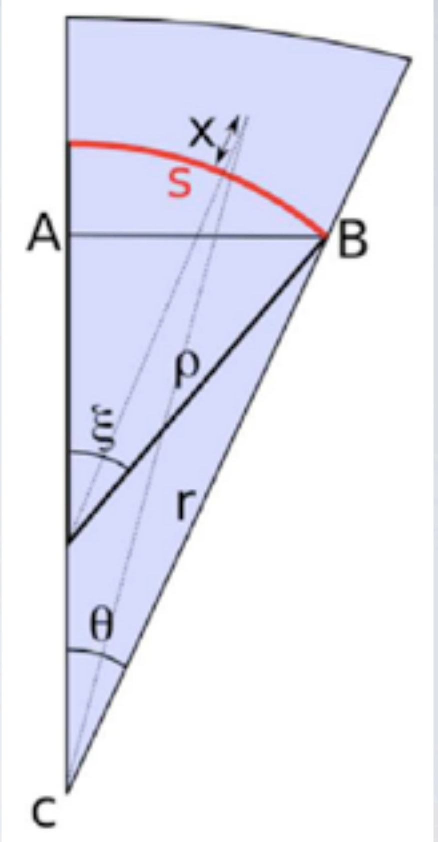
FFAG MUON DECAY RING FOR LOW ENERGY NF

Y. Mori, JB. Lagrange
Kyoto University

MUON DECAY RING WITH SCALING FFAG

- Fixed field is not an advantage for storing the particles in the ring. But, if a beam is largely momentum spread like muon beam, then,
- Advantages
    - Zero chromaticity
        - No resonance crossing for large momentum range : not for non-scaling FFAG
        - Large acceptance in longitudinal & transverse directions : $\Delta p/p > 10\%$
        - Orbit configurations for different beam momentum are similar.
- Issue : race-track ring
    - Require FFAG optics for a long straight section with small dispersion.
        - Beyond ordinary FFAG optics which is only for circular ring.
- Need advancement on scaling FFAG optics.

ZERO CHROMATICITY CIRCULAR RING



- Betatron oscillation (cylindrical coordinate)

$$\frac{d^2x}{d\theta^2}+\frac{r^2}{\rho^2}\left(1-K\rho^2\right)x=0$$

$$\frac{d^2z}{d\theta^2}+\frac{r^2}{\rho^2}\left(K\rho^2\right)z=0$$

$$K=-\frac{1}{B\rho}\frac{\partial B}{\partial r}$$

- Zero chromaticity : constant betatron tunes during acceleration

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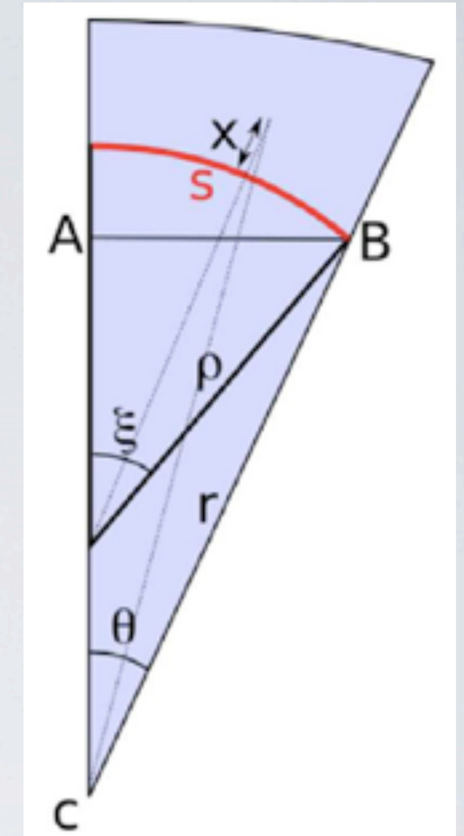
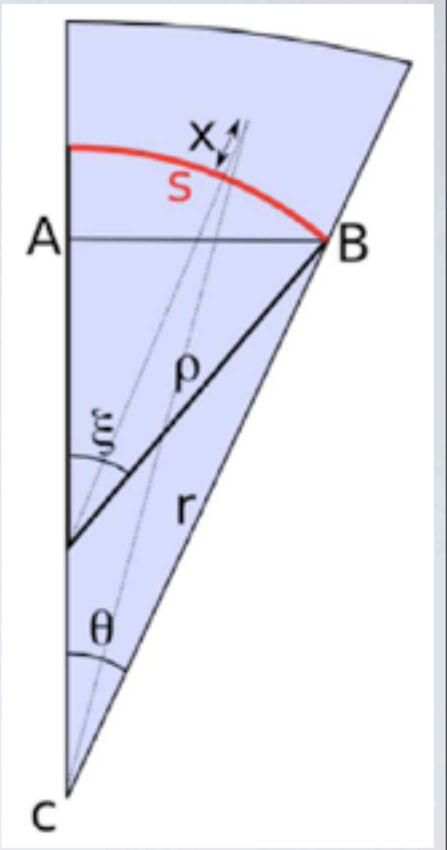


- Zero chromaticity : constant betatron tunes during acceleration

$$\frac{d\left(r^2/\rho^2\right)}{dp}=0$$

$$\frac{d\left(K\rho^2\right)}{dp}=0$$

ZERO CHROMATICITY CIRCULAR RING



- Betatron oscillation (cylindrical coordinate)

$$\frac{d^2x}{d\theta^2} + \frac{r^2}{\rho^2}\left(1 - K\rho^2\right)x = 0$$

$$\frac{d^2z}{d\theta^2} + \frac{r^2}{\rho^2}\left(K\rho^2\right)z = 0$$

$$K = -\frac{1}{B\rho}\frac{\partial B}{\partial r}$$

- Zero chromaticity : constant betatron tunes during acceleration

$$\frac{d\left(r^2/\rho^2\right)}{dp} = 0$$

$$\frac{d\left(K\rho^2\right)}{dp} = 0$$

$$\longrightarrow \begin{cases} r \propto \rho \\ \dfrac{r}{B}\left[\dfrac{\partial B_z}{\partial r}\right]_{z=0} = k \end{cases}$$

$$\Rightarrow \quad B_z = B_0\left(\frac{r}{r_0}\right)^k f(\theta)$$

AG FOCUSING LATTICE OF SCALING FFAG RING

$$B_z = B_0 \left(\frac{r}{r_0} \right)^k f(\theta)$$

- AG focusing : FODO lattice
- Radial sector
  - F: positive bend
  - D:negative bend

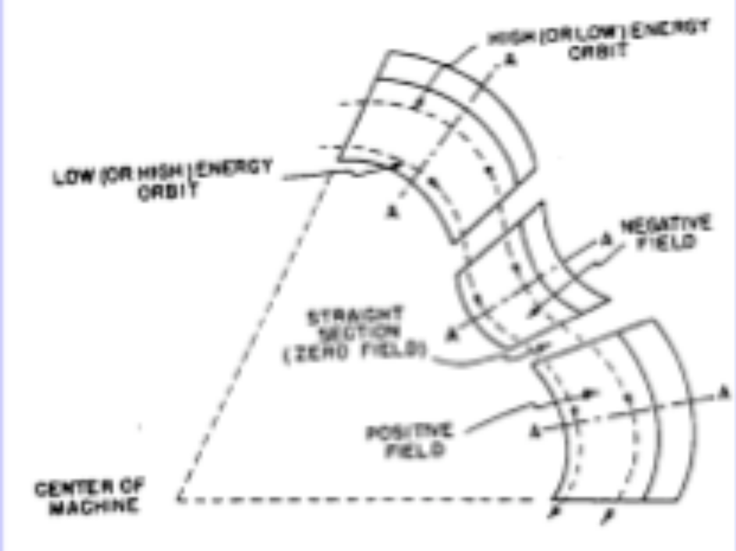


FIG. 2. Plan view of radial-sector magnets.

- Spiral sector
  - F: positive bending
  - D: edge focusing

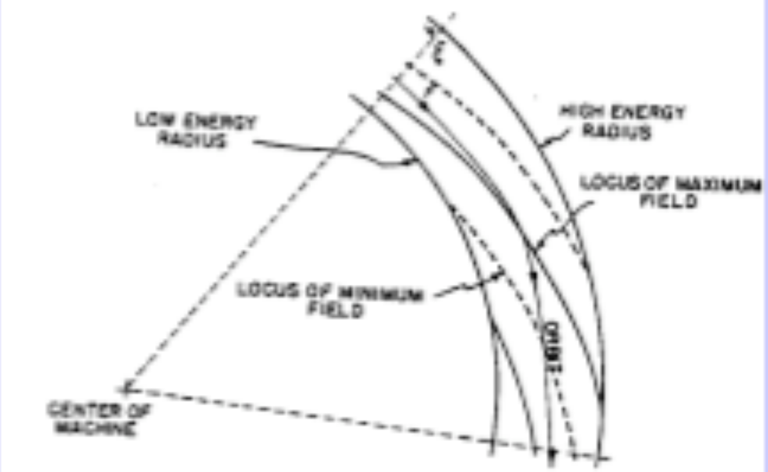


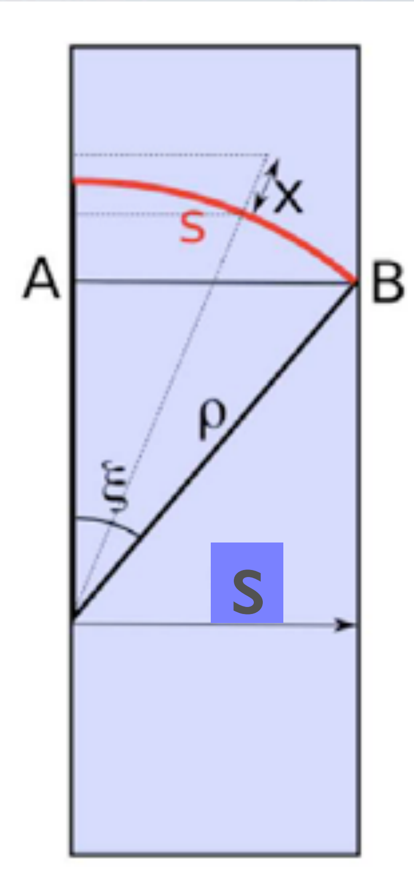
FIG. 3. Spiral-sector configuration.

ZERO CHROMATICITY STRAIGHT LINE

- Betatron oscillation (linear coordinate)

$$\frac{d^2x}{ds^2}+\frac{1}{\rho^2}\left(1-K\rho^2\right)x=0$$

$$\frac{d^2z}{ds^2}+\frac{1}{\rho^2}\left(K\rho^2\right)z=0 \qquad K=-\frac{1}{B\rho}\frac{\partial B}{\partial x}$$



- Zero chromaticity : constant betatron tunes for various beam momentum

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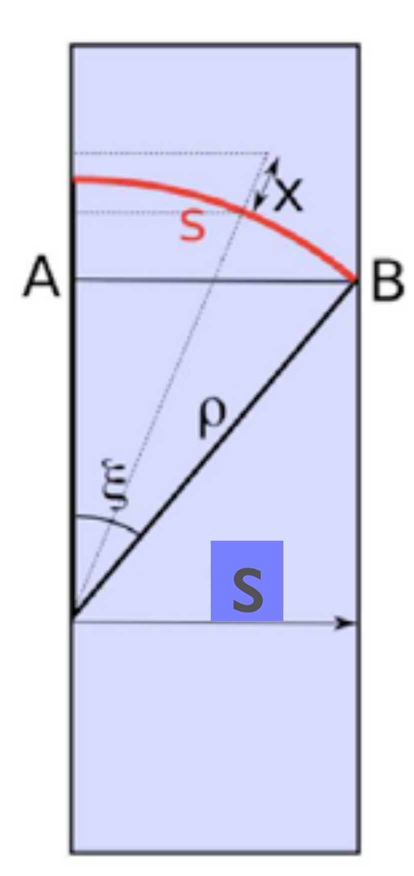

- Zero chromaticity : constant betatron tunes for various beam momentum

$$\frac{d\left(1/\rho^2\right)}{dp}=0$$

$$\frac{d\left(K\rho^2\right)}{dp}=0$$

ZERO CHROMATICITY STRAIGHT LINE

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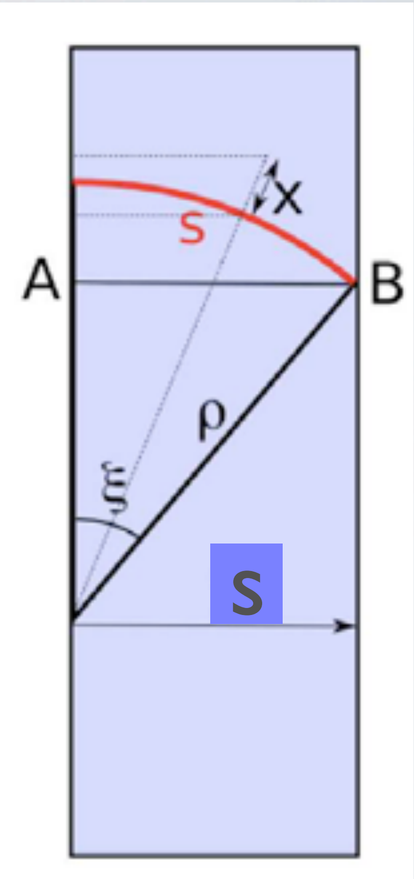


- Zero chromaticity : constant betatron tunes for various beam momentum

$$\frac{d\left(1/\rho^2\right)}{dp}=0$$

$$\frac{d\left(K\rho^2\right)}{dp}=0$$

$$\longrightarrow \begin{cases} \rho = const. \\ \frac{1}{B}\left[\frac{\partial B_z}{\partial x}\right]_{z=0} = \frac{n}{\rho} \end{cases}$$

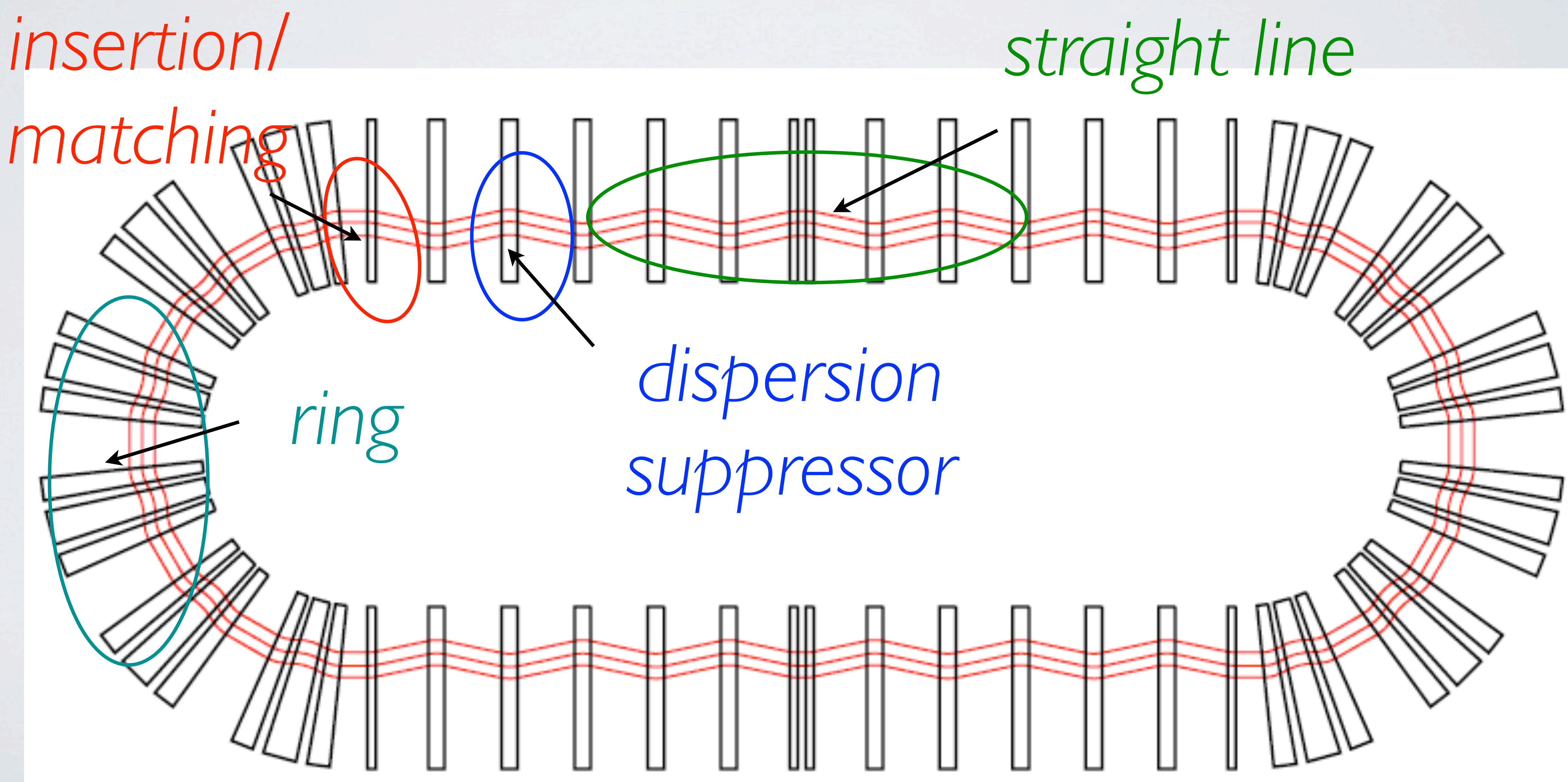
$$\Rightarrow \quad B_z = B_0 \exp\left[\frac{n}{\rho}x\right]$$

RACE-TRACK SCALING FFAG

- **Symmetric circular scaling FFAG**
- Pros/
  - Zero chromaticity : Large acceptance → Essential for storage ring such as muon decay ring.
    - Difficulty in non-scaling FFAG
- Cons/
  - Large dispersion:orbit excursion becomes large
    - Large horizontal aperture magnet
    - Large horizontal aperture rf cavity → Low frequency rf system is needed.
  - Short straight section
    - Small space for injection/extraction → Kicker/septum require large aperture.
    - Small space for rf cavity → High gradient rf is needed.
- **We need a long straight line with very small dispersion still keeping "Zero-chromaticity".**
- Is it possible to make a race-track scaling FFAG ring ?
  - Keeping a scaling law : zero chromaticity
  - Reducing dispersion : dispersion suppressor
  - Making a good match with circular FFAG ring : insertion

RACE-TRACK SCALING FFAG

- Optics issues:



OPTICS

- Scaling(zero-chromatic) FFAG straight line (JB. Lagrange)
  - Example
    - Proton beam
    - Energy range : E=80~200MeV

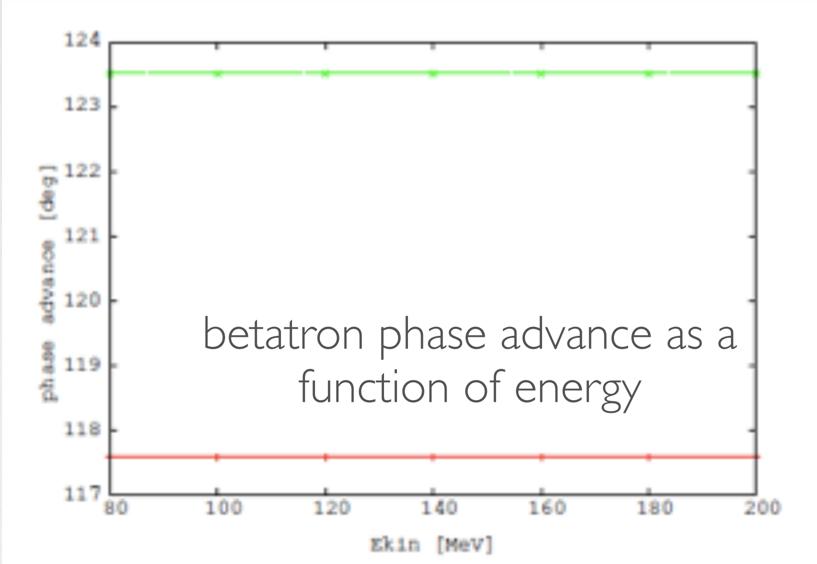
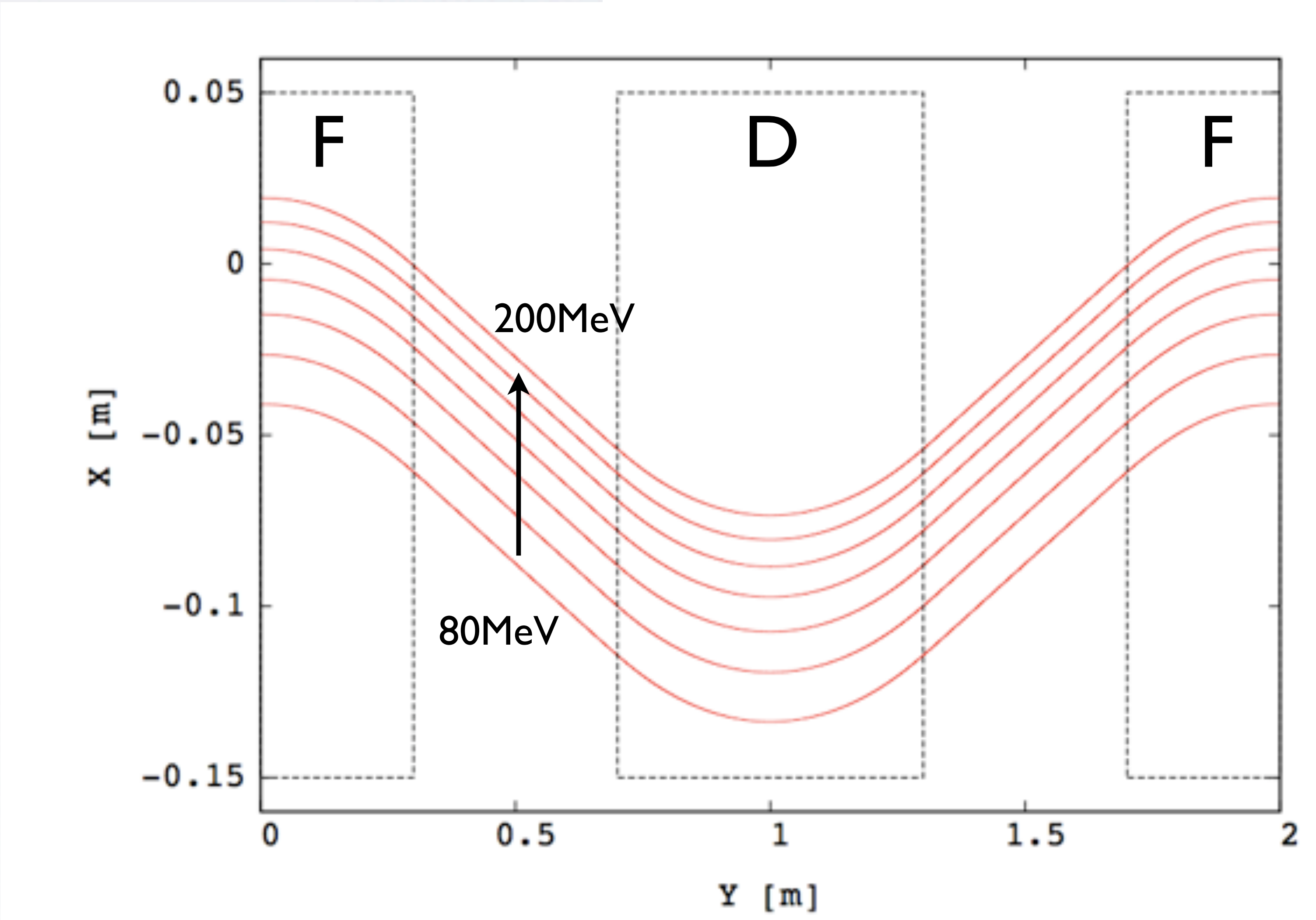


Table 1: Tracking parameters

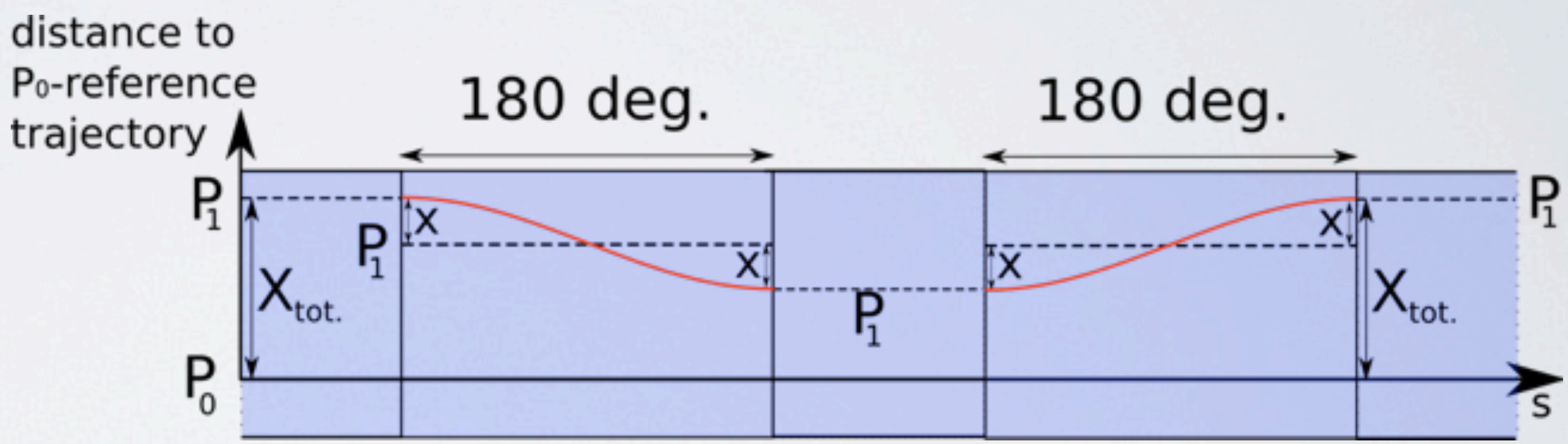
Length of the magnets	60 cm
Drift	40 cm
Kinetic energy range	80 to 200 MeV
Field index	17
Local curvature radius	2.1 m
Step size	1 mm
Phase advances:	
horizontal μ_x	104.8 deg.
vertical μ_z	112.5 deg.



DISPERSION SUPPRESSOR/ INSERTION MATCHING

- Dispersion suppressor
  - Successive π-cells in the horizontal plane can suppress the dispersion.
  - Help to reduce the size of apertures of the magnet and rf cavity.

$$x = \ln\left(\frac{P_1}{P_0}\right)\left(\frac{\rho_0}{n_0} - \frac{\rho_1}{n_1}\right)$$



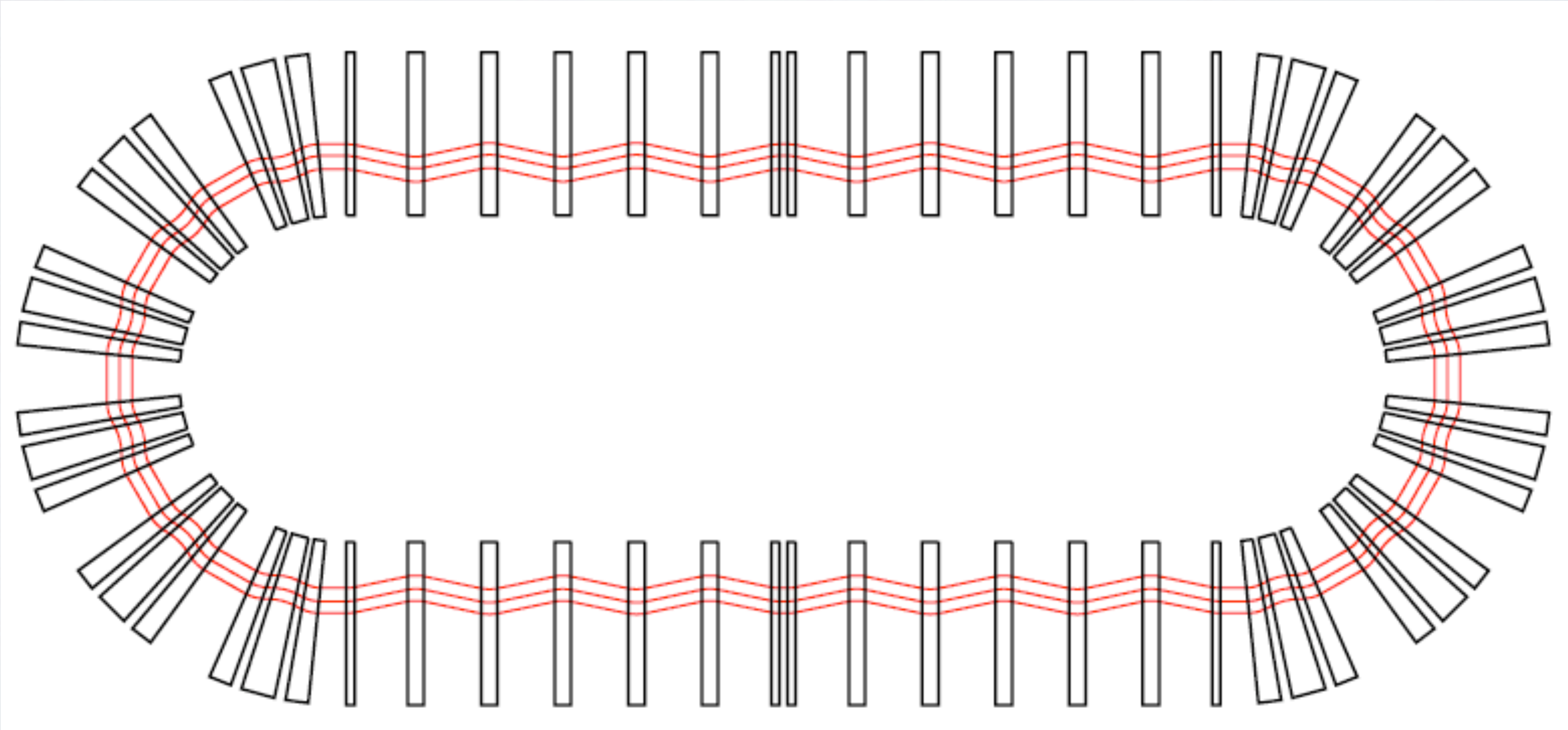
- Insertion matching
  - Matching condition for closed orbit between ring and straight line

ring — linear line

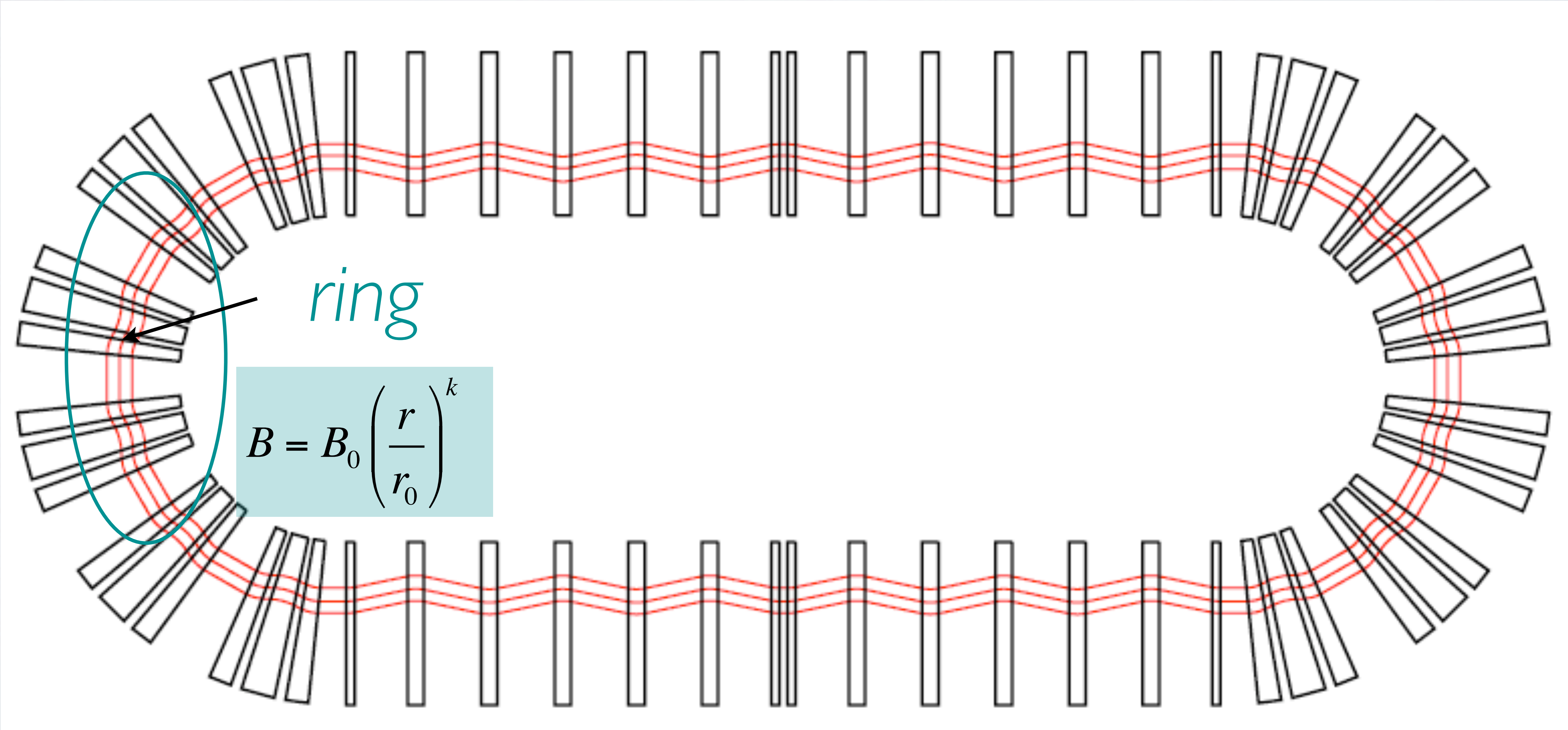
$$\left(1+\frac{x}{r_m}\right)^{k+1} = \exp\left(\frac{n}{\rho}x\right) \quad \Rightarrow \quad \frac{k+1}{r_m} = \frac{n}{\rho}$$

← 1st order

RACE TRACK FFAG RING

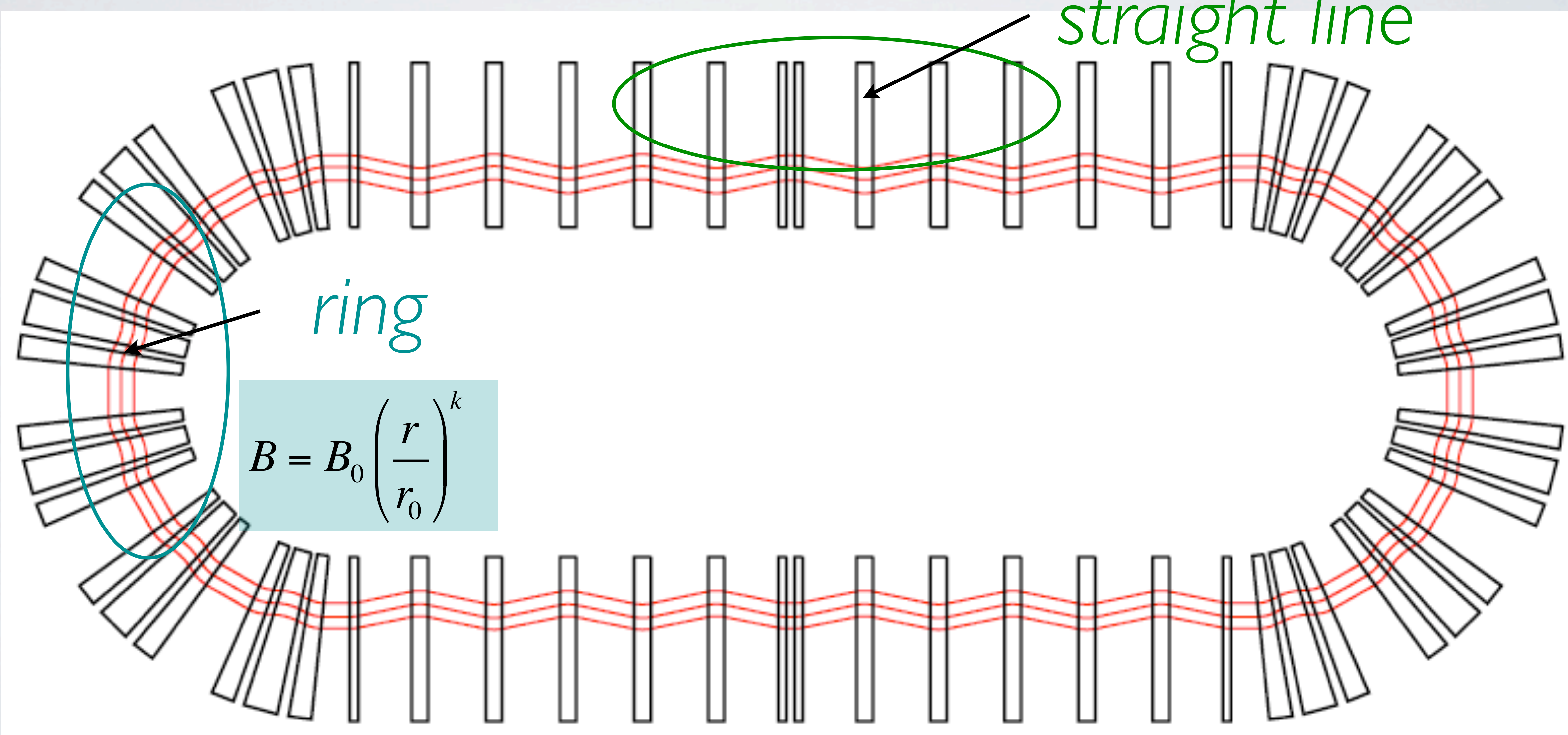


RACE TRACK FFAG RING



RACE TRACK FFAG RING

$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$

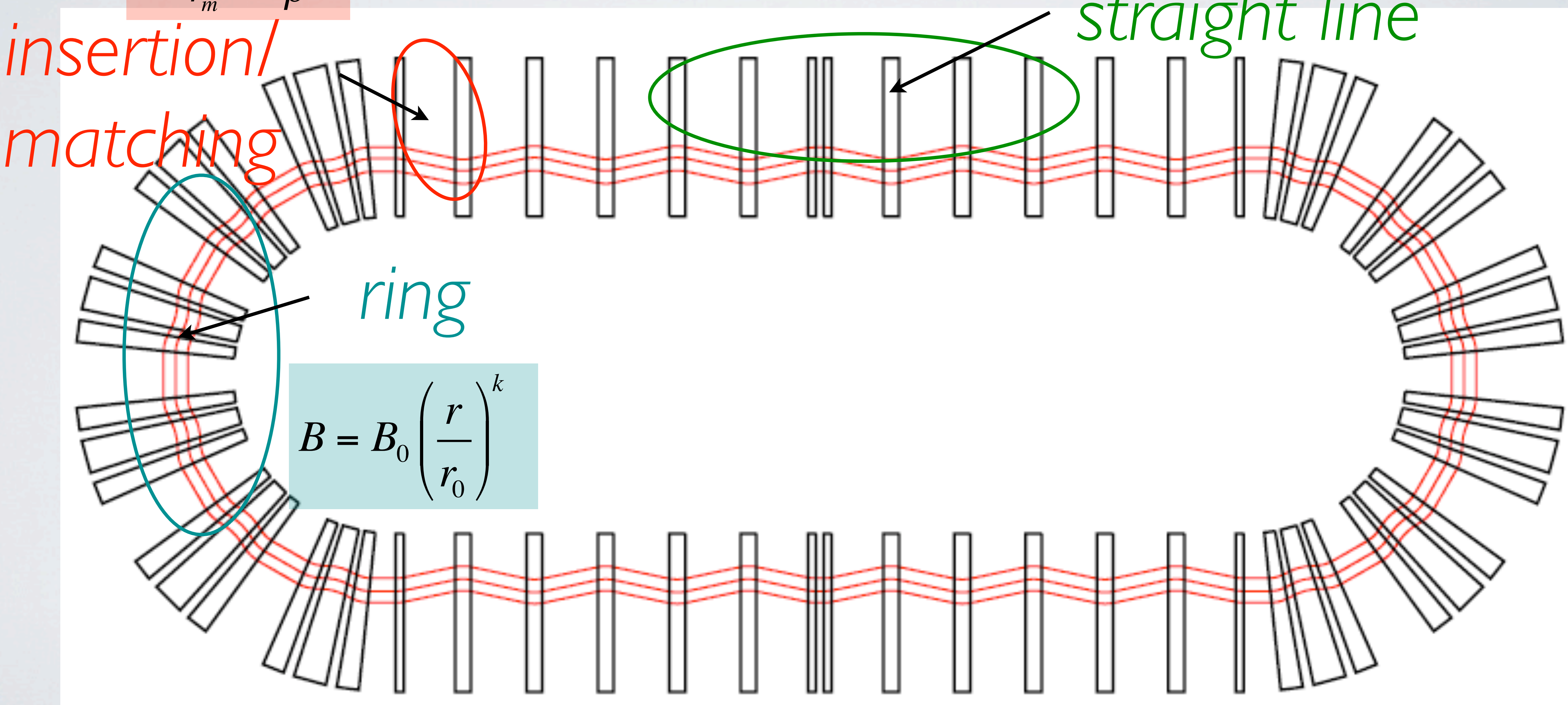


$$B = B_0 \left(\frac{r}{r_0}\right)^k$$

RACE TRACK FFAG RING

$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$

$$\frac{k+1}{r_m} = \frac{n}{\rho}$$



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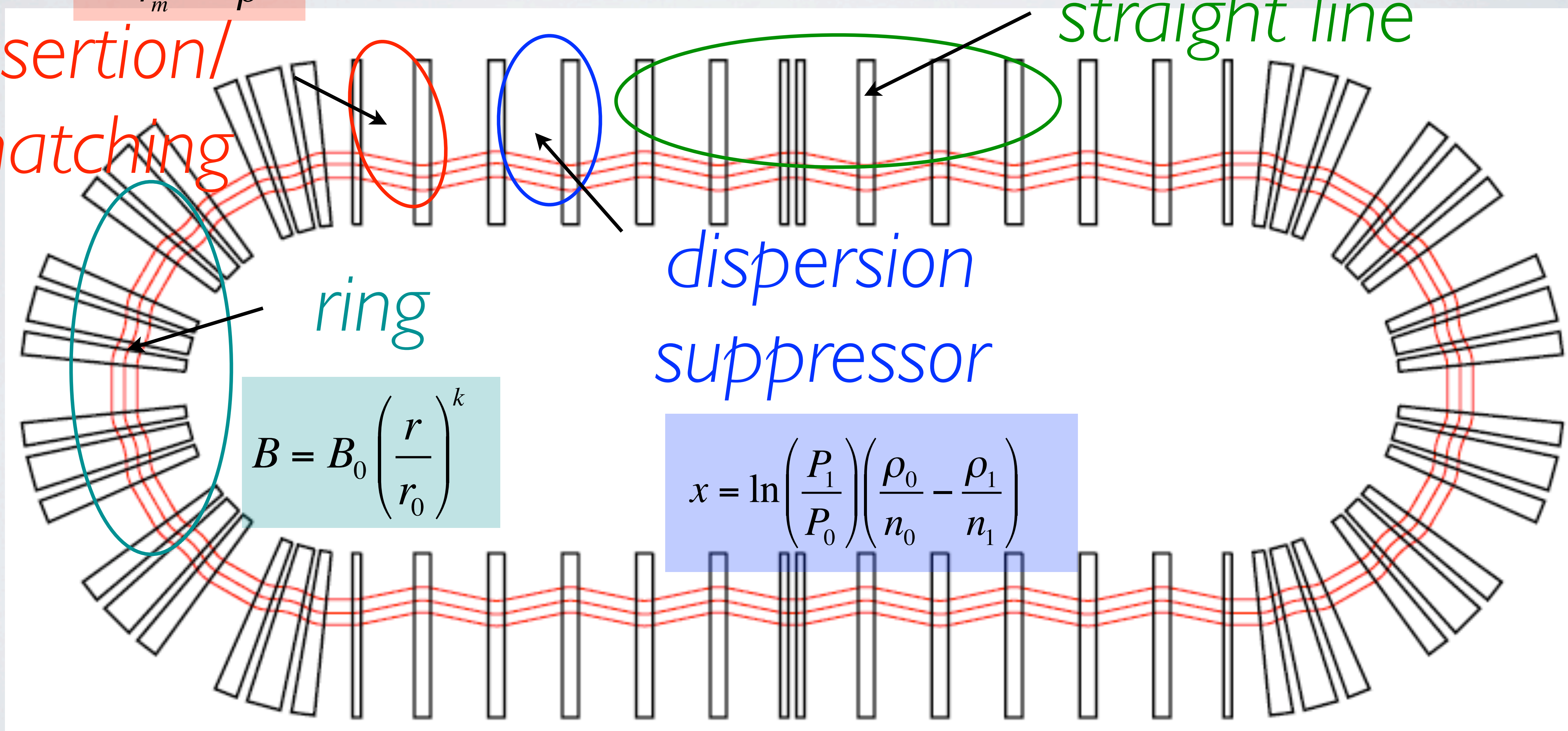


$$B = B_0 \left(\frac{r}{r_0}\right)^k$$

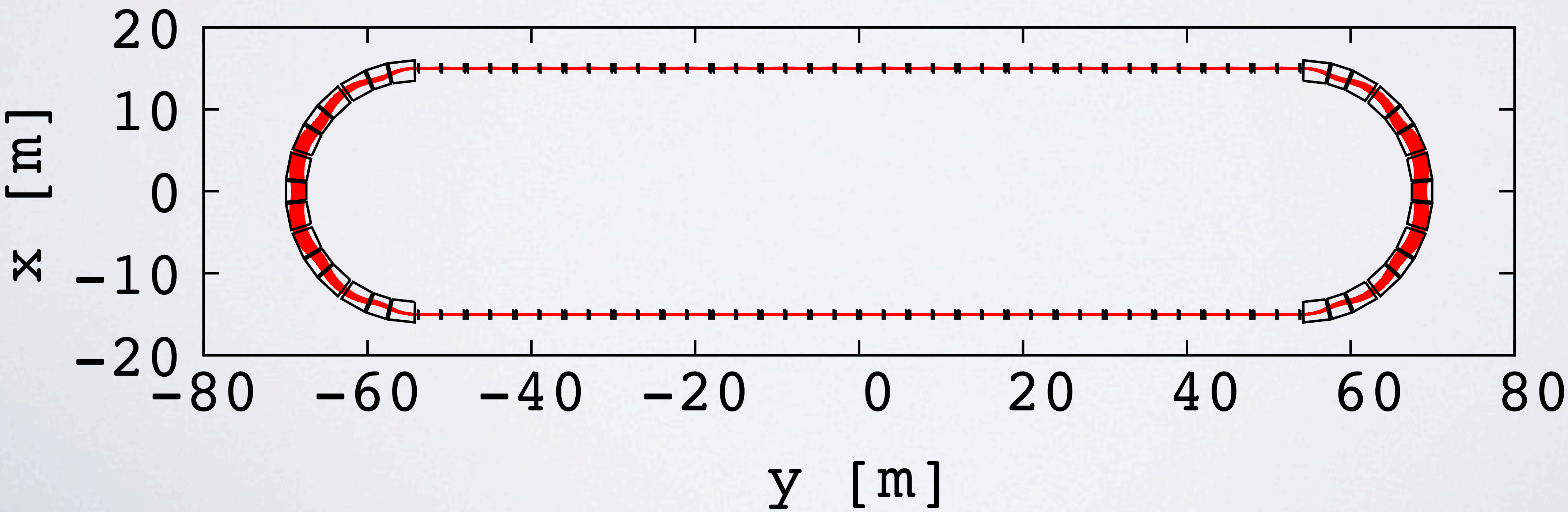
$$x = \ln\left(\frac{P_1}{P_0}\right)\left(\frac{\rho_0}{n_0} - \frac{\rho_1}{n_1}\right)$$

PRELIMINARY DESIGN OF MUON DECAY FFAG RING FOR LENF

- Basic parameters aimed
  - energy 3GeV (1GeV)
  - SS/circumference 70%
  - bending angle in SS <10 mrad
  - max. magnetic field <1.8T
  - Acceptance
  - Transverse $\varepsilon_{h,v}{}^{*}$> 15,000mm.mrad(normalized: a half of that for NF)
  - Longitudinal dp/p > 20%
- Design strategy
  - Optics matching between arc and straight
    - Dispersion matching: Closed orbits between arc and straight has to be matched for small dispersion in the straight section.
    - Optical transparency: Arc should be transparent to straight.
- Total phase advance/half arc should be mx2π. (m:integer)

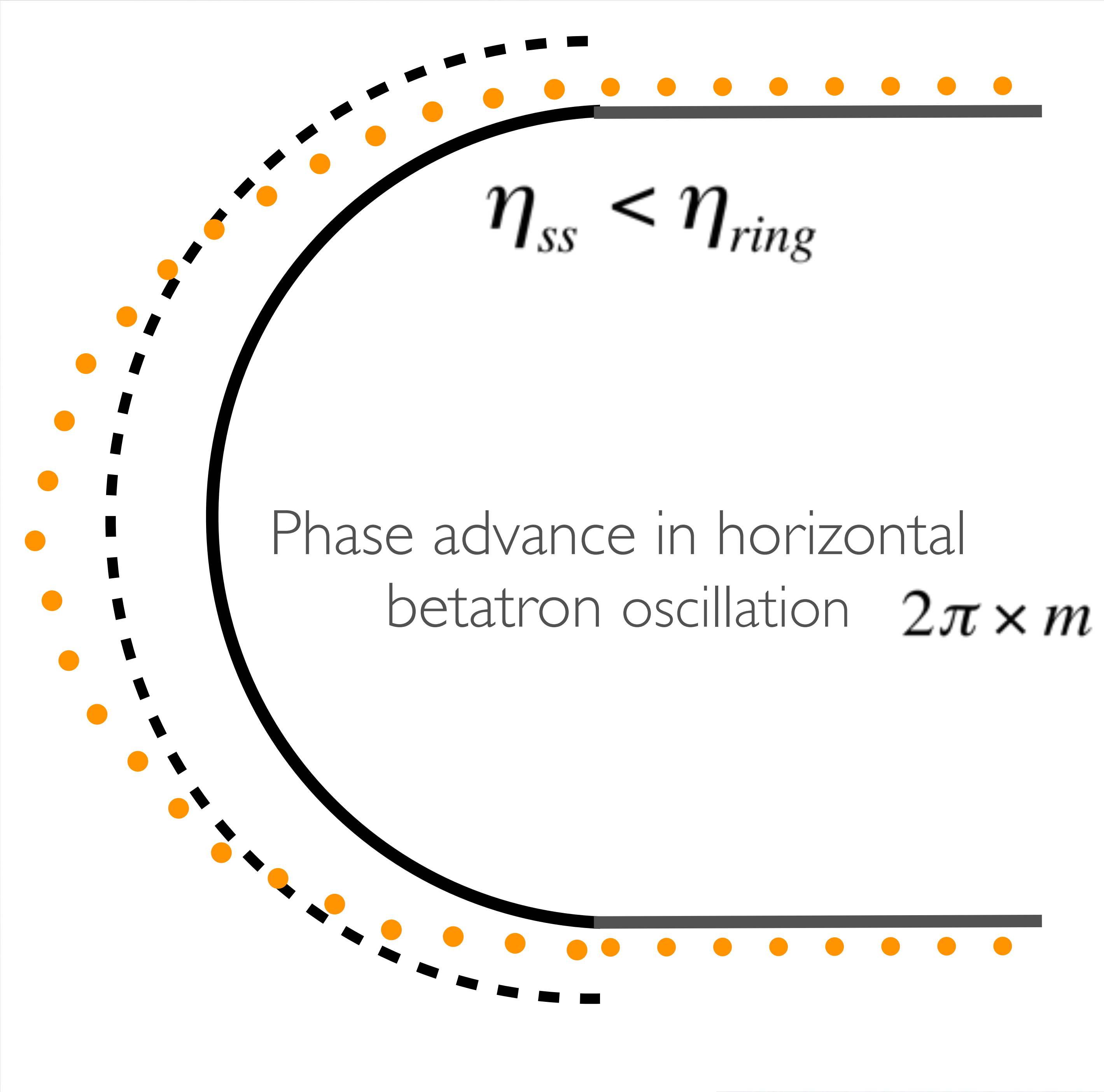
3GEV FFAG MDR

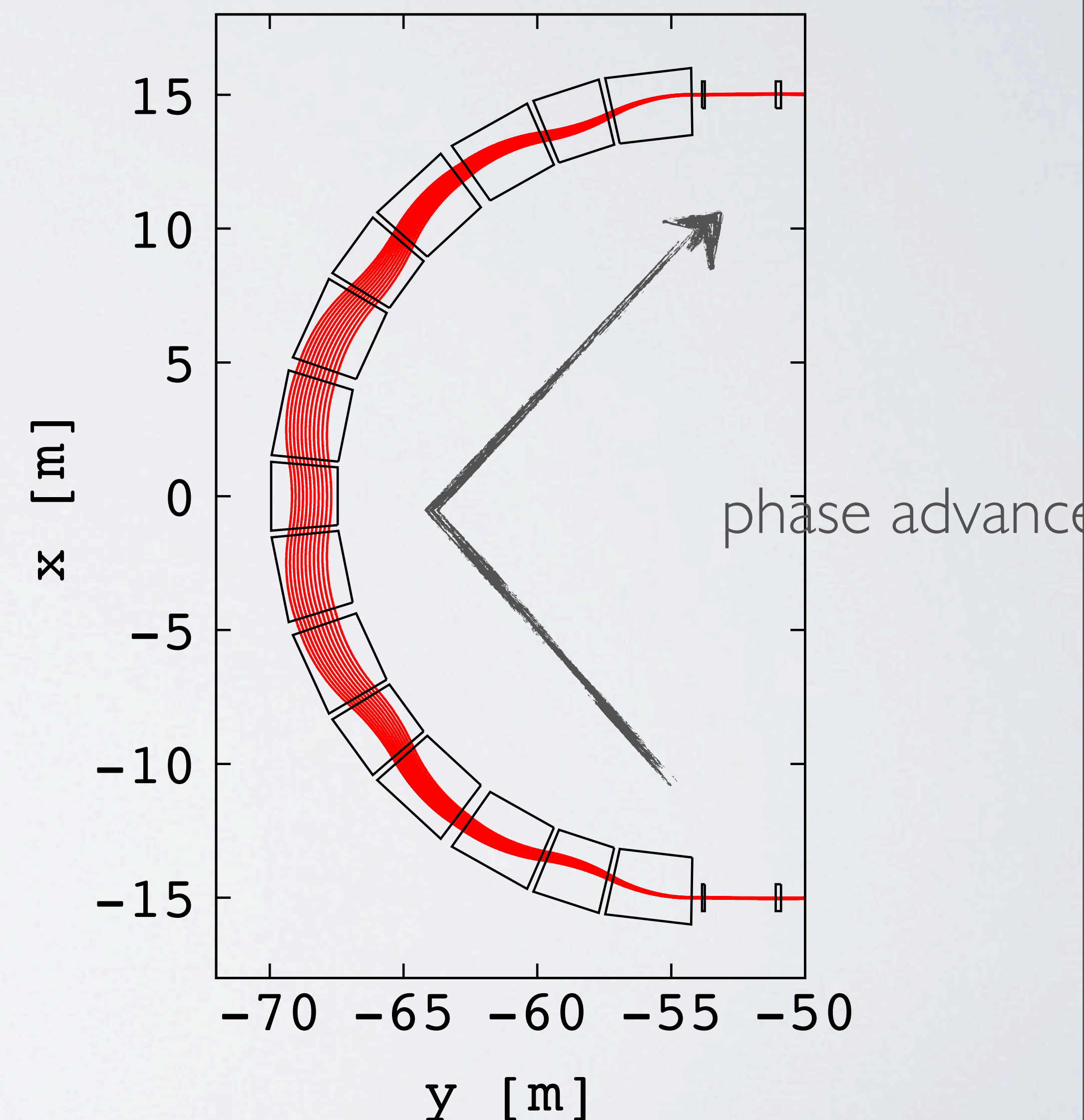
	Circular Section	Straight Section
Type	FDF triplet	FD doublet
k-value or m-value	2.56	3.9m−1
Opening angle/Radius or Cell length	36 deg./15m	6m
Packing factor	0.9	0.07
Average periodic cell dispersion	4.21 m	0.26 m
Horizontal phase advance	72 deg.	13 deg.
Vertical phase advance	36 deg.	15 deg.
Number of cells in the ring	10	36
Total circumference	310 m	
Ring Tune	(3.30,2.52)	
Useful part	35%	



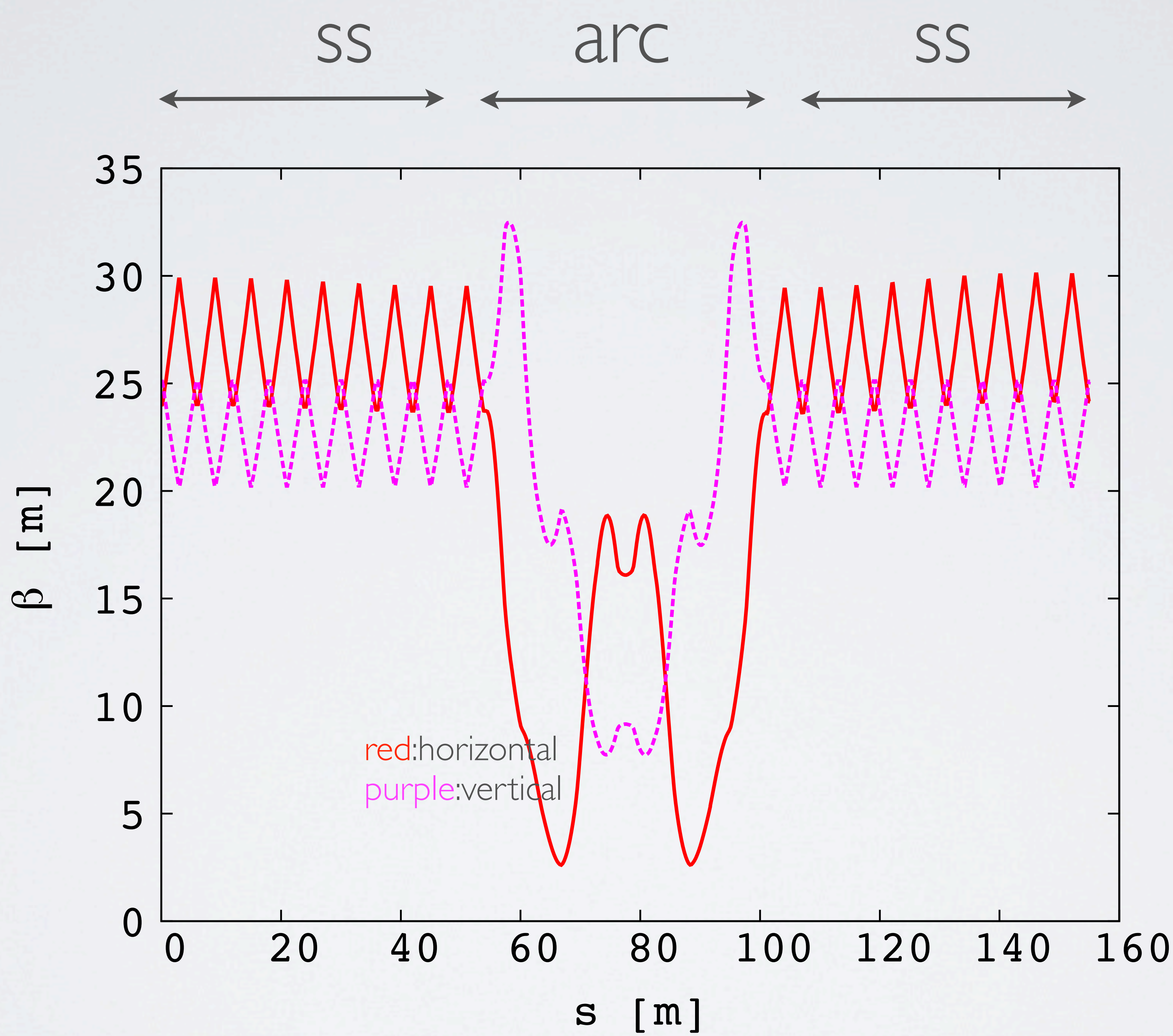
DISPERSION MATCHING

- Phase advance in horizontal betatron oscillation/btw. straight and ring →2πxm





BEAM PARAMETERS

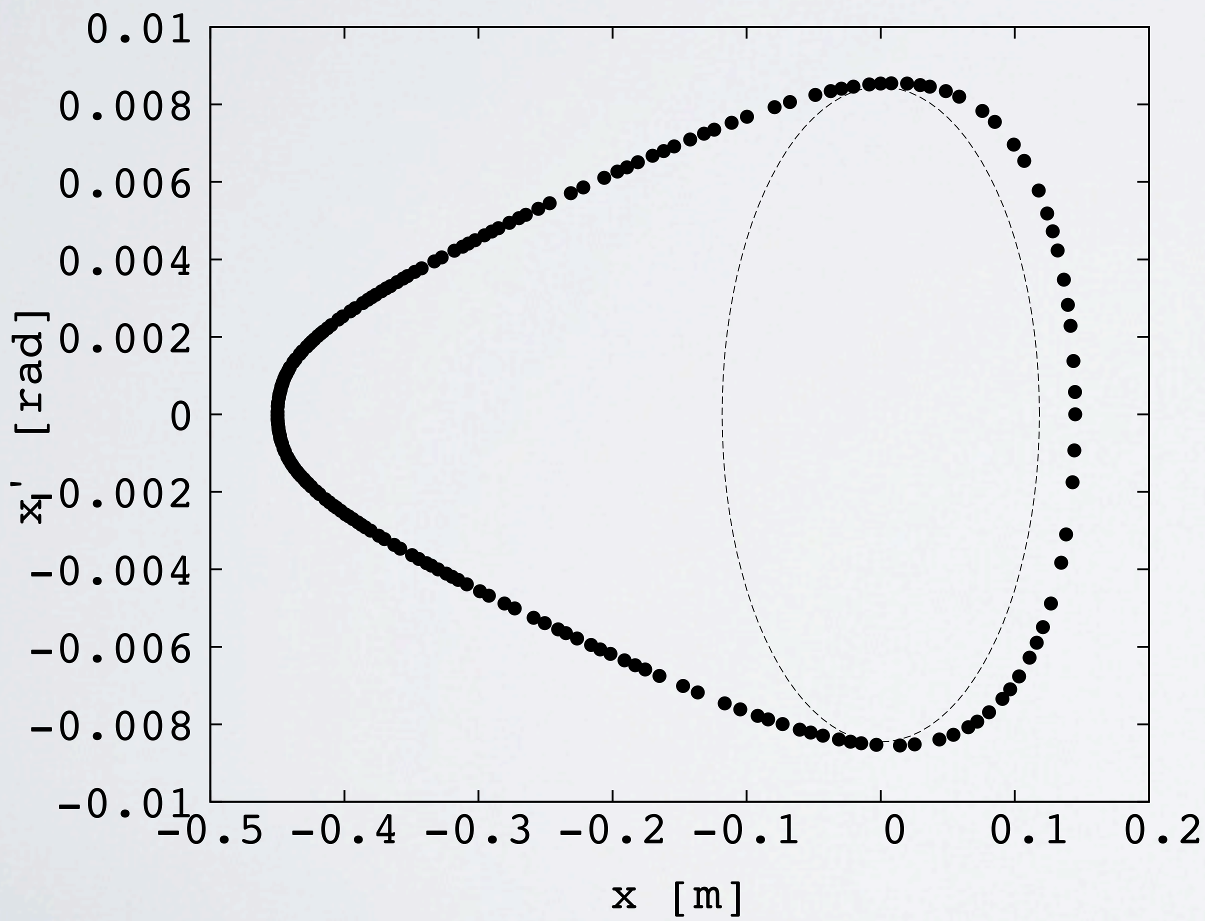


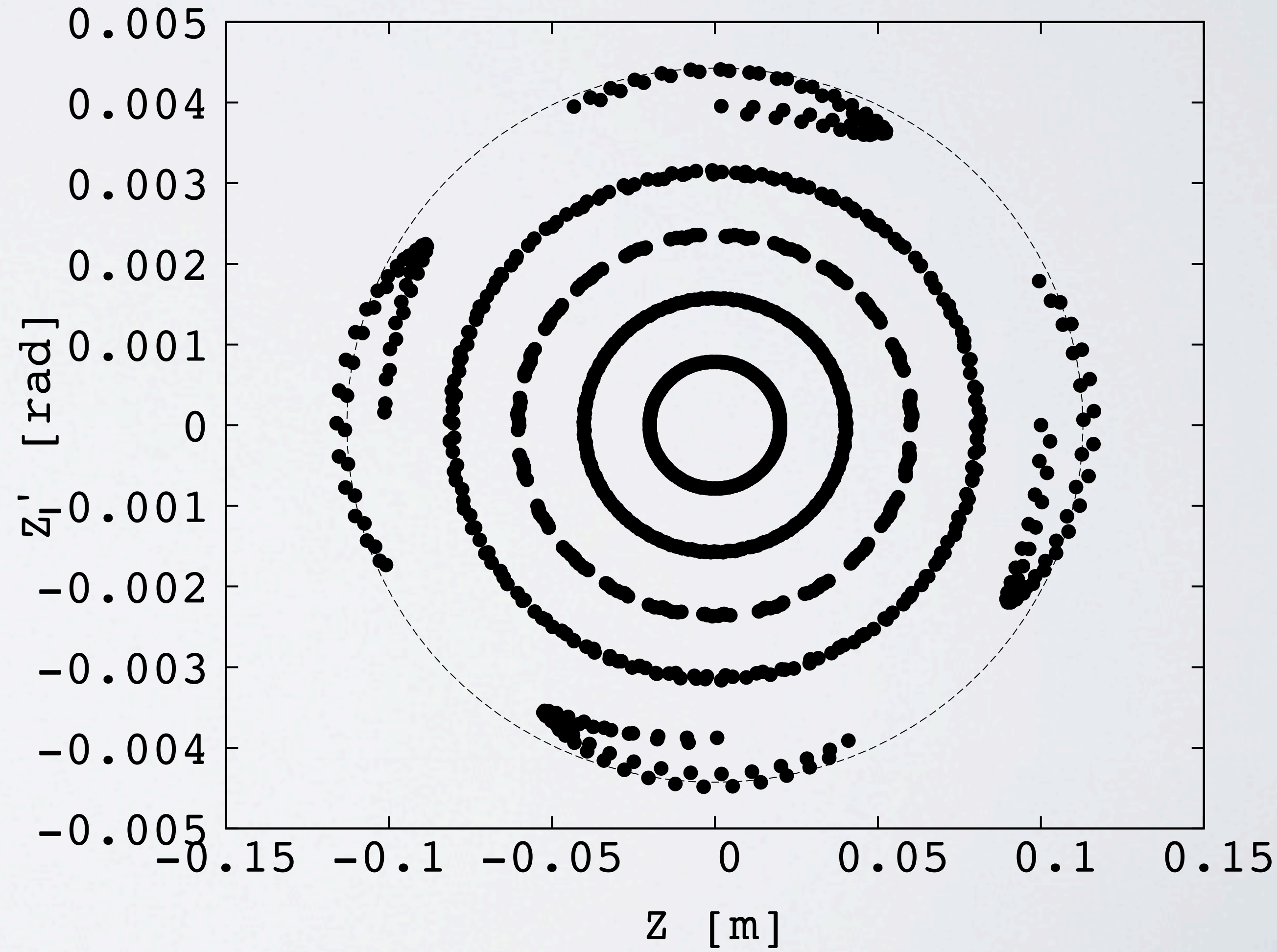
betafunctions

ACCEPTANCE

- Estimated by single-particle tracking

$\varepsilon_h^* \sim 30{,}000$mm.mrad(normalized) ε_v^* $10{,}000 \sim 15{,}000$mm.mrad(normalized)

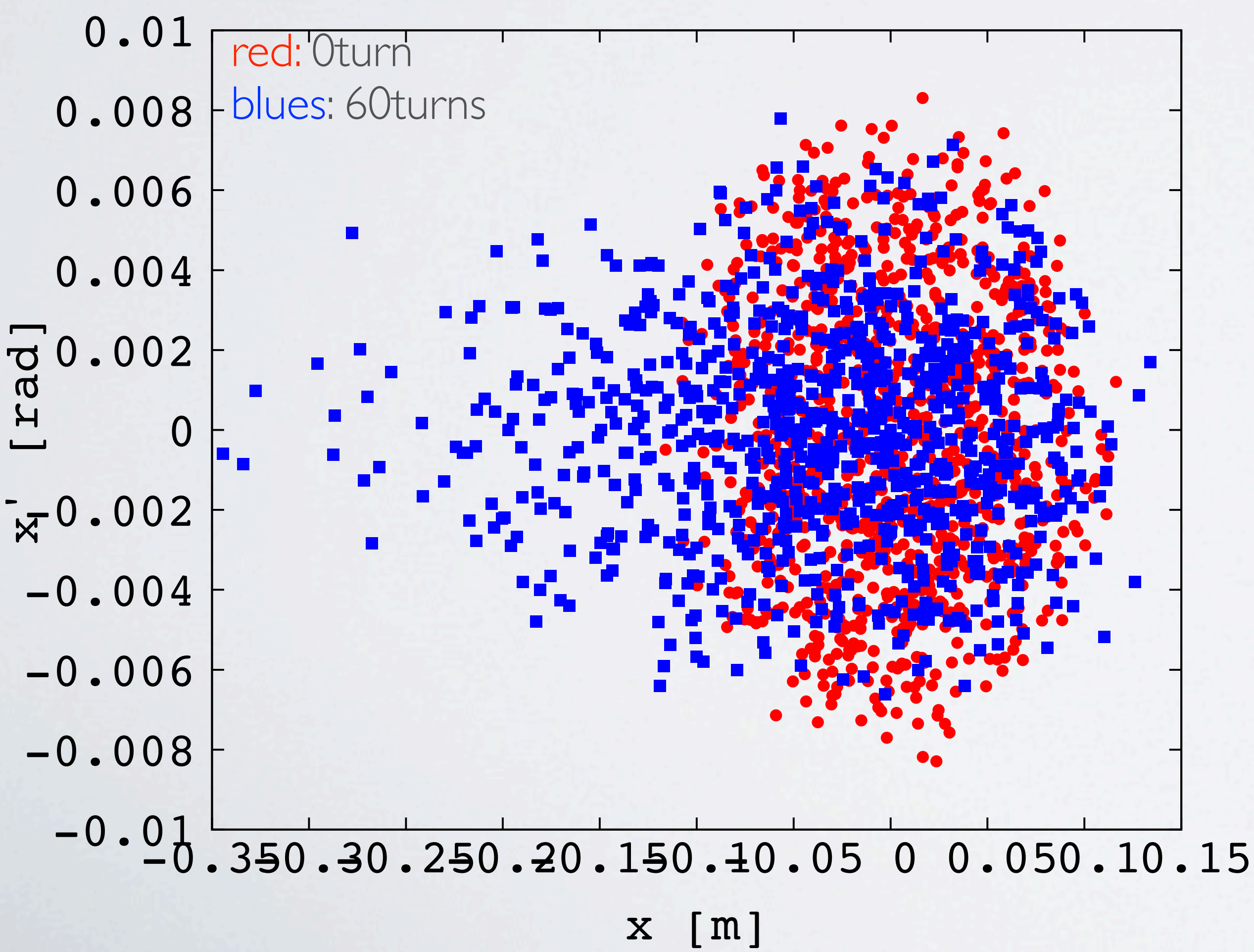


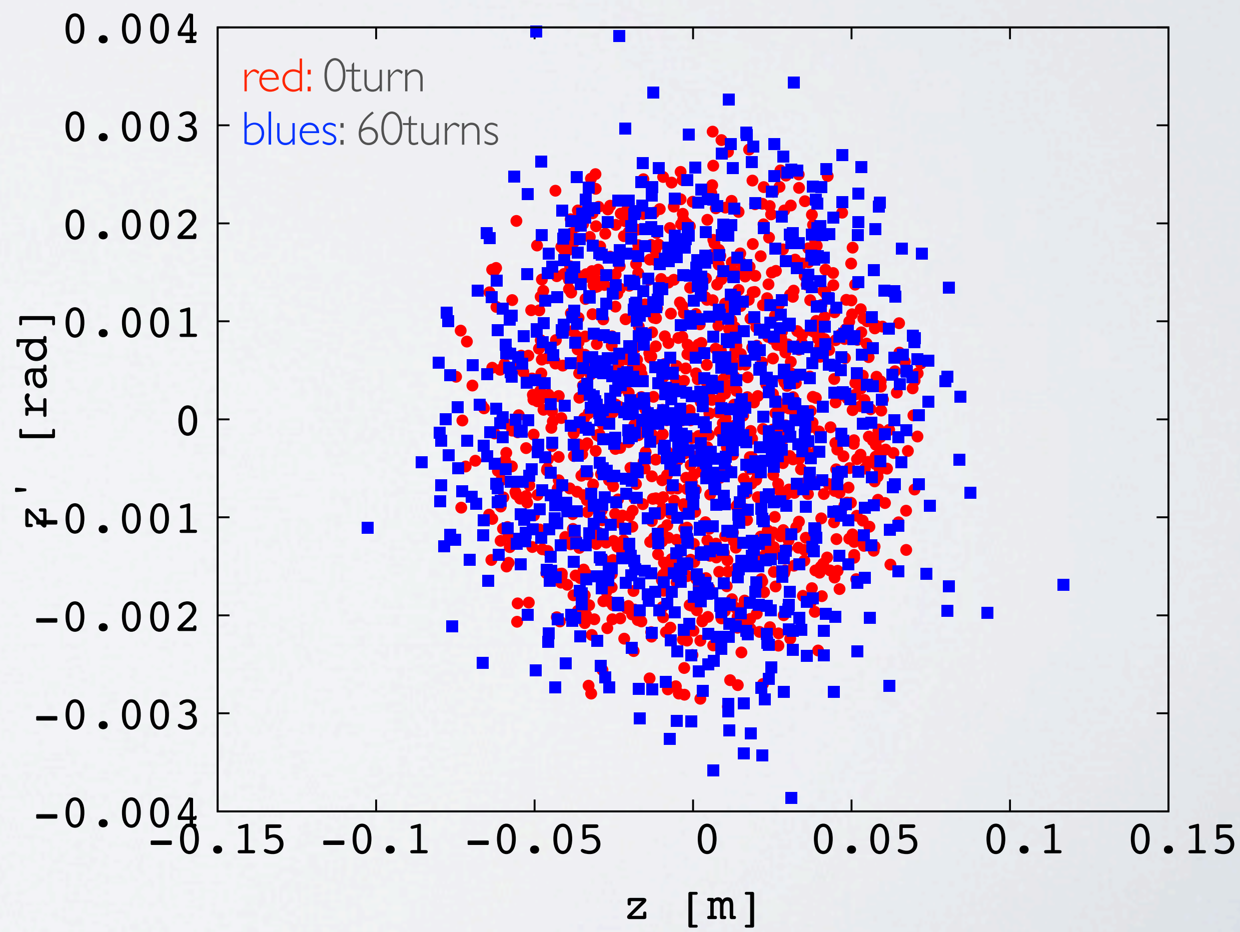


cf. $\varepsilon_{h,v}^* = 30{,}000$mm.mrad(normalized) for NF

6-D TRACKING

- Emittance after 60 turns
  - horizontal : ~30,000mm.mrad(normalized)
  - Magnetic field : body: rk(arc),exp(mx)(ss), edge: Enge(linear)
  - vertical : ~10,000mm.mrad(normalized) some emittance growth probably caused by improper initial tune/energy optimization
  - momentum : Δp/p=20%, homogeneous→ no dispersion matched injection



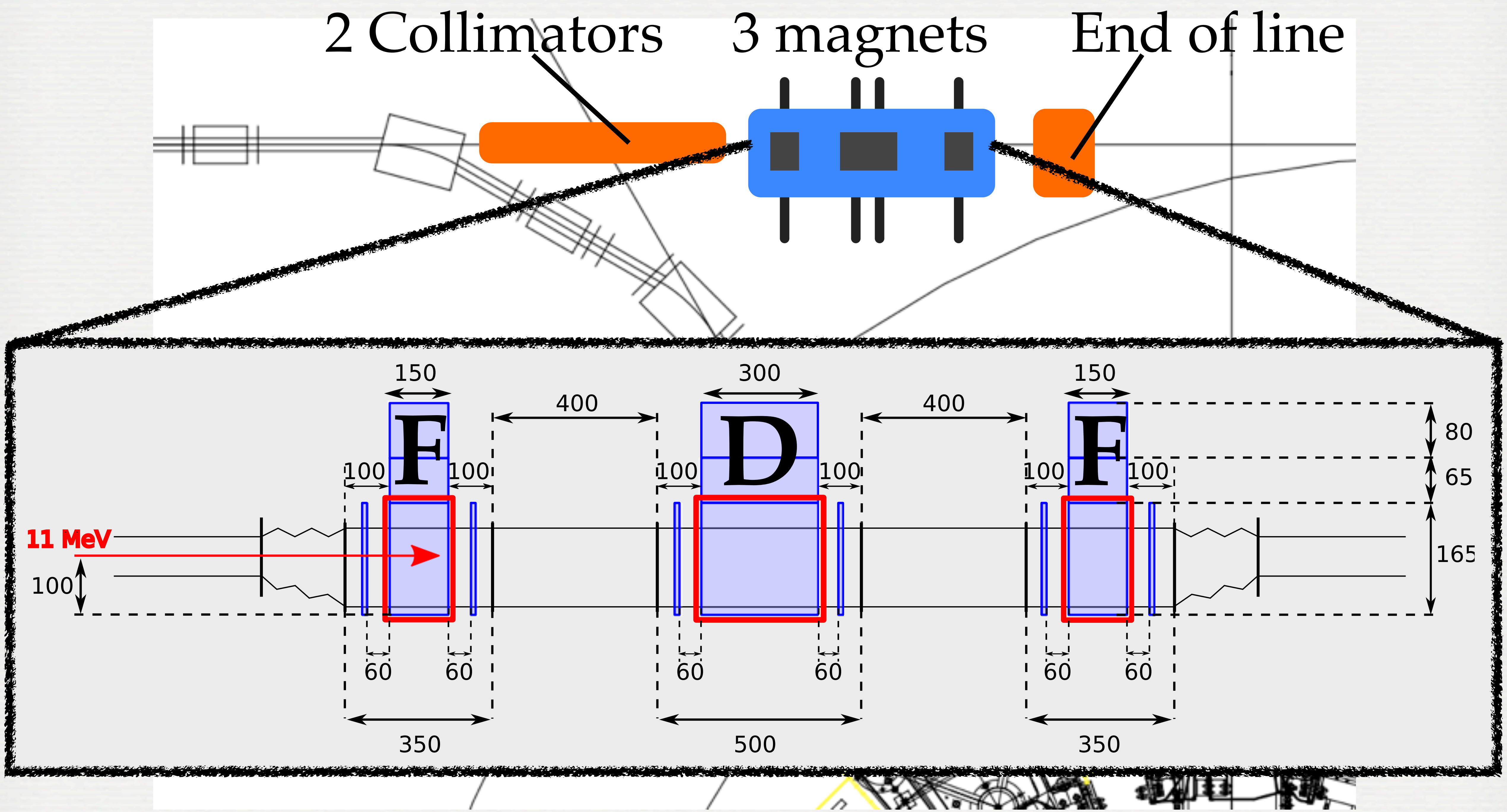


EXPERIMENT OF SCALING FFAG STRAIGHT LINE

JB Lagrange
FFAG'11

- Clarify the FFAG straight line experimentally with π-section
  - Dispersion suppressor
  - Insertion matching
- Momentum range
  - 0.0811 - 0.1441 GeV/c
  - H- ion beam

Layout of the experiment

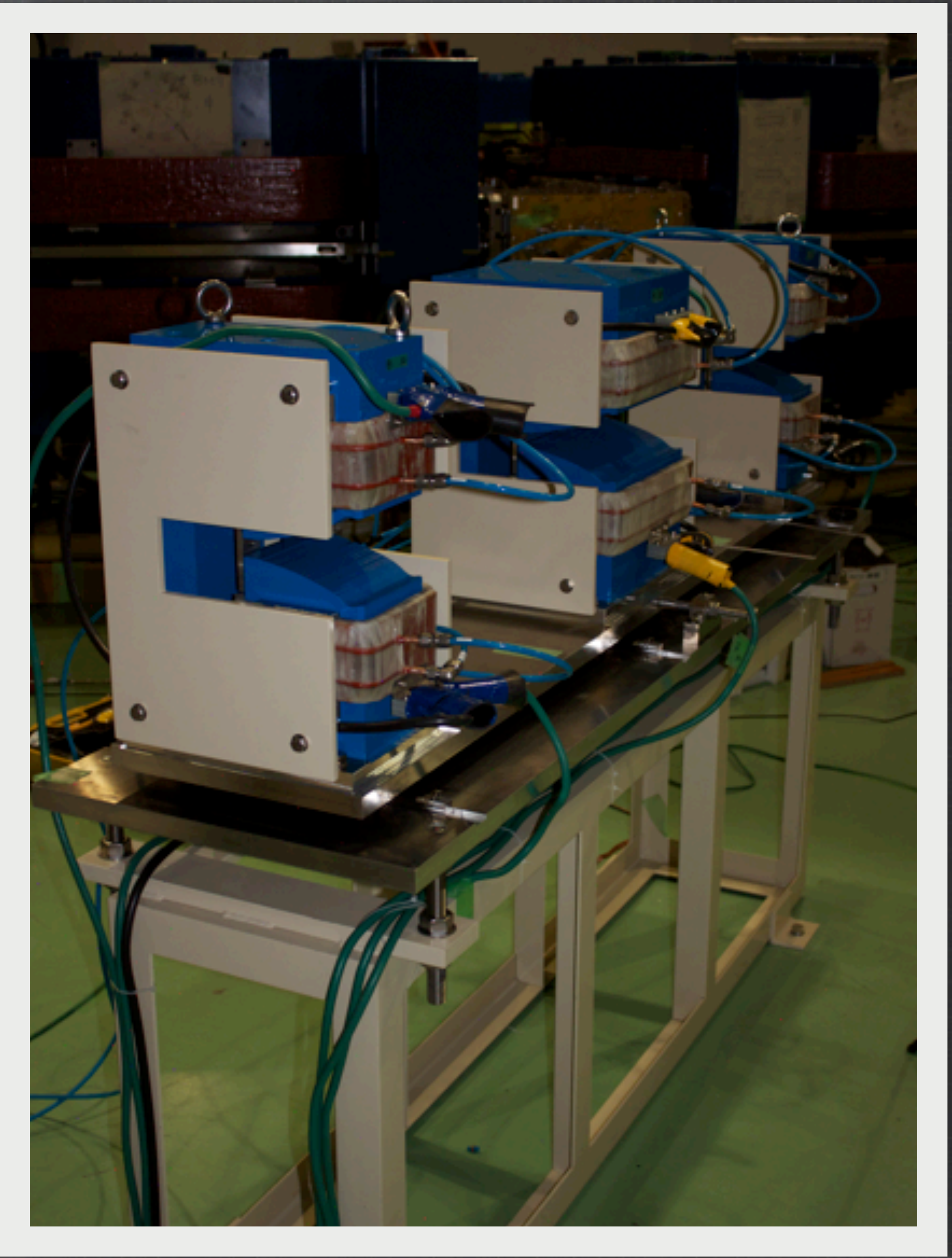


2011年10月18日火曜日

FIELD MEASUREMENT

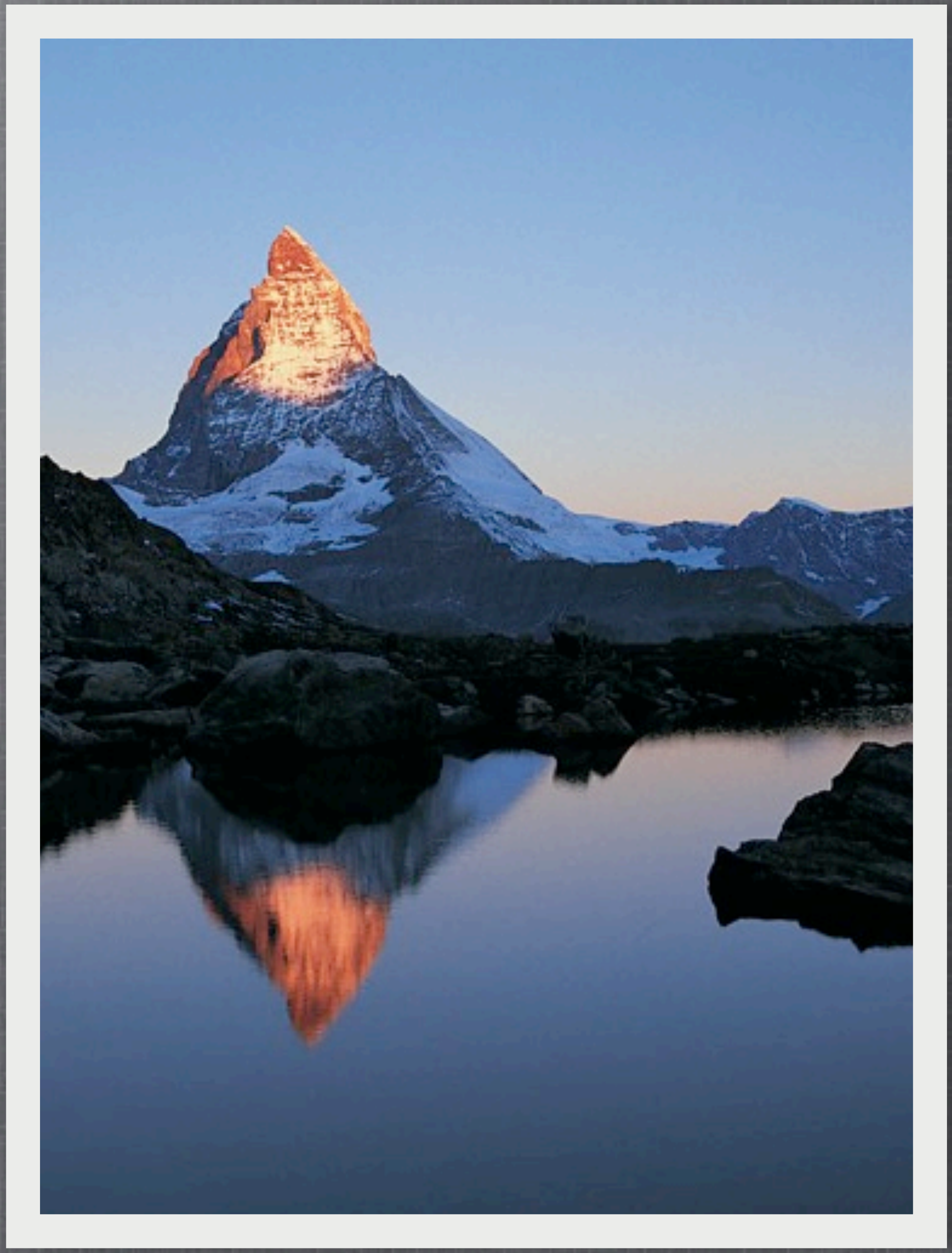
- Magnets delivered early August 2011
- Measurement of the field in the good field region done last week

→ Measurement field map created and tracking in this map



Picture of the Straight Cell prototype.

THANK YOU FOR YOUR ATTENTION



SUMMARY

- Muon decay ring with scaling(zero-chromatic) FFAG lattice has been studied.
  - Zero chromaticity is essential to have large acceptance especially for largely momentum spread beam.
  - Race track ring can be realized with zero-chromatic FFAG straight whose field configuration is, $B \sim \exp\left(\frac{n}{\rho}x\right)$
- A preliminary design for 3GeV MDR has been done.
  - Specifications
    - total circumference ~ 310m
    - energy 3GeV (1GeV)
    - SS/circumference 70%
    - bending angle in SS <10 mrad
    - maximum B field ~ 1.6T
    - Emittance/Accepatnce $A_h^* \sim 30{,}000$mm.mrad, $A_v^* \sim 10{,}000$mm.mrad, $\Delta p/p=20\%$
      - Enough horizontal acceptance but need optimization for vertical one.
      - Need to reduce dispersion in arc. → Increase the number of cells in the arc.
  - Better performance can be expected.
- Need further study on optics optimization, inj/ext, etc.
- Experiment of FFAG straight line is in progress.