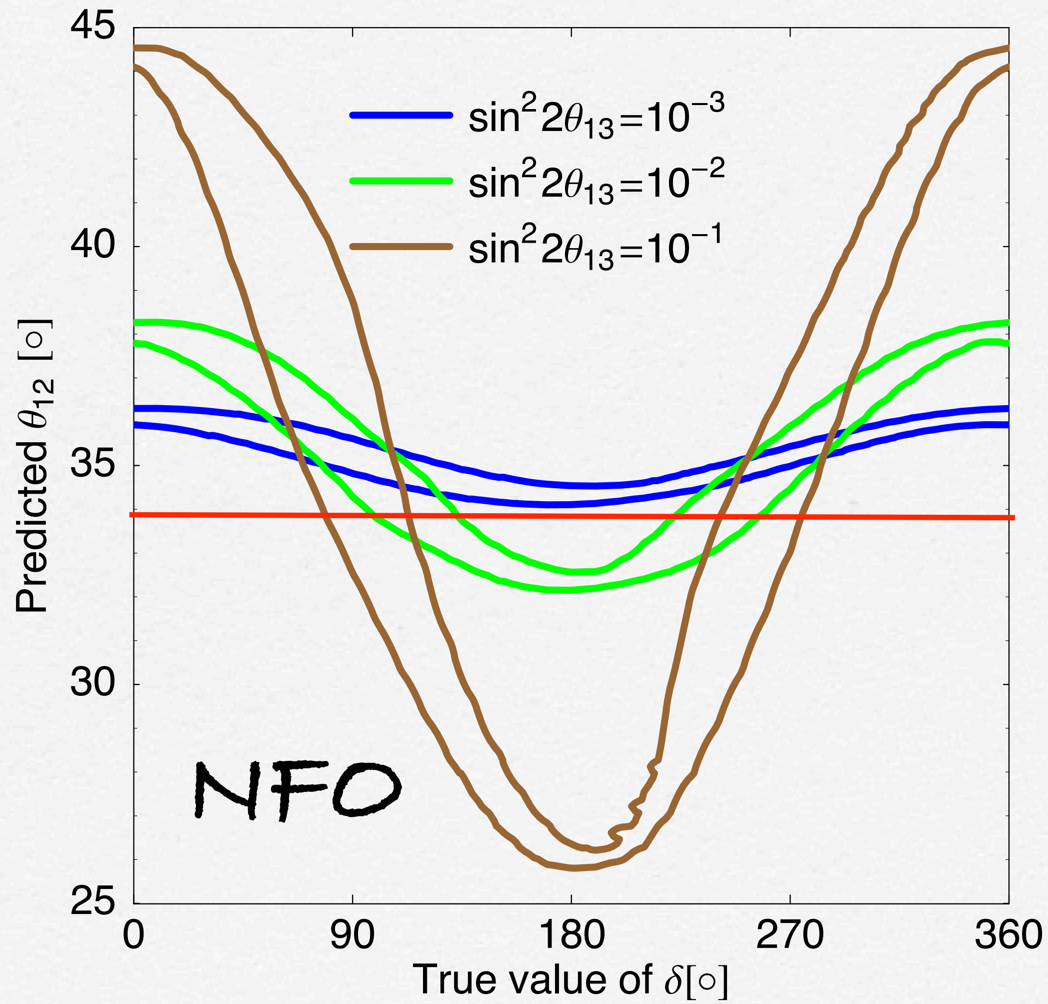
$$\phi = \frac{1+\sqrt{5}}{2} \qquad \tan\theta_{12} = \frac{1}{\phi} \qquad \theta_{12} = 31.7^o$$

Large Charged Lepton Mixing Corrections U_e to the Rescue

- BM, TBM, GR might only apply to neutrino mixing and $U_{PMNS} = U_e U_\nu^\dagger$ implies $\theta_{13} \approx \theta^e_{12}/\sqrt{2}$
- Leading to solar sum rules: (Antusch, King)
- Bimaximal $\theta_{12} = 45^o + \theta_{13} \cos\delta$ c.f. QLC $\theta_{12} + \theta_C = 45^o$
- Tri-bimaximal $\theta_{12} = 35^o + \theta_{13} \cos\delta$
- Golden ratio $\theta_{12} = 32^o + \theta_{13} \cos\delta$
- c.f. Experiment $\theta_{12} = 34^o \pm 1^o$ $\theta_{13} = 9^o \pm 1^o$
- e.g. Bimaximal sum rule implies $\cos\delta \approx -1$

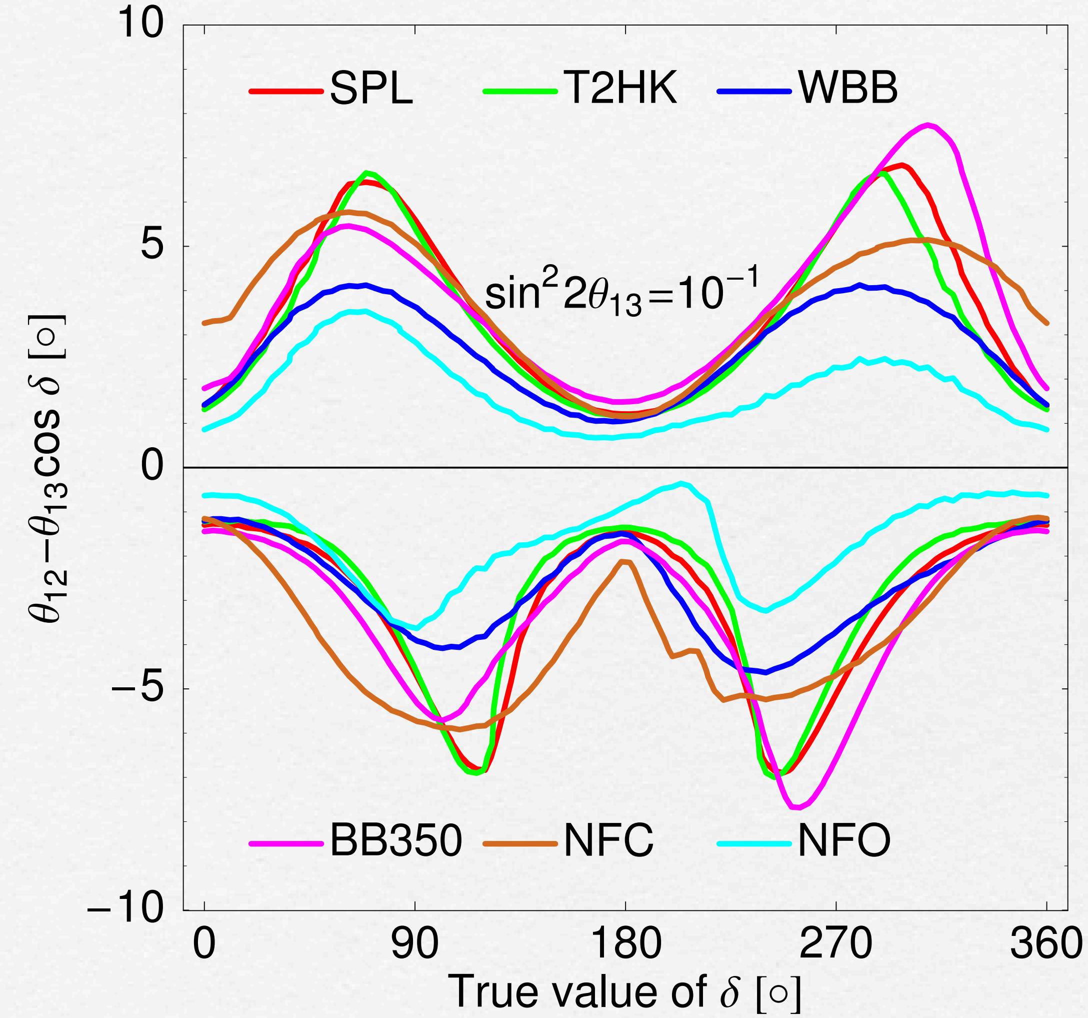
Tri-bimaximal Sum Rule prediction

$\delta \approx 90^o \;\; or \;\; 270^o$



- Antusch, Huber, King, Schwetz

Achievable Precision on $\theta_{12} - \theta_{13}\cos\delta$



Setup	Ref.	Baseline	Detector	Beam
SPL	[29]	130 km	440 kt WC	4 MW superbeam, 2 y (ν) + 8 y ($\bar{\nu}$)
T2HK	[29]	295 km	440 kt WC	4 MW superbeam, 2 y (ν) + 8 y ($\bar{\nu}$)
WBB	[30]	1300 km	300 kt WC	1.5 MW superbeam, 5 y (ν) + 5 y ($\bar{\nu}$)
BB350	[32]	730 km	440 kt WC	$5 \times 1.1 \cdot 10^{18}$ ^{18}Ne + $5 \times 2.9 \cdot 10^{18}$ ^{6}He
NFC	[33]	4000 km	50 kt MID	50 GeV, 4×10^{21} μ^- + 4×10^{21} μ^+
NFO	[33]	4000+7500 km	2×50 kt MID*	20 GeV, 4×10^{21} μ^- + 4×10^{21} μ^+

King; Pakvasa, Rodejohann, Weiler

Tri-Bimaximal Parametrisation

$$U_{\rm PMNS} \approx \begin{pmatrix} \frac{2}{\sqrt{6}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix} P$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s)\ , \qquad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a)\ , \qquad \sin\theta_{13} = \frac{r}{\sqrt{2}}$$

$$s = -0.03 \pm 0.03 \qquad a = -0.02 \pm 0.10 \qquad r = 0.22 \pm 0.02$$

s = solar a = atmospheric r = reactor

TB mixing corresponds to s=r=a=0

Tri-maximal Hydras

- Tri-bimaximal (s=a=r=0)

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P,$$

- Tri-bimaximal-reactor (s=a=0)

$$U_{TBR} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} r e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1 + r e^{i\delta}) & \frac{1}{\sqrt{3}}(1 - \frac{1}{2} r e^{i\delta}) & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}}(1 - r e^{i\delta}) & -\frac{1}{\sqrt{3}}(1 + \frac{1}{2} r e^{i\delta}) & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

- Tri-maximal 1 (s=0, a=r.cosδ)

$$U_{\mathrm{TM}_1} = P' \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} r e^{-i\delta} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}(1 - \frac{3}{2} r e^{i\delta}) & \frac{1}{\sqrt{2}}(1 + r e^{-i\delta}) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}(1 + \frac{3}{2} r e^{i\delta}) & -\frac{1}{\sqrt{2}}(1 - r e^{-i\delta}) \end{pmatrix} P$$

- Tri-maximal 2 (s=0, a=-r/2.cosδ)

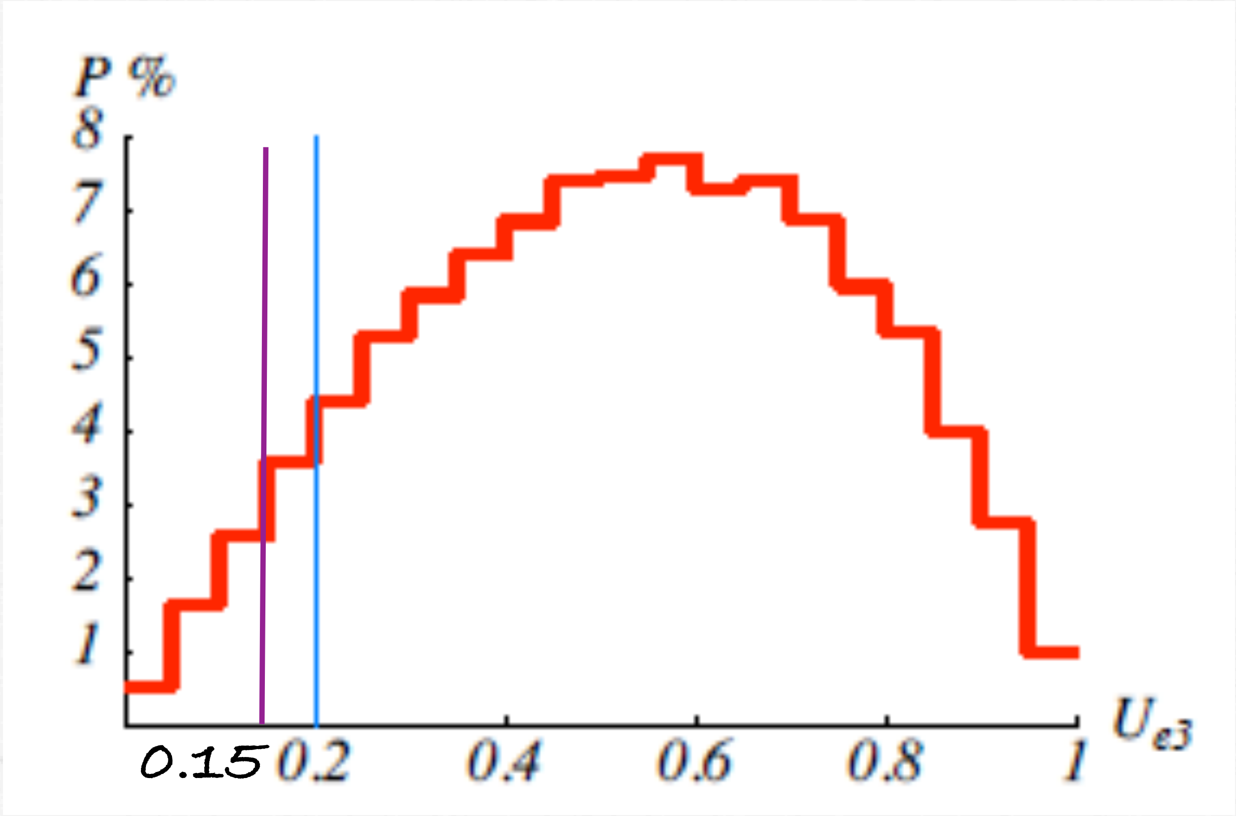
$$U_{\mathrm{TM}_2} = P' \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} r e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1 + \frac{3}{2} r e^{i\delta}) & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}(1 - \frac{1}{2} r e^{-i\delta}) \\ -\frac{1}{\sqrt{6}}(1 - \frac{3}{2} r e^{i\delta}) & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}(1 + \frac{1}{2} r e^{-i\delta}) \end{pmatrix} P$$

Anarchy

Hall, Murayama

- Anarchy: all angles are "large" and unpredicted, so expect $\sin\theta_{13} \sim 0.5$
- Hence larger reactor angle is good news
- Problem is that reactor angle is not that large...
- Also Anarchy not very predictive c.f. landscape

Altarelli, Feruglio, Masina



$$U_{e3} = 0.15 \pm 0.02$$

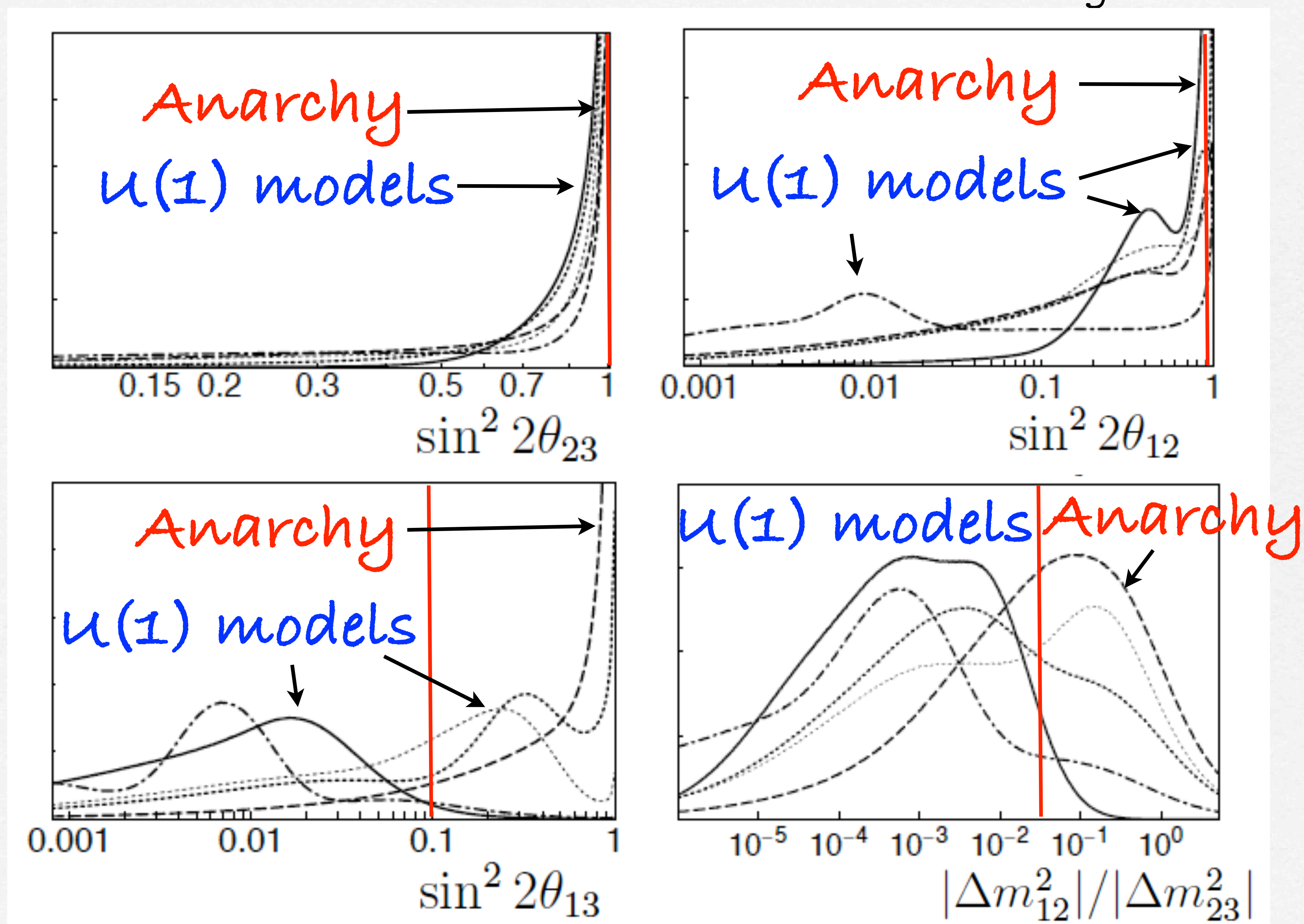
So far no symmetry or dynamics... now we consider...

- U(1) Family Symmetry
- See-saw mechanism
- Sequential dominance
- Constrained sequential dominance 2
- Discrete Family Symmetry
- Grand Unified Theories

U(1) family symmetry

- lepton (Quark) generations labelled by U(1) family symmetry
- $\sin^2 2\theta_{13}$ may peak at lower values

Hirsch and King



See-saw mechanism

P.Minkowski, PLB67(1977)421 ...

Possible type II contribution

Dirac matrix

$$\begin{pmatrix}\overline{\nu_L} & \overline{\nu_R^c}\end{pmatrix}\begin{pmatrix}0 & m_{LR}\\ m_{LR}^T & M_{RR}\end{pmatrix}\begin{pmatrix}\nu_L^c\\ \nu_R\end{pmatrix}$$

$$M^{\nu} = m_{LR}.\frac{1}{M_{RR}}.m_{LR}^T$$

Light Majorana matrix

Heavy Majorana matrix

- ❑ Neutrinos are light because RH neutrinos are heavy
- ❑ No explanation of neutrino mixing
- ❑ Need to add another ingredient e.g. Sequential Dominance

Sequential dominance (SD)

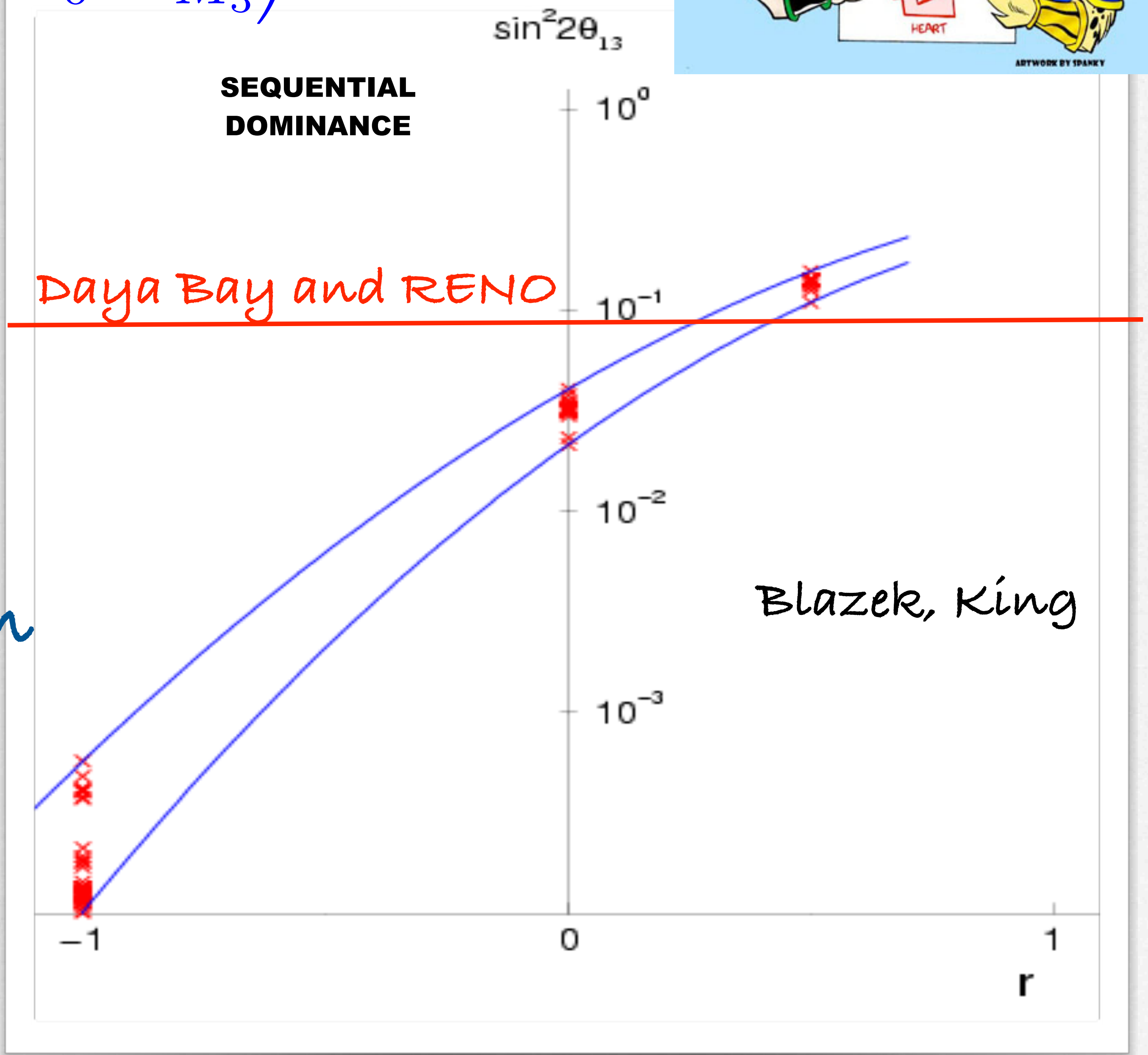
$$m_{LR} = \begin{pmatrix} - & a & d \\ - & b & e \\ - & c & f \end{pmatrix} \quad M_{RR} = \begin{pmatrix} - & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$$

- RH neutrinos contribute sequentially in see-saw

$$m_1 \ll m_2 \ll m_3$$
$$m_2 \sim (a+b+c)^2/M_2$$
$$m_3 \sim (d+e+f)^2/M_3$$

- Mixing angles depend on ratios of Yukawas

$$\tan\theta_{23} \approx e/f$$
$$\tan\theta_{12} \approx a/(b-c)\sqrt{2}$$
$$\theta_{13} \approx d/e\sqrt{2} + O(m_2/m_3)$$



Constrained sequential dominance 2

$$m_{LR} = \begin{pmatrix} - & a & d \\ - & b & e \\ - & c & f \end{pmatrix} \quad M_{RR} = \begin{pmatrix} - & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$$

Antusch, King, Luhn, Spinrath

Suppose $a = b/2 \quad c = d = 0 \quad e = f$

Then
- Trimaximal1 solar mixing
- leptogenesis phase = oscillation phase
- Reactor angle and sum rule predictions

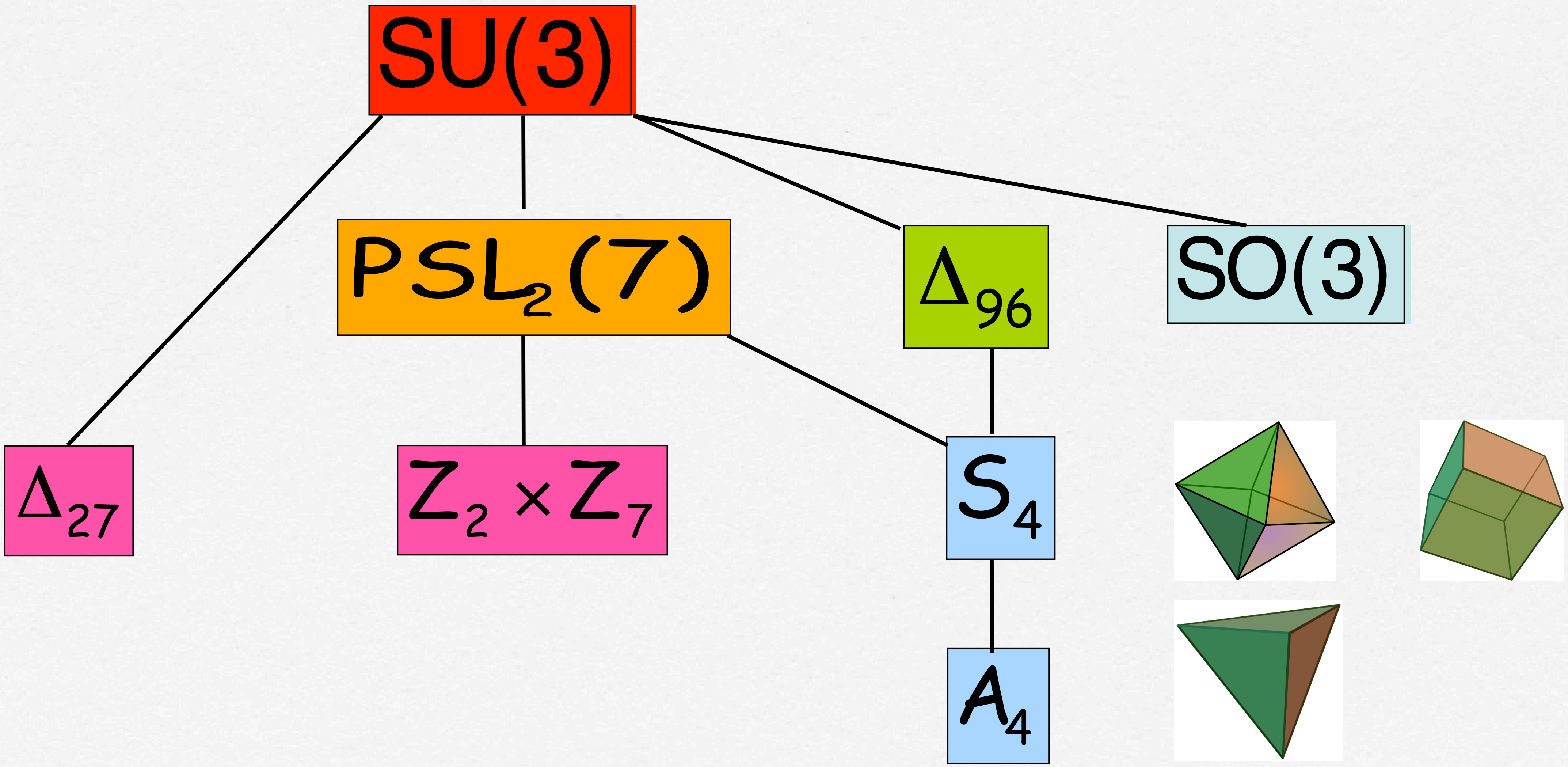
$$\theta_{13} = \frac{\sqrt{2}}{3}\frac{m_2}{m_3} \sim 5^o \qquad \theta_{23} = 45^o + \sqrt{2}\theta_{13}\cos\delta$$

Can such sum rules be tested in neutrino experiments?

Ballett, King, Luhn, Pascoli, Schmidt

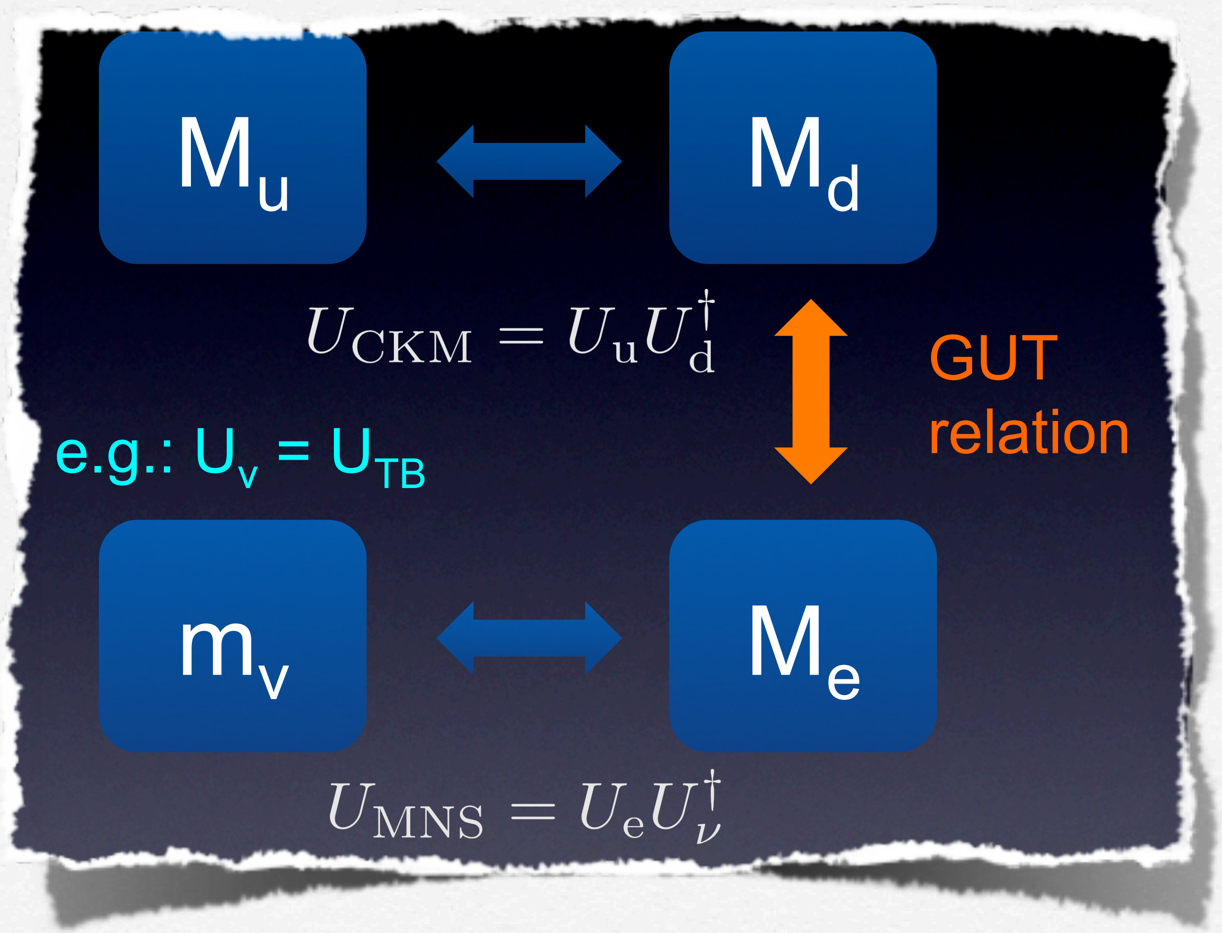
Discrete Family Symmetry

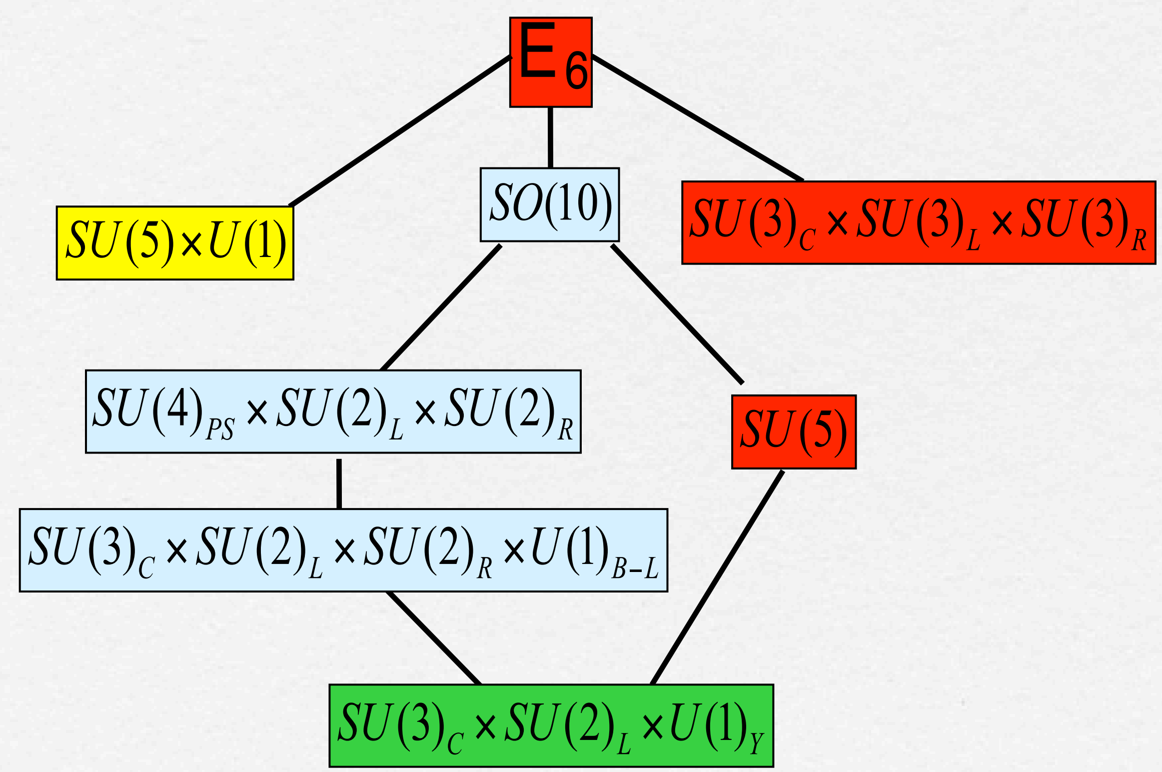
- Relations such as e=f suggest permutation sym
- More generally discrete family symmetry



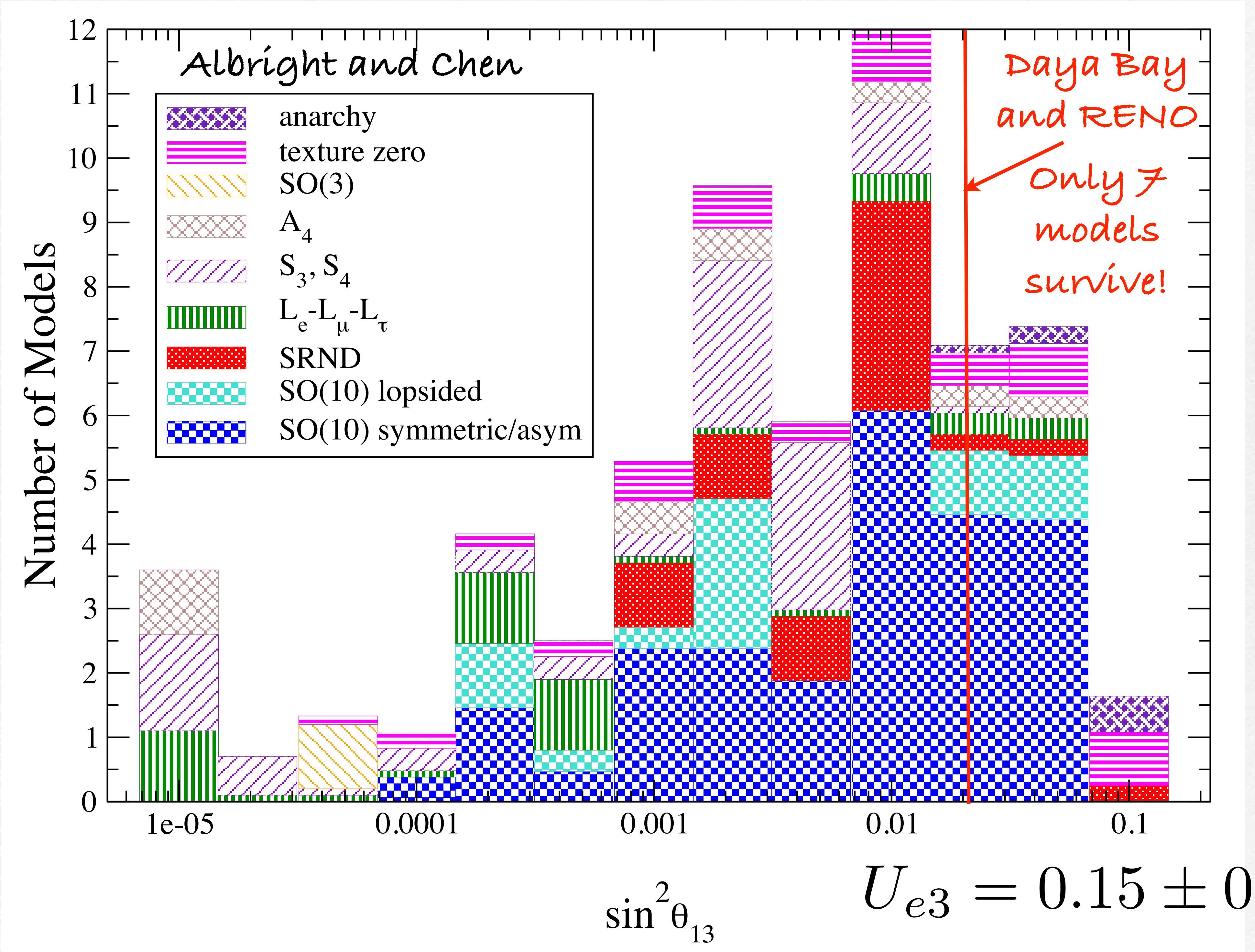
Grand Unified Theories

- Quarks and leptons may be related via GUTs





Models Survey c.2006



$$U_{e3} = 0.15 \pm 0.02$$

Now there are many more HYDRAS

- H.-J. He, F.-R. Yin, arXiv:1104.2654; Z.-Z. Xing, arXiv:1106.3244; N. Qin and B. Q. Ma, Phys. Lett. B **702** (2011) 143 [arXiv: 1106.3284]; Y. j. Zheng and B. Q. Ma, arXiv:1106.4040; S. Zhou, arXiv:1106.4808; T. Araki, arXiv:1106.5211; N. Haba, R. Taka- hashi, arXiv:1106.5926; D. Meloni, arXiv: 1107.0221; S. Morisi, K. M. Patel and E. Peinado, arXiv:1107.0696; W. Chao, Y.-J. Zheng, arXiv:1107.0738; H. Zhang, S. Zhou, arXiv: 1107.1097; X. Chu, M. Dhen, T. Hambye, arXiv:1107.1589; P. S. B. Dev, R. N. Mohapatra, M. Severson, arXiv:1107.2378; R. d. A. Toorop, F. Feruglio, C. Hagedorn, arXiv:1107.3486; S. Antusch, V. Maurer, arXiv:1107.3728; Q. H. Cao, S. Khalil, E. Ma and H. Okada, arXiv:1108.0570; D. Marzocca, S. T. Petcov, A. Romanino and M. Spinrath, arXiv:1108.0614; S. F. Ge, D. A. Dicus and W. W. Repko, arXiv:1108.0964; F. Bazzocchi, arXiv:1108.2497; Y. Shimizu, M. Tanimoto and A. Watanabe, arXiv:1105.2929; X. G. He and A. Zee, arXiv:1106.4359; S. F. King and C. Luhn, arXiv: 1107.5332

Distinguished by sum rules…

Summary of Sum Rule Predictions

- Quark-Lepton Complementarity $\theta_{12} + \theta_C = 45^o$
- Solar sum rules
  - Bimaximal $\theta_{12} = 45^o + \theta_{13} \cos\delta$
  - Tri-bimaximal $\theta_{12} = 35^o + \theta_{13} \cos\delta$
  - Golden Ratio $\theta_{12} = 32^o + \theta_{13} \cos\delta$
- Atm. sum rules
  - Tri-bimaximal-reactor $\theta_{23} = 45^o$
  - Trimaximal1 $\theta_{23} = 45^o + \sqrt{2}\theta_{13} \cos\delta$
  - Trimaximal2 $\theta_{23} = 45^o - \frac{\theta_{13}}{\sqrt{2}} \cos\delta$

Now that θ_{13} is measured these predict $\cos\delta$

Conclusions

- Simple patterns of mixing such as Bimaximal, Tri-bimaximal, Golden Ratio are ruled out by Daya Bay and RENO
- However they may be rescued by invoking large charged lepton corrections leading to solar sum rules involving the CP phase delta
- Other patterns consistent with Daya Bay and RENO have been proposed such as Tri-bimaximal-reactor mixing and two versions of Trimaximal mixing, leading to atmospheric sum rules also involving the CP phase
- Many models based on discrete family symmetry and GUTs proposed before Daya Bay and RENO have been killed but many other "Hydras" have emerged, distinguished by the above solar and atm. sum rules
- It is vital to measure the mixing angles and the CP phase delta to good precision to discriminate between the different models and decide if the Universe is based on GUTs and Family symmetry or if Anarchy Rules
- Some models unpredictive due to NLO corrections - resemble Anarchy