

Nuclear and Particle Physics - Lecture 11

Parity and charge conjugation conservation

1 Introduction

We have seen the interaction vertices for QED and QCD and know that these do not change the types of particles, i.e. the numbers of quarks and leptons. As we will see, the weak interaction also conserves all these quantities. QED and QCD also conserve the fermion flavour, meaning for quarks u , d , etc, and for leptons e , μ , etc. It turns out the weak force does not conserve flavour.

Another important property of the weak interaction is that it does not conserve the parity (P) or charge conjugation (C) quantum numbers. Let's look at what P and C conservation gives us in strong and EM interactions and how we know it is not conserved in some weak processes.

2 Symmetries and conservation laws

There is a very deep connection in physics between symmetries and conserved quantities. For example, we can do an experiment today or tomorrow. Assuming we remove all the external factors which depend on when we do it, we should get the same answer. This is because there is a time translation symmetry; there is no absolute time scale. This symmetry actually leads to conservation of energy. Similarly, symmetry under translation in each of three directions in space leads to conservation of the three components of momentum. Symmetry of direction in space (as space is isotropic) leads to conservation of angular momentum.

Parity and charge conjugation are further examples. Parity is an operation which takes a vector, e.g. the space position $\mathbf{r}$ and reflects it through the origin to make it $-\mathbf{r}$. Charge conjugation takes every particle and replaces it with its antiparticle (and vice versa). It seems obvious from everyday experience that things will work the same after a parity operation; a mirror-image of any machine will function in the same way. It is maybe not so obvious that we would expect systems to be the same after swapping particles and antiparticles; however, if you think about the EM force between any pair of particles, it goes as $Q_1 Q_2$ so swapping the signs of Q_1 and Q_2 in fact keeps all the forces the same and since the masses are the same, this has no effect.

Parity and charge conjugation are somewhat different from the other symmetry cases because they are not continuous symmetries, but are discrete. Take the example of time translation invariance which leads to energy conservation. Because any translation is possible, then any energy is allowed. It turns out that for a discrete transformation symmetry, then there is a discrete quantum number which is conserved. Hence, there is still a connection between a symmetry and a conserved quantity, even for the discrete case.

What are these discrete eigenvalues for parity and charge conjugation? Consider the case for the parity operator, $\hat{P}$, which reflects polar vectors through the origin $\mathbf{r} \rightarrow -\mathbf{r}$. For an eigenstate of parity, where

$$\hat{P}\psi = P\psi$$

it is obvious that applying the parity operation a second time must return everything to its original state, so

$$\hat{P}^2\psi = P\hat{P}\psi = P^2\psi$$

and since this must be just ψ again, then $P^2 = 1$ and the eigenvalues of parity must be ± 1 . The same argument holds for the charge conjugation operator; this changes particles to antiparticles

so a second application must return everything back to the original state. Again, the eigenvalues of $\hat{C}$ must be $C = \pm 1$.

Hence, we can look at parity (or charge conjugation) conservation either in terms of the whether there is a symmetry or in terms of whether the quantum number of P or C is conserved. It is a little unfortunate that the same word, “parity”, is used to mean both the symmetry and the conserved quantity. Compare to time translation (the symmetry) and energy (the conserved quantity), which never get confused. It will hopefully usually be clear from context which is meant. It is often hard to see a situation in terms of both views, so we will use either the symmetry or the conserved quantity, depending on which is easier, for any case. What we find is that QED and QCD conserve P and C , or equivalently respect $\hat{P}$ and $\hat{C}$ symmetries, whereas the weak force does not. We will first consider some cases where we can understand things easily in terms of the conserved quantum numbers and later in terms of symmetries.

3 Conserved quantities; decays of mesons

As was stated in a previous lecture, of the nine ground state meson flavour combinations, only the three flavour neutral states, i.e. those made out of combinations of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$, specifically the π^0 , η and η' , can decay electromagnetically. This interaction does not change flavour and so the others, such as $u\bar{d}$, have no lower mass state to go to. The excited states, in contrast, can all decay strongly as there is always at least one lower mass state of the same net flavour. The mesons all have well-defined parity values which turn out to be -1 , i.e. they are parity eigenstates

$$\hat{P}|\pi^+\rangle = -|\pi^+\rangle$$

Along with being spin 0, this is why they are written as $J^P = 0^-$.

In contrast, only the flavour neutral mesons are $\hat{C}$ eigenstates as these are the only ones which are their own antiparticles. For the others, for example the charged pion

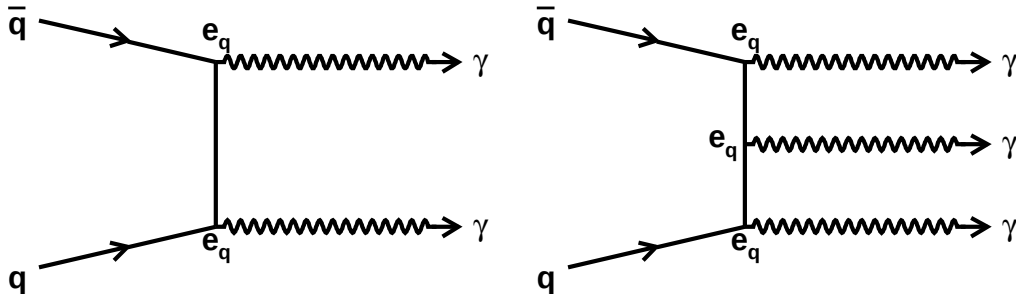
$$\hat{C}|\pi^+\rangle = \hat{C}|u\bar{d}\rangle = |\bar{u}d\rangle$$

and so is clearly not a $\hat{C}$ eigenstate. However, the three flavour neutral ground state mesons are their own antiparticles and for them, it turns out that $C = +1$, i.e.

$$\hat{C}|\pi^0\rangle = +|\pi^0\rangle$$

Hence both P and C are well defined so they are written as $J^{PC} = 0^{-+}$. This information comes from P and C conservation in the flavour neutral meson decays so let's take a look at how this works.

Consider the π^0 first. The π^0 decays almost 99% of the time to two photons; this is obviously an EM decay. Comparing the Feynman diagrams for two and three photon decays



then we might think the three photon decay has an extra vertex and hence the decay rate will be down by a factor of $\alpha \sim 1/137$, which accounts for the other 1% of the decays. This is *not* the case. The photon is its own antiparticle so it is also a $\hat{C}$ eigenstate. Its charge conjugation value is $C_\gamma = -1$, which is clear if you think of an arbitrary system of charges with their resulting $\mathbf{E}$ and $\mathbf{B}$ fields. Under a charge conjugation operation, all the charges become the opposite and so all the field values becomes reversed

$$\mathbf{E} \rightarrow -\mathbf{E}, \quad \mathbf{B} \rightarrow -\mathbf{B}$$

The photon consists of these fields, so it must be that

$$\hat{C}|\gamma\rangle = -|\gamma\rangle$$

This means a system of n photons has an overall C value of $(-1)^n$. Hence, two (or any even number of) photons have $C_{\gamma\gamma} = +1$ while three (or any odd number of) photons have $C_{\gamma\gamma\gamma} = -1$. Thus, the three photon π^0 decay would change C from $+1$ to -1 , which is not allowed for an EM decay as it is a conserved quantity. In fact, the three photon decay is not 1% of the two photon decay, but has never been observed and is known to be less than 10^{-7} of the two photon rate. (As an aside, going back to positronium in lecture 5, C conservation is also the reason why each of the hyperfine split ground states only goes to two or three photons.)

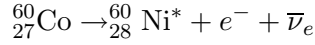
Now consider the η . For identical reasons to the above, it can decay to two photons but not three. However, since $m_\eta > 3m_\pi$, it is also heavy enough to decay to two pions, e.g. $\pi^+\pi^-$, or three pions, e.g. $\pi^+\pi^-\pi^0$. However, the former does not occur. Consider the two pion decay; the parity for any orbital angular momentum state is the same as its exchange symmetry, i.e. $P_L = (-1)^L$, and the intrinsic parity of the pions is $P_\pi = -1$ (i.e. $J^P = 0^-$), so the total parity of a two pion state is $P = P_\pi P_\pi P_L = (-1)^L$. The η and the pions have no spin, so L must be zero for this decay. Hence for this decay, the two pion final state parity would be $P_\pi P_\pi P_L = +1$, while the η itself has intrinsic parity, like the pions, of $P_\eta = -1$. Obviously, adding a third pion but keeping $L = 0$ gives a total P of $P_\pi P_\pi P_\pi P_L = -1$ which is the same as the η . Hence, the two pion decay would violate parity conservation, whereas the three pion decay would not. The two pion decay has never been observed; the experimental limit on the rate for two pion decays is at most 10^{-3} of the three pion rate.

In fact, this argument is valid for any of the ground state, $J^P = 0^-$, meson decays as they all have well-defined parity; none of them can decay to two pions (or indeed any two of the ground state mesons) without violating parity. In fact, the first decay observed which violated parity was the decay $K^+ \rightarrow \pi^+\pi^0$, which has a rate roughly three times that of the three pion decay. This cannot be a strong or EM decay as the strange quark disappears; it is indeed a weak decay. At the time, the significance of this was not understood and it was thought there were two kaon-like particles of the same mass but with different intrinsic parities, one of which decayed to two pions and one to three pions. It was only when the weak force was seen not to respect the $\hat{P}$ symmetry that people realised there was only one charged kaon. We will look at this symmetry violation next.

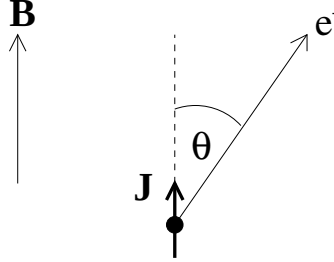
4 Symmetries; Co decay

The above are examples of when we have considered the P and C quantum numbers and asked if they are conserved in a decay or not. Now let's look at a case where we can ask whether the parity symmetry is valid or not. As was said above, it seems obvious to us from everyday experience that parity is a valid symmetry. However, our everyday experience is really based only on the EM force. It turns out the weak force does not obey this symmetry.

The classical experiment which actually established parity was not a good symmetry was using the beta decay of ${}^{60}_{27}\text{Co}$



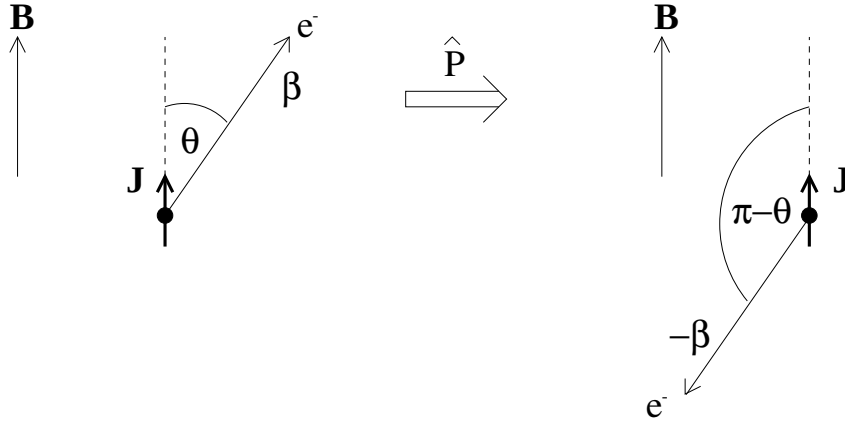
This is a weak interaction as it involves a the neutrino, which has no EM or strong charge. The cobalt nuclei have spin $J = 5$. They were cooled to temperatures around 0.01K and placed in a magnetic field. At such low temperatures, the magnetic moments of the nuclei cause the nuclear spins to align themselves with the magnetic field direction, so the nuclei are all polarised along the field direction. The experiment then simply consisted of measuring the angular distribution of the emitted electrons.



It was found that the distribution was

$$1 - \beta \cos \theta$$

where $\beta = v/c$ was the electron velocity, meaning more electrons were emitted in a direction opposite the magnetic field than along it. This sounds perfectly reasonable so why is this a big problem? One way to think of parity as a symmetry is that, if the symmetry holds, the rate for any process must be equal to the rate for the parity inverted process. We know that $\mathbf{r} \rightarrow -\mathbf{r}$, $\mathbf{p} \rightarrow -\mathbf{p}$ and $\boldsymbol{\beta} \rightarrow -\boldsymbol{\beta}$ (these are all called “polar” vectors). However, not all vectors act the same way; e.g. $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, so $\mathbf{L} \rightarrow \mathbf{L}$ under parity. This is an example of an “axial” vector. Spin $\mathbf{S}$ and total angular momentum $\mathbf{J}$ are also axial vectors. Hence, under parity, the velocity of the electron is reversed but the spin of the nucleus is not.



Does this make sense with regard to the magnetic field? Yes; since $\mathbf{B} = \nabla \times \mathbf{A}$, then $\mathbf{B}$ is itself an axial vector like $\mathbf{L}$. Hence, the nuclear spin is still aligned along the magnetic field, which had better be true as this alignment is due to the EM force which we believe conserves parity.

In the parity inverted experiment, the electron comes off at $\pi - \theta$ (and also $\phi + \pi$ but that is not important here) so if parity is a symmetry, then the rate for electrons at θ must equal the rate at $\pi - \theta$. However, a distribution of $1 - \beta \cos \theta$ clearly does not have the same rate, since

$$1 - \beta \cos(\pi - \theta) = 1 + \beta \cos \theta \neq 1 - \beta \cos \theta$$

Hence, this simple observation is all that is required to conclude that parity is not a symmetry of the weak interactions.

One thing to note is the velocity dependence. For the electrons emitted with $v \rightarrow 0$, the distribution becomes isotropic, which clearly then is parity symmetric. In contrast, for electrons with $v \rightarrow c$, the maximum parity violation occurs. This speed dependence is very characteristic of the weak interactions, as we shall see in the next lecture.

We actually have already seen an example of a case where parity is conserved, i.e. e^+e^- annihilation to muons, where the muons are produced with a $1 + \cos^2\theta$ distribution; this is clearly symmetric under $\theta \rightarrow \pi - \theta$, so this distribution is consistent with parity conservation, as expected for EM interactions. Indeed, we even saw there is a correction visible in the data which breaks this forward-backward symmetry and this is due to the weak force.

5 CP , T and CPT

We have seen weak interactions violate parity conservation and indeed they also do not conserve C . However, it was initially thought that they did conserve the combined value of CP (or PC). In fact, this is not quite true, although CP violation arises from a very different source. It is usually a very small effect which has only been observed in the decays of two particles, namely the K_L^0 and B^0 mesons. We will discuss this further later in the course.

Another operator we might consider is time reversal, $\hat{T}$, which is a good symmetry if the rate of a reaction and its inverse reaction are equivalent. It corresponds to the time equivalent of a parity operation, i.e. reflect the time coordinate through the time axis origin. It is actually quite hard to measure T conservation (or violation) in weak decays because no particles are $\hat{T}$ eigenstates. Similarly, $\hat{T}$ symmetry is hard to see as almost all reactions have strong or EM interactions in at least one of the two directions, which swamp the effect being looked for. Do we think it is conserved or violated in weak interactions? The answer is that it is also thought not to be a conserved quantity because of a very basic theory called the CPT theory. It has been shown that at a very fundamental level, any quantum field theory must be invariant under the operation of $\hat{C}$, $\hat{P}$ and $\hat{T}$ all together (in any order). This is a very general argument just based on quantum mechanics and relativity and it seems not possible to actually write down any field theory which does not satisfy it. Hence, if CPT is conserved but CP is violated, then there must be a compensating T violation to cancel out the CP violation and preserve CPT . Hence, the weak interaction is also expected to violate T but experimental evidence for this is not (yet) uncontested.