Muon Storage Rings

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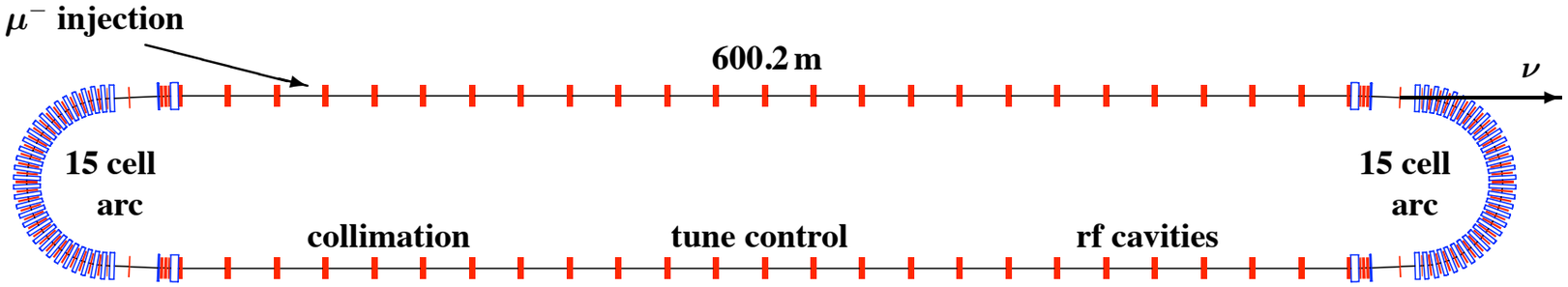
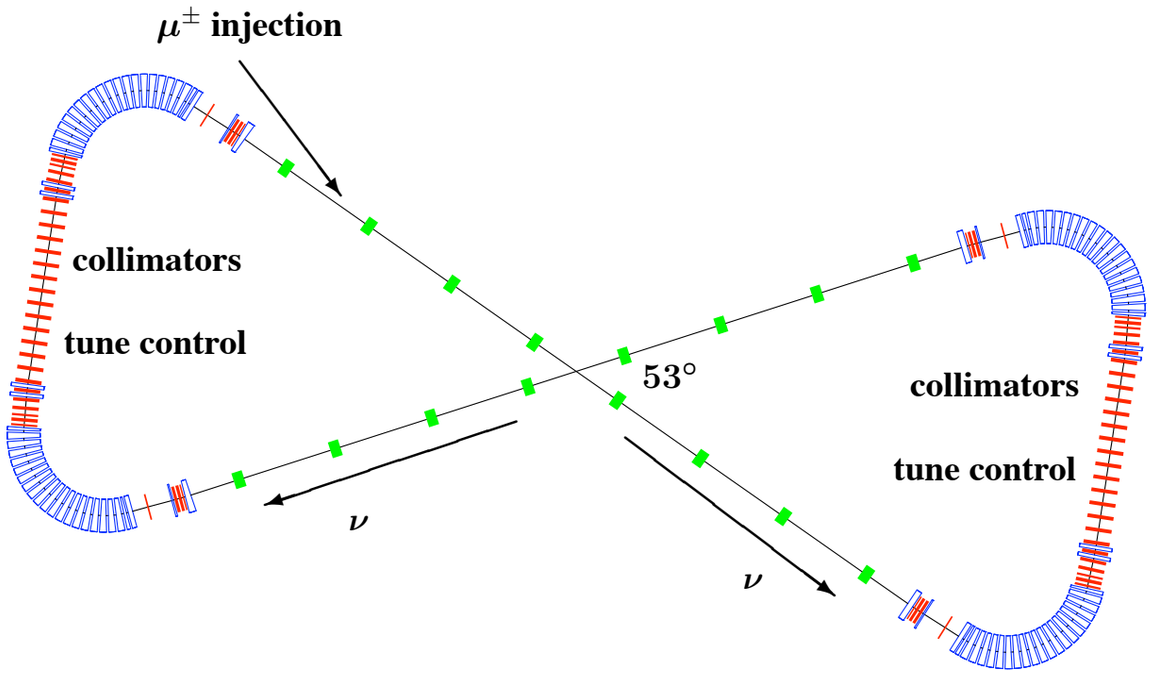
Design points:

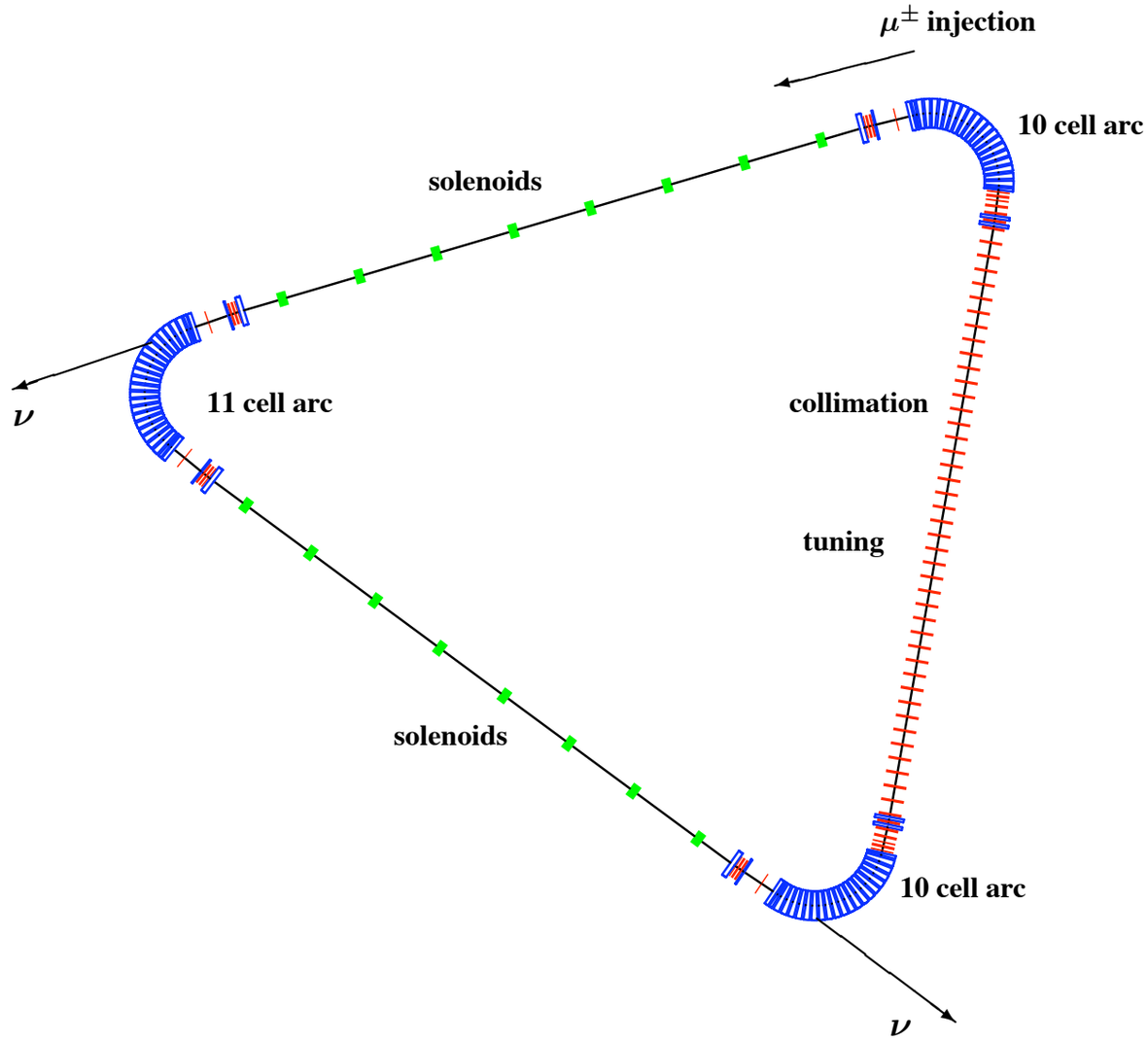
- Efficiency $\eta = \dfrac{\text{length of production straights}}{\text{circumference}}$
- Tunnel depth
- $\chi = \dfrac{\text{muon rms divergence}}{\text{rms opening angle of decay neutrinos}}$
- MW levels of beam power
- Fitting equally spaced muon bunch trains into the ring and maintaining $\gtrsim$100 ns gaps between μ^+ and μ^-

Summary Paper WEPFP099, PAC'09

Racetrack	
Production straights	600 m
Efficiency η	37.25%
Depth at 18°	233 m
Depth at 36°	444 m
χ at 25 GeV	0.12



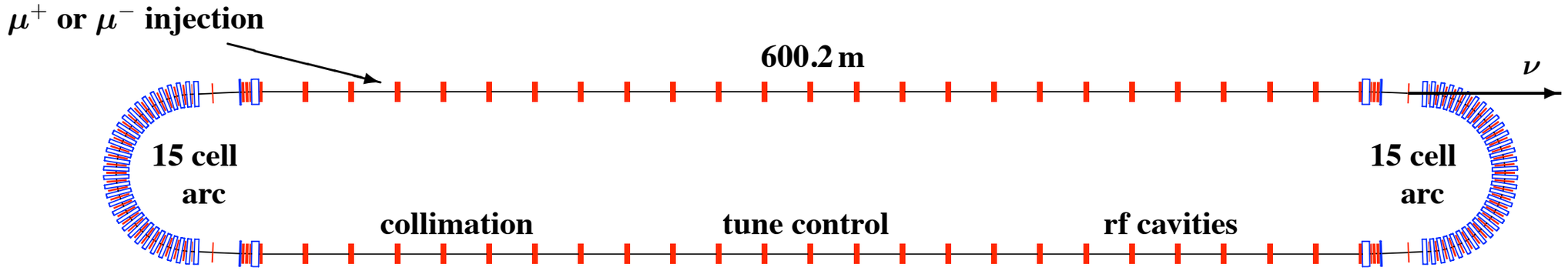




Bowtie	
Production straights	469 m
Efficiency η	$2 \times 29.2\%$
Maximum depth	312 m
χ at 25 GeV	0.12

Triangle	
Production straights	398.5 m
Efficiency η	$2 \times 24.8\%$
Maximum depth	493 m
χ at 25 GeV	0.12

Bunch Merging in the 25 GeV Muon Decay Ring (IDS Baseline)



- Ring circumference 1608.802 m to match proton driver ring (802 m)
- Duration of a train of 80, 201.25 MHz bunches is 397.5 ns
- Three muon bunches per pulse
- Ring revolution period 5.37 ns
- Gaps between bunches 1391 ns
- Fit counter-rotating, opposite sign bunch train, leaving 497 ns between bunch trains
- Preserve 100 ns gap between neutrino/anti-neutrino bursts at detectors.

- Simple formula for time variations:

$$\text{Revolution period } T = 2\pi R/\beta c$$

$$\Longrightarrow \frac{\Delta T}{T} = \frac{\Delta R}{R} - \frac{\Delta \beta}{\beta} = \left(\frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}\right)\frac{\Delta p}{p}.$$

- For 25 GeV, $\gamma \approx 236.6$, $\gamma_t \approx 14.5$

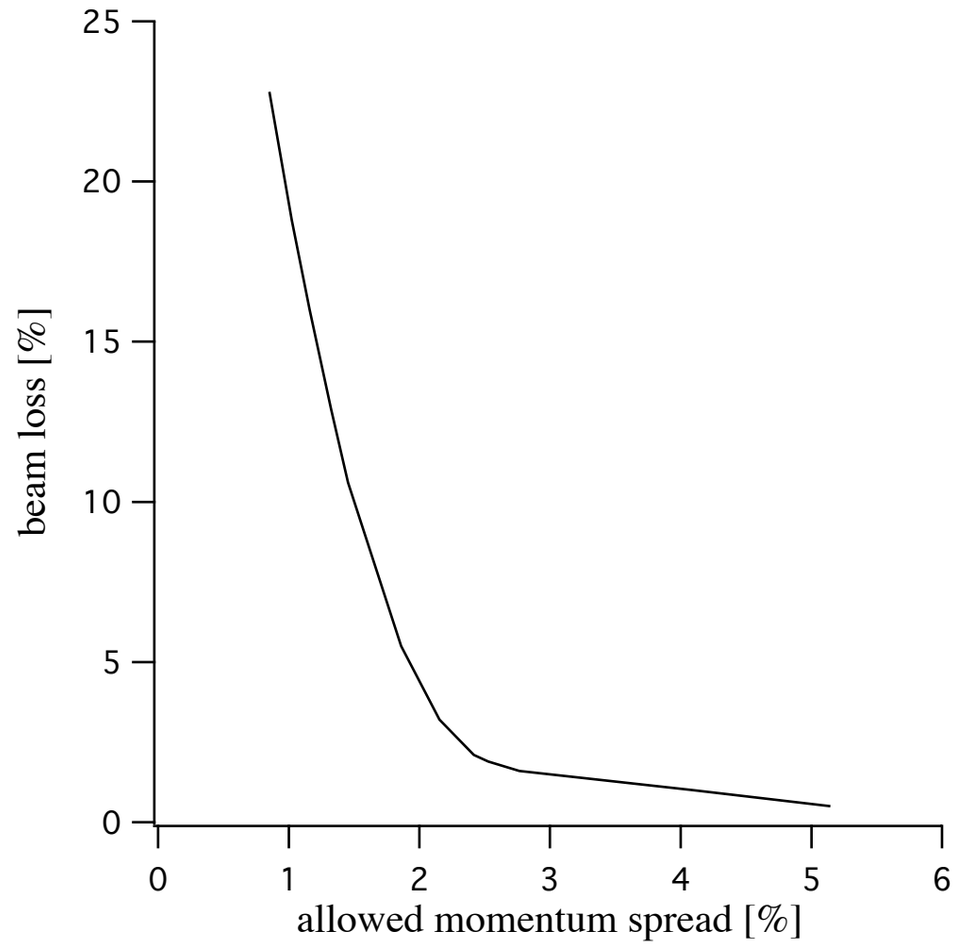
- For momentum spread of

$$\frac{\Delta p}{p} = \pm 5\% \text{ max}$$

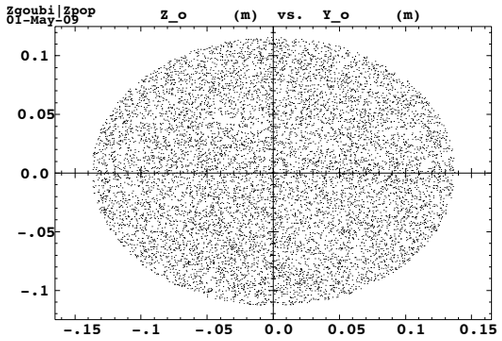
this gives

$$|\Delta T| \lesssim 1.27\,\text{ns per revolution}$$

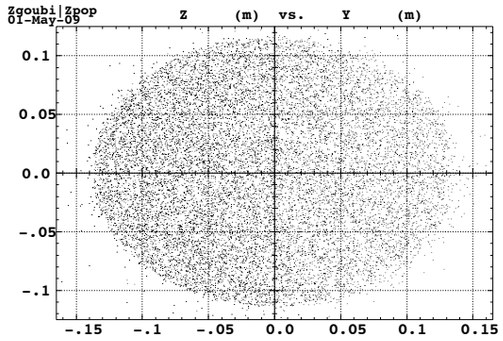
- Inter-bunch gap of 497 ns will reduce to 100 ns in 156 revolutions, corresponding to 1.6 e-folding times.



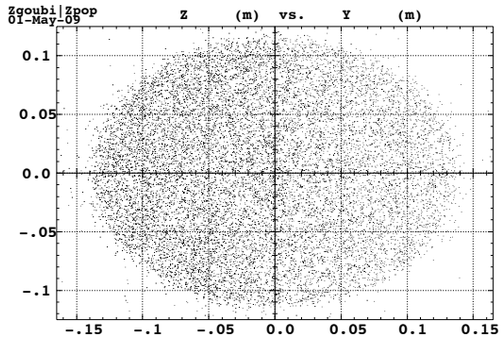
For 5% momentum spread, would need rf.
If can collimate beam at (say) 2.5%, could avoid use of rf.



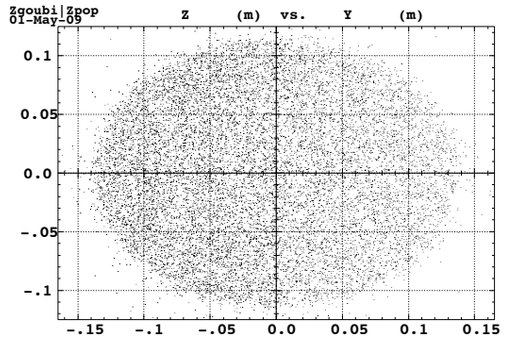
(a) xy-input



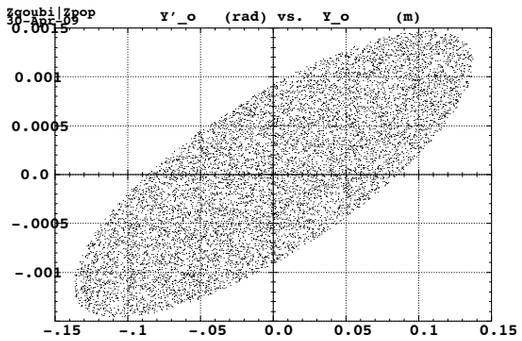
(b) xy-50 turns



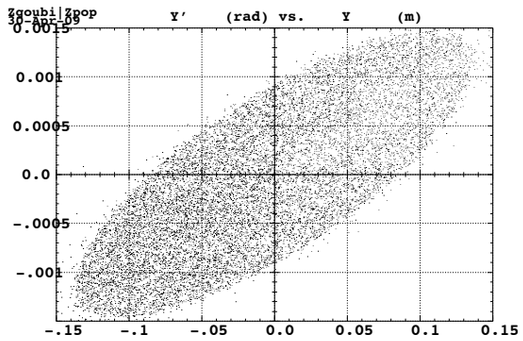
(c) xy-100 turns



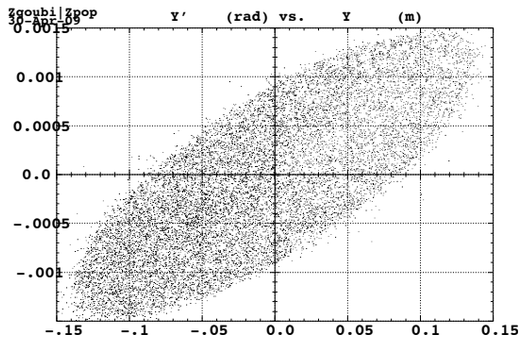
(d) xy-156 turns



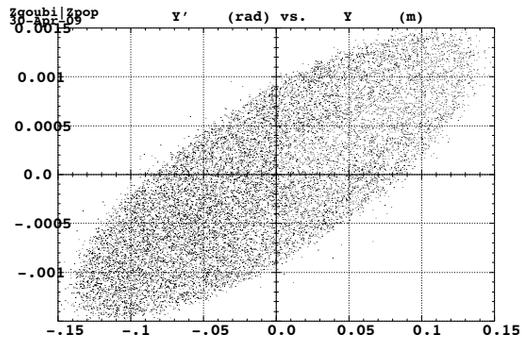
(e) xx'-input



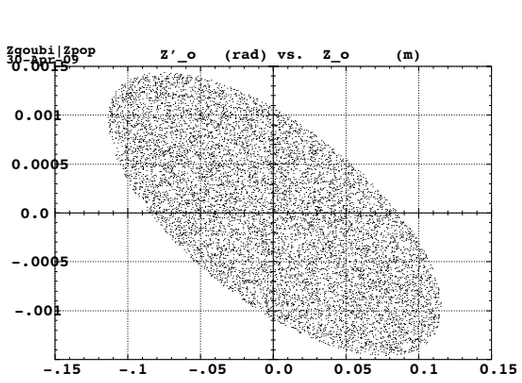
(f) xx'-50 turns



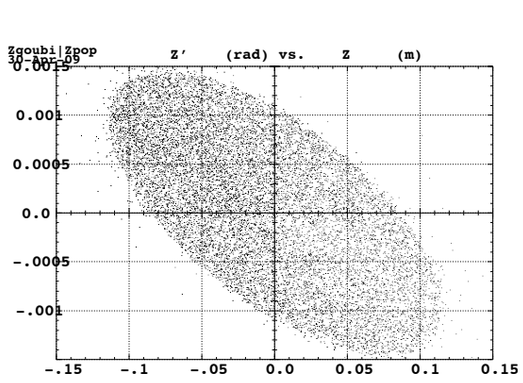
(g) xx'-100 turns



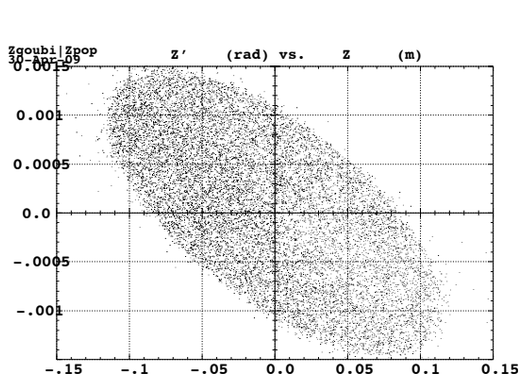
(h) xx'-156 turns



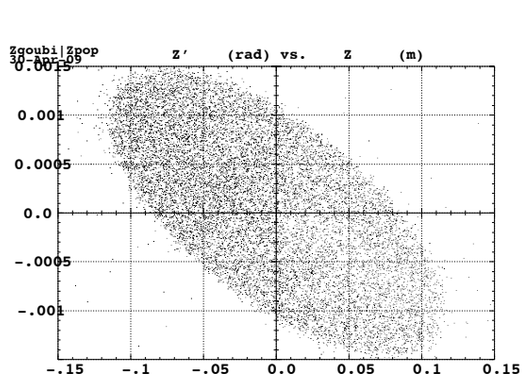
(i) yy'-input



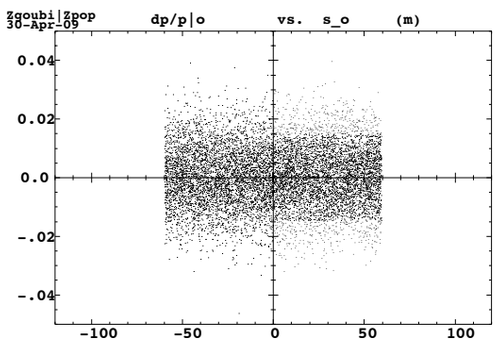
(j) yy'-50 turns



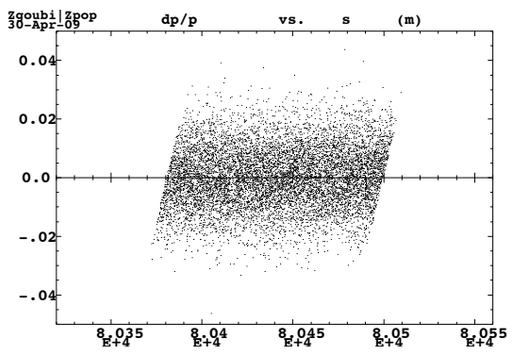
(k) yy'-100 turns



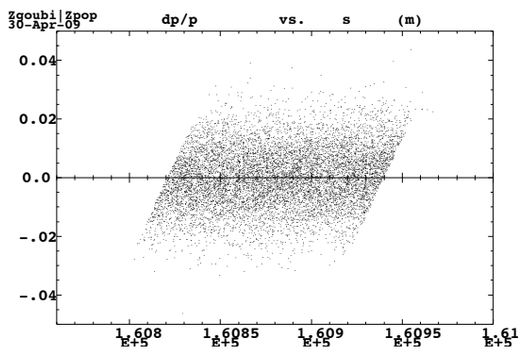
(l) yy'-156 turns



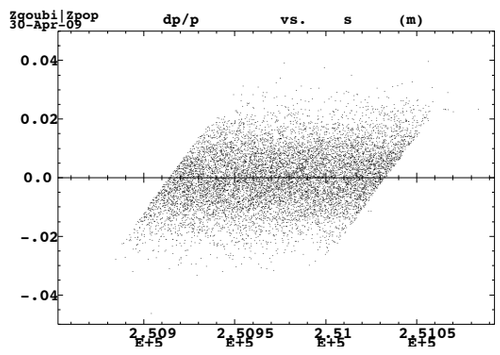
(m) s-$\frac{\Delta p}{p}$-input



(n) s-$\frac{\Delta p}{p}$-50 turns

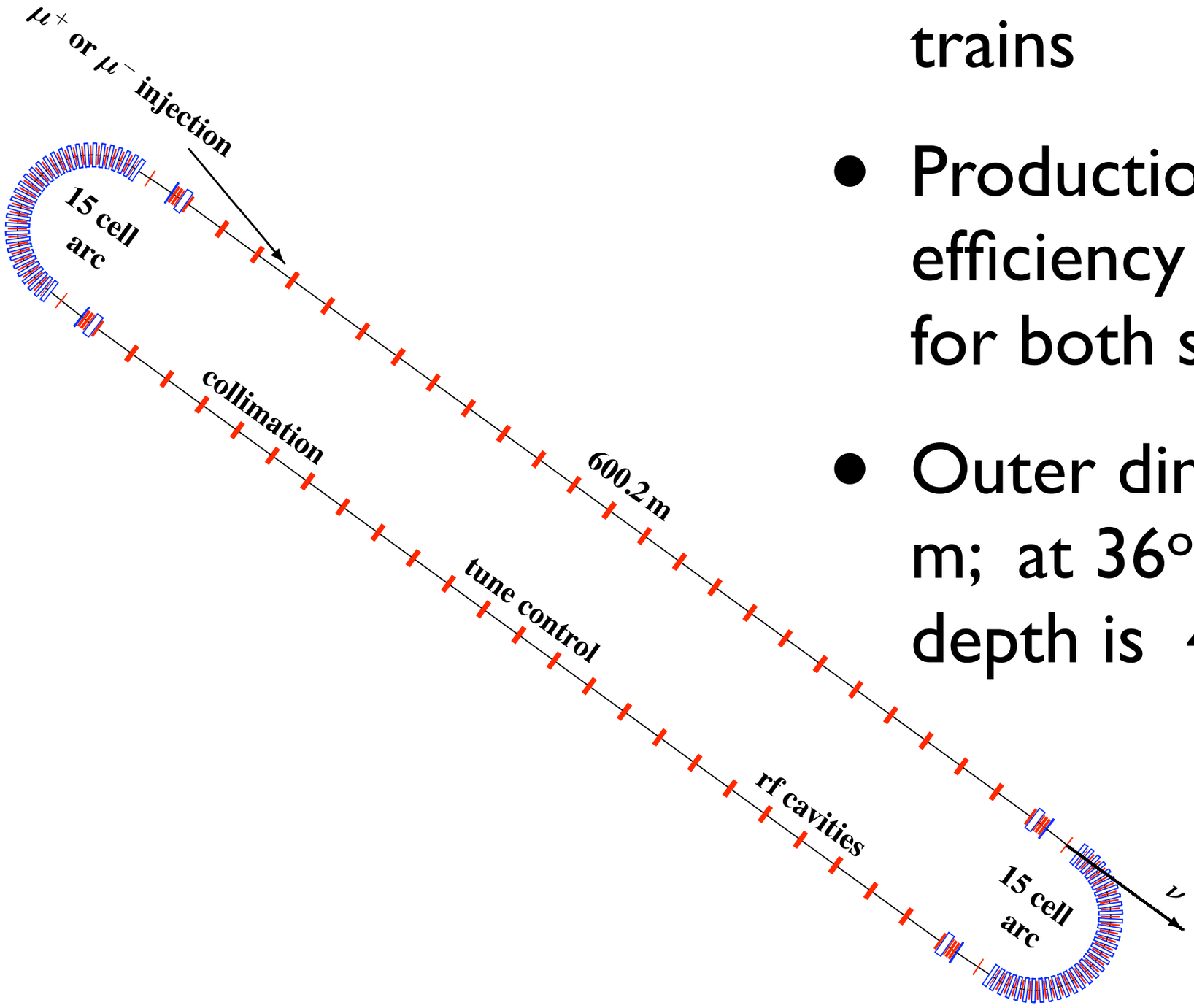


(o) s-$\frac{\Delta p}{p}$-100 turns



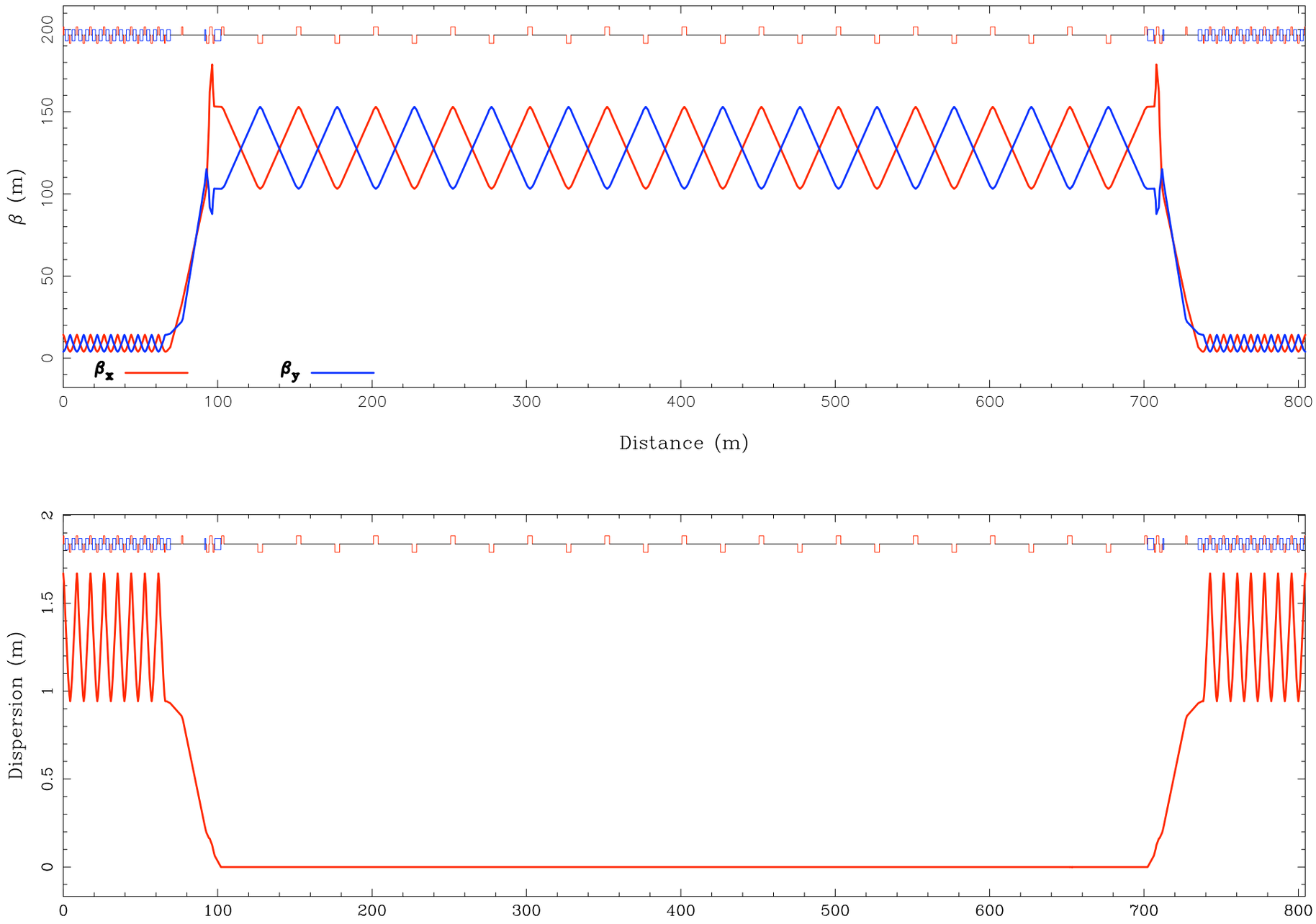
(p) s-$\frac{\Delta p}{p}$-156 turns

Reducing Size of Racetrack Decay Rings



- Ring designed for 5 bunch trains
- Production straight 599.4 m, efficiency 37.25% (doubled for both sign muons)
- Outer dimension 755 m x 82 m; at 36° into Earth, tunnel depth is 444 m

- 15 cell arcs, each cell 8.8 m arc lengths 132 m
- Matching sections 36.5 m at start and end of each straight.



- L_A=length of each arc
- L_M=length of each matching section
- L_ν=length of each production straight

$$\text{Efficiency } \eta = \frac{2L_\nu}{2L_A + 4L_M + 2L_\nu} \quad \Longrightarrow \quad L_\nu = (L_A + 2L_M)\frac{\eta}{1-\eta}$$

$$\text{Tunnel depth is} \quad d = \left(L_\nu + 2L_M + \frac{2}{\pi}L_A\right)\sin\theta$$

$\theta = 18^\circ$ for $\sim 4000\,\text{km}$ detector, $\theta = 36^\circ$ for $\sim 7500\,\text{km}$ detector.

Circumference is

$$C = 2L_A + 4L_M + 2L_\nu \geq 2N \times 500\,\text{ns} \times \beta c = 300N\,\text{m}$$

For uniform spacing of muon bunch trains, require:

$$T_p\left(n+\frac{k}{N}\right)=T_\mu\left(m+\frac{l}{N}\right),$$

where

T_p, T_μ are the revolution periods for (10 GeV) protons and (25 GeV) muons respectively

N is the number of proton bunches= number of μ^+ or μ^- bunch trains

k, l, m, n are integers; $k, l = 0, \ldots, n-1$

For 3 bunches, take $\dfrac{T_\mu}{T_p}=\dfrac{5}{4}$.

n	k	m	l
0	0	0	0
1	2	1	1
3	1	2	2

Storage ring circumference is

$$C = 1.25 \times \text{driver circumference} = 1005.506\,\text{m}$$

Possibilities:

- Length of matching section effectively fixed
- Reduce arc cells to 10 $\implies$ magnetic field increases to 6.42 T
- Then $2L_A + 4L_M = 322\,\mathrm{m}$, so for $C = 1005.506\,\mathrm{m}$, $L_\nu = 341.75\,\mathrm{m}$ and $\eta = (2\times)34\%$
- Tunnel depths are 150 m (18°) and 280 m (36°)
- Certainly need RF - need to provide additional space in the arcs
- For target number of neutrino events, would need to run for ∼1 extra year.

Note: Engineers confirm deeper tunnel is far cheaper than extra operating costs