Nuclear and Particle Physics - Lecture 6
The strong force and QCD

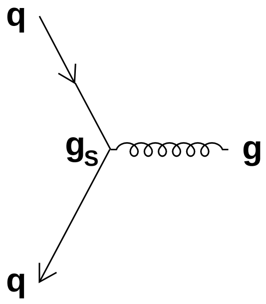
1 Introduction

We will now start to consider the first new force we will look at, namely the strong force. This has some similarities to and some differences from QED. The SM theory of the strong force is called Quantum ChromoDynamics (QCD). It is called "chromo" (from the Greek for colour) because the strong charge equivalent is called colour and is labelled by red, blue and green, although these have *nothing* to do with colours of light; they are simply convenient labels.

2 Gluons

The bosons of the strong field, equivalent to the photons of QED, are called gluons and there are eight distinct gluon particles, compared to only one photon in QED. All eight are massless, like the photon and so (in principle) have an infinite range, although in practise things are more complicated, as we will see below.

Of all the twelve fundamental fermions, the gluons only interact with the six quarks and not the six leptons. Another way to say the same thing is that the quarks have strong (i.e. colour) charges whereas the leptons are colour uncharged. I already said that in some ways, QCD is similar to QED and one similarity is in the allowed Feynman diagram vertices. QCD says there is only one type of quark-gluon interaction vertex allowed and it looks just like QED



which is a quark radiating or absorbing a gluon. The quarks in this diagram can be time-reversed, just as we did for the electrons in QED (as this is a fermion property coming from the Dirac equation and is nothing to do with any particular force) to get $q\overline{q}$ annihilation and production from the same vertex term. Note, the vertex strength, i.e. the charge equivalent, is labelled by g_S rather than e; this is the standard notation for the strong charge value of a quark. In addition, in the same way as the fine structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$$

is dimensionless, we can form a strong force equivalent

$$\alpha_S = \frac{g_S^2}{4\pi\hbar c} \sim 0.1 \gg \alpha$$

where the exact value is not well defined, but is generally much bigger that α; this is why the strong force is called "strong".

3 Colours

So far, everything looks very similar to QED. However, QCD is in fact very different as a simple look around shows; we don't have gluon lights and strong force power stations. This is because the nature of the QCD charge is very different to QED. In QED, charge can be positive or negative and these add in a simple and obvious way to give a total charge which is always conserved. However, in QCD, there are three types of charge, not one, and these are labelled by red, blue and green as mentioned above. Each type of charge is separately conserved, but adding these is much more complicated. Each quark actually comes with one g_S unit of one of the three types of charge and each quark exists with all three types, e.g. u_r, u_b and u_g, so in fact there are three u quarks, three d quarks, etc. All three of each flavour of quark have exactly the same mass. Hence, there are really not 6 fundamental quarks, but 18. The resulting three antiquarks come with the opposite types of colour (the equivalent of negative), e.g. $\overline{u}_{\overline{r}}$, $\overline{u}_{\overline{b}}$ and $\overline{u}_{\overline{g}}$.

This would still not make QCD so different from QED until we consider the gluons. These also carry colour charge; in fact each gluon carries one g_S unit of charge of one colour and one g_S unit of charge of an anticolour. For example, one of the eight gluons is $g_{r\overline{b}}$ and its antiparticle is $g_{b\overline{r}}$, which is another of the eight. Different combinations of colour and anticolour give the eight gluons mentioned previously. However, all combinations of r, b and g with $\overline{r}$, $\overline{b}$ and $\overline{g}$ would imply nine. Six of these are the combinations like $r\overline{b}$ above and are straightforward. However, there are three combinations, $r\overline{r}$, $b\overline{b}$ and $g\overline{g}$ which we might think have no total colour charge. However, adding colour charge is not quite so simple as this. Let's use an analogy; with two spin 1/2 particles, each with $s_z = \pm 1/2$, then there are four combinations, namely

$$|+1/2,+1/2\rangle \equiv\uparrow\uparrow, \quad |+1/2,-1/2\rangle \equiv\uparrow\downarrow, \quad |-1/2,+1/2\rangle \equiv\downarrow\uparrow, \quad |-1/2,-1/2\rangle \equiv\downarrow\downarrow$$

However, they do not all correspond to definite total spin values, i.e. total spin eigenstates, which can be $S = 0$ or $S = 1$. For these we have to take particular combinations of the above, specifically

$$\begin{aligned}
|S=1,S_z=+1\rangle &= |+1/2,+1/2\rangle &&= \uparrow\uparrow \\
|S=1,S_z=\ \ 0\rangle &= \tfrac{1}{\sqrt{2}}(|+1/2,-1/2\rangle + |-1/2,+1/2\rangle) &&= \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\
|S=1,S_z=-1\rangle &= |-1/2,-1/2\rangle &&= \downarrow\downarrow \\
|S=0,S_z=\ \ 0\rangle &= \tfrac{1}{\sqrt{2}}(|+1/2,-1/2\rangle - |-1/2,+1/2|\rangle) &&= \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)
\end{aligned}$$

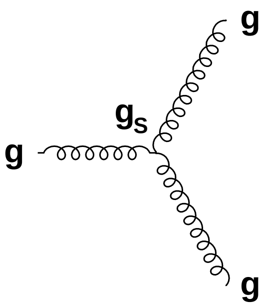
Hence, although the states $|+1/2,-1/2\rangle$ and $|-1/2,+1/2\rangle$ have $S_z = 0$, they still have some amount of total $S = 1$ as well as $S = 0$. By analogy, the states $r\overline{r}$, $b\overline{b}$ and $g\overline{g}$ have zero individual colour charges but we can still have non-zero total colour if they are combined correctly. Explicitly, the combinations needed are

$$C=1: \quad \frac{1}{\sqrt{6}}(r\overline{r} + b\overline{b} - 2g\overline{g}), \qquad C=1: \quad \frac{1}{\sqrt{2}}(r\overline{r} - b\overline{b}), \qquad C=0: \quad \frac{1}{\sqrt{3}}(r\overline{r} + b\overline{b} + g\overline{g})$$

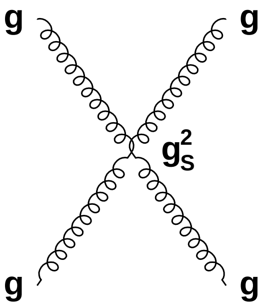
Hence, of these three, there is one total colour zero combination $C = 0$, the so-called "colourless" state, and two (plus the previous six) states with total colour of one unit, called "coloured" states. For some reason, the Universe is built so this one colourless gluon does not interact, i.e. the strong force requires $C = 1$ not $C = 0$. Hence, there are only eight interacting gluons, not nine. We will meet this colourless combination again in the next lecture.

4 Self interactions

We now have the situation where we have said gluons carry colour charge, which would be equivalent to photons being electrically charged. Gluons can therefore act like quarks and radiate other gluons; this means there are more allowed Feynman diagrams.



and there is even another vertex allowed with four gluons which has strength g_S^2



This means the gluon force fields have very different properties to electromagnetic force fields. One big difference is that the gluon force does not fall off with distance; it appears to stay roughly constant for large distances.

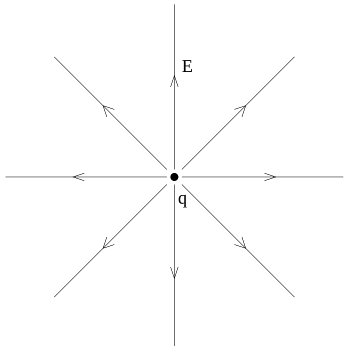
$$F_{QED} \propto \frac{1}{r^2}, \qquad F_{QCD} \approx \text{constant} \approx 14\,\text{tonnes}$$

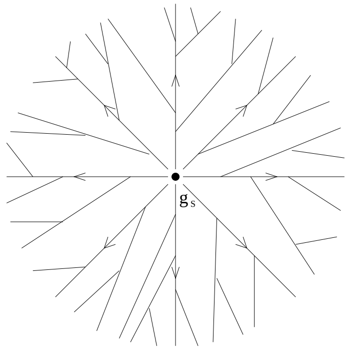
or equivalently in terms of potentials

$$V_{QED} \propto \frac{1}{r}, \qquad V_{QCD} \propto r$$

This can roughly be thought of in terms of field lines; a single EM charge has field lines spreading out from the charge. The number of field lines doesn't change so the density (and hence field strength they represent) falls off as $1/r^2$. In QCD, the radiated gluons can (and do) radiate off more gluons, so the field lines can increase; their number is not fixed. It turns out the field line density stays approximately constant, and hence so does the force.

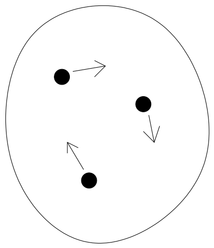
However, this means the energy density is approximately constant everywhere so the total energy integrated over all space is infinite; i.e. it takes an infinite energy to get a single colour charge by itself. This clearly cannot happen as we never have infinite energy available. Hence, we never see single quarks or gluons but only the bounds states called hadrons, i.e. the mesons and baryons, mentioned before. This property of the QCD field is called "confinement"; the quarks are confined within hadrons. This is why giving an exact value of α_S is not possible; we never

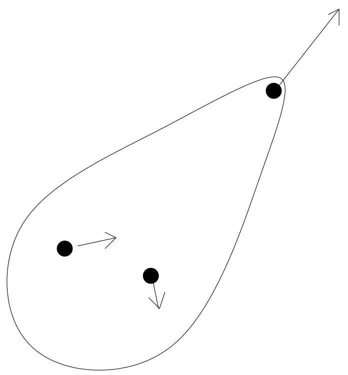




have a state with a non-zero strong charge on it. Consider how difficult it would be to measure e if only neutral atoms were available.

The other consequence is less easy to understand intuitively, but the QCD field has totally the opposite behaviour to QED at short distances also, namely the force goes to zero. This means that if a quark is hit, e.g. by a photon from outside the hadron, for a short time such that it does not move very far, then it acts as if it is not in a bound state at all. This is called "asymptotic freedom"; asymptotic as it only holds exactly for infinitessimally small times or distances and freedom as the quarks act as if free, not bound. One analogy often used is the "bag" (or "balloon") model, where the quarks are pictured as being within an infinitely strong balloon. When they are moving around the middle, there is no force on them and they act as if free. However, if given a big kick (e.g. by a photon) then they hit the balloon wall and get bounced back in; they can never escape. E.g for a proton with three quarks





That ends our quick overview of QCD. You are probably thinking this is all very wierd and complicated. You are right; it is. However, the whole structure actually results from a single change between QCD and QED, made within an overall principle called gauge invariance. This is beyond the scope of this course, but is a basic physical principle defining all three of the forces we will consider. The most commonly known manifestation of it is that it says there cannot be an absolute value of EM potential, by definition; we can only be sensitive to differences in the potential. However, this principle applied to QCD gives all the above features; they are not arbitrarily added in ad hoc to make the theory agree with experiment.

In fact, gauge invariance requires one more feature, which is the universality of the colour charge. In QED, the charge on a fundamental particle could in principle take any value (although we in fact see only $\pm e/3$, $\pm 2e/3$ and $\pm e$). However, gauge invariance in QCD says the gluons must have the same colour charges as the quarks, or it doesn't work. This then means all quarks (and gluons) must have the same charge g_S, not $g_S/3$ or any other value. We will see the results of this in the next couple of lectures.